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Polytechnic



First Year Second Semester
(All Branches)



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West Bengal Polytechnic

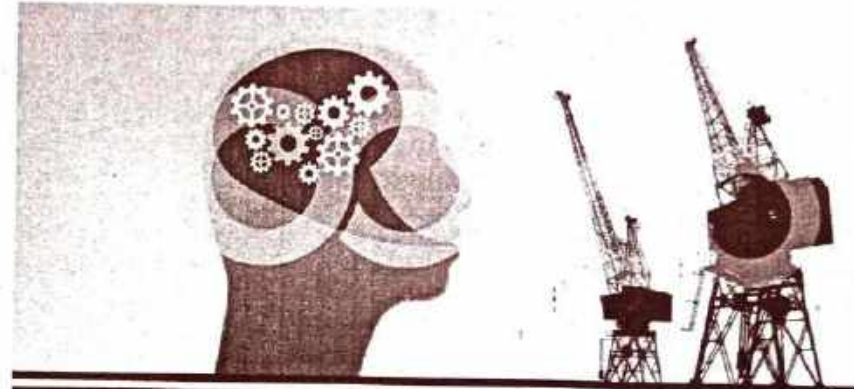
According to New Syllabus

First Year Second Semester
(All Branches)



Yearwise
Chapterwise
Chapter at a glance

Dual Solutions



POLYTECHNIC

First Year Second Semester
(All Branches)

10 Years WBSCTE Question Papers & Solutions

Chapter at a Glance

Chapterwise

&

Yearwise

*Dual
Solutions*

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Foreword.....

Matrix is the most popular name among the engineering students. Since 2003 Matrix is providing real, useful and handy engineering books of all subjects from 1st to 4th years. We are glad to know from various social network sites that we get millions of admirers, they believe that Matrix is the "Best Friend" of them. This affection makes us more sincere and passionate to expand our hand for Polytechnic students. Our research team and expert editorial board have designed a new concept of learning for Polytechnic students. You may find our innovation, idea, dedication and commitment in this Book.

What is the specialty?

This is a single book which contains all subjects for a particular semester. Each subject is organised in chapterwise, according to Polytechnic new syllabus and every chapter contains the Polytechnic questions with solutions, yearwise, in sequential order. It is a real helpful guide for examination purpose. Students will get weightage of the chapters and will be more focused. We believe the quality of answers is trustworthy and reliable.

Features:

- It is easy, lucid, compact, complete yet comprehensive.
- It consists of some years question answers upto 2022.
- Chapter is introduced in brief and compact form known as "Chapter at a glance".

Greatest Scientist Dr. MADY is with us:

We have launched Dr. MADY's invention of series of science & scientific experimental kits known as "MADY KIT". These kits are for students, teachers, parents and anyone who are interested in science. MADY KITS are designed exclusively to mix and blend education and fun. These kits are mainly designed to impart knowledge because they provide detailed and subtle facts on various aspects of life and physical sciences. The inquisitive mind is able to explore different branches of science including chemistry, biology, physics, mathematics, astronomy etc. using these kits.

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After the Covid 19 pandemic the whole education system has been changed. Students are getting tired a long time online classes and online exams. Online study materials cannot be substitute of the physical books. We have provided books to the students even in the pandemic period. As we know during the pandemic period all raw materials price increase intensely, specially in paper materials, we have no other option else to increase our book price according to the market situation.

Matrix Educare Pvt. Ltd. expresses extreme gratitude and thanks to the Professors, authors, well-wishers and in-house staff without their whole hearted effort such massive operation would not have been possible. Matrix Educare Pvt. Ltd. wishes in advance the best of luck to the students preparing for the coming semester examinations.

ANIRBAN GUPTA
on behalf of MATRIX EDUCARE PVT. LTD.
Email: info@matrixeducare.com
1st August, 2022

Syllabus

Engineering Mathematics-II

Unit – 1: Determinants & Matrices

1.1 Determinant:

- 1.1.1 Definition and expansion of determinants of order 2 & 3. Minors and cofactors.
- 1.1.2 Elementary properties of Determinants (statements only) & simple problems
- 1.1.3 Chios Method for 4th order determinant.
- 1.1.4 Solution of linear simultaneous equations (up to 3 unknowns) by Cramer's Rule.

1.2 Matrix:

- 1.2.1 Definition of Matrix and its order.
- 1.2.2 Different types of Matrices.(rectangular, square, row, column, upper triangular, lower triangular, diagonal, scalar, identity, null).
- 1.2.3 Equality of two matrices.
- 1.2.4 Addition, subtraction, multiplication of a matrix by a scalar and multiplication of two matrices.
- 1.2.5 Transpose of a matrix, symmetric & skew symmetric matrices, simple problems.
- 1.2.6 Singular & non-singular matrices, adjoint and inverse of a matrix of order 3.

Unit – 2: Co-ordinate Geometry (only 2-dimension)

2.1 Coordinate System

- 2.1.1 Cartesian & Polar Coordinate system & their relations.
- 2.1.2 Distance between two points, internal & external division of a line segment & simple problems.
- 2.1.3 Area of a triangle and condition of collinearity.

2.2 Straight Line

- 2.2.1 Gradient (slope) of a straight line.
- 2.2.2 Equations of straight line in various standard forms (no proof) (slope-intercept form, slope-point form, intercept form, two point form) & simple problems.
- 2.2.3 Angle between two straight lines- conditions of parallelism and perpendicularity & simple problems.
- 2.2.4 Perpendicular distance from a given point to a line, perpendicular distance between two parallel lines.

2.3 Circle:

- 2.3.1 Equation of a circle - centre-radius form, diameter form, simple problems.
- 2.3.2 General equation of a circle and its centre and radius. Simple problems.

2.4 Conic Section:

- 2.4.1 Definition of conic section, vertex, axis, eccentricity, focus, directrix, latus rectum & problem.
- 2.4.2 Standard equations of parabola and ellipse, simple problems.

Unit – 3: Integral Calculus

3.1 Indefinite integral

- 3.1.1 Definition of Integration as inverse process of differentiation.
- 3.1.2 Rules for integration: (sum, difference, scalar multiple).
- 3.1.3 Integration of standard functions.
- 3.1.4 Integration by substitution.
- 3.1.5 Integration by parts.
- 3.1.6 Integration by partial fraction.

3.2 Definite Integral

- 3.2.1 Definition of definite integral & simple problems.
- 3.2.2 Properties of definite integral with simple problems.
- 3.2. Application of definite integral -i) area of bounded region ii) Volume and surface area of solid generated by revolving an area about x and y-axis.

Unit – 4: Ordinary Differential Equation

- 4.1 Definition of ordinary differential equation, order & degree.
- 4.2 Solution of Differential equation of First order and first degree .
 - 4.2.1 Separation of Variables.
 - 4.2.2 Homogeneous type.
 - 4.2.3 Exact type.
 - 4.2.4 Linear type..
- 4.3 Solution of Linear Second order Differential equations with constant coefficients
 - 4.3.1 Complementary Functions (C.F)
 - 4.3.2 Particular Integral for polynomial function, e^{ax} , $\sin ax$ and $\cos ax$, $[F(-a^2) \neq 0]$, $e^{ax}V$ where V is a polynomial function. Simple problem.

Unit – 5: Partial Differentiation

- 5.1 Definition & meaning of partial derivative.
- 5.2 Evaluation of partial derivatives.
- 5.3 Definition & examples of homogeneous functions.
- 5.3 Euler's theorem (1st order) on Homogeneous functions for 2 variables (without proof). Problems.

Unit – 6: Statistics & Probability

6.1 Statistics

- 6.1.1 Definition & examples of frequency distribution.

6.1.2 Measure of Central Tendency (mean, median, mode) for ungrouped and grouped frequency distribution.

6.1.3 Measures of dispersion-Standard deviation, Simple problems.

6.2 Probability

6.2.1 Definition of random experiment, sample space, event, occurrence of events & types of events (eg. Impossible, Mutually exclusive, Exhaustive, Equally likely).

6.2.2 Classical definition of probability, simple problems.

APPLIED PHYSICS-II

Unit – 1: Wave motion and its applications

Simple Harmonic Motion (SHM): definition, expression for displacement, velocity, acceleration, time period, frequency etc. study of vibrations of cantilever and determination of its time period, Free, damped and forced vibrations with examples.

Wave motion, transverse and longitudinal waves with examples (Sound and light waves) definitions of wave velocity, frequency and wave length and their relationship, equation of a plane progressive wave. Principle of superposition of waves and beat formation.

Acoustics of buildings- reverberation, reverberation time, methods to control reverberation time, noise, coefficient of absorption of sound, Ultrasonic waves – Introduction and properties, engineering and medical applications of ultrasonic.

Unit – 2: Optics

Basic optical laws: reflection and refraction, refractive index, Images and image formation by thin lenses, lens & lens maker's formula, (no deduction) power of lens, magnification simple numerical problems. Total internal reflection, Critical angle and conditions for total internal reflection, applications of total internal reflection in optical fiber:

Optical Instruments; simple and compound microscope, astronomical telescope (refracting, Ray Diagram and formula for magnification). Interference and diffraction of light (Qualitative ideas only).

Unit – 3: Electrostatics

Coulombs law, unit of charge, Electric field, Electric lines of force and their properties, Electric flux, Electric potential and potential difference, Gauss law (statement only) Application of Gauss law to find electric field due to a charged sphere.

Capacitor and its working, types of capacitors, Capacitance and its units. Capacitance of a parallel plate capacitor (formula only), Series and parallel combination of capacitors formula (related numerical problems), dielectric and its effect on capacitance, dielectric break down.

Unit – 4: Current Electricity

Electric Current and its units, Direct and alternating current, resistance and its units, Specific resistance, Conductance, Specific conductance, Series and parallel combination of resistances. Factors affecting resistance of a wire, carbon resistance and colour coding. Ohm's law, Kirchhoff's laws, Wheatstone bridge, Carrey Foster Bridge and its applications, Concept of terminal potential difference and Electro motive force (EMF). Heating effect of current, Electric power, electric energy and its units (related numerical problems).

Thermoelectric effect: Seebeck & Peltier effects.

Unit – 5: Electromagnetism

Magnetic field and its origin, units Lorentz force (force on moving charge in magnetic field). Biot- Savart law, Application to Straight Conductor & circular loop; concept of magnetic dipole. Force on current carrying conductor, Torque on rectangular coil placed in magnetic field concept of electromagnetic induction, Faraday's Laws, Moving coil galvanometer; principle, construction and working. Conversion of a galvanometer into ammeter and voltmeter.

Types of magnetic materials; dia, para and ferromagnetic with their properties.

Unit – 6: Semiconductor Physics

Energy bands in solids, Types of materials (insulator, semi-conductor, conductor), intrinsic and extrinsic semiconductors, p-n junction, junction diode and V-I characteristics, Diode as rectifier- half wave and full wave rectifier (Centre taped) & circuit symbol.

Transistor, Block diagram types (pnp and npn) & circuit symbol, transistor as an amplifier CE mode (Circuit diagram and concept).

Photocells, Solar cells and LED working principle and engineering application.

Unit – 7: Modern Physics

Bohr's atom model and concept energy levels, ionization and excitation potentials, X-rays, Production (Coolidge tube) continuous and characteristic-X-rays, soft and hard X-rays, and use,

Laser: spontaneous and stimulated emission; Laser light; He-Ne laser elementary characteristics, applications of lasers.

Fiber Optics: Introduction to optical fibers, mechanism of light propagation, applications.

Nanoscience and nanotechnology (Introduction only).

INTRODUCTION TO IT SYSTEMS

Unit 1:

Basic Internet skills: Understanding browser, efficient use of search engines, awareness about Digital India portals (state and national portals) and college portals.

Number system and codes: Binary, octal, hexadecimal and decimal Number systems and their inter conversion, BCD numbers (8421-2421), gray code, excess-3 code, cyclic code, code conversion, ASCII, EBCDIC codes. Binary addition and subtraction, signed and unsigned binary numbers, 1's and 2's complement representation. 2. Boolean Algebra: Basic logic circuits: Logic gates (AND, OR, NOT, NAND, NOR, Ex-OR, ExNOR and their truth tables), Universal Gates, Laws of Boolean algebra, De-Morgan's theorem.

General understanding of various computer hardware components –

CPU, Memory (types), Display (modern), Keyboard, Mouse, HDD, SSD and other Peripheral Devices.

Types of software.

Unit 2:

Overview of Operating Systems

- What is an OS
- Brief history.

Background and Basics

- Computer System review
- Types of OS
- Architecture
- Basic Oss
- Batch
- Multi-programmed batch
- Timesharing
- Computer System Structures
- Operating System Structures.

Unit 3:

Algorithm and Flowcharts

- Algorithm
- Definition
- Characteristics.

Advantages and disadvantages

- Flowchart
- Definition
- Define symbols of flowchart
- Advantages and disadvantages.

Examples

Unit 4:

HTML 5:

HTML – Introduction, HTML – Elements, HTML – Tags, HTML – Text, HTML – Formatting, HTML – Pre, HTML – Attributes, HTML – Font, HTML – Text Links, HTML – Comments, HTML – Lists, HTML – Images, HTML – Image Links, HTML – Tables, HTML – Bgcolor, HTML – Color Codes, HTML – Color Chart, HTML – Background, Web Forms, HTML – Forms,

HTML – Input, HTML – Text Fields, Hidden Fields, HTML – Password, HTML – Reset, HTML – Submit, HTML – Checkboxes, HTML – Radio, HTML – Select, HTML – Hidden Fields, HTML – Upload, HTML – Textarea, Special Tags, HTML – Body, HTML – Meta, HTML – Style, HTML – Div, HTML – Layouts, HTML – Frames, Formatting Tags, HTML – Bold, HTML – Paragraphs, HTML – Headings, HTML – Line Breaks.

CSS: CSS Introduction, CSS Syntax, CSS Id & Class, CSS How CSS Styling, Styling Backgrounds, Styling Text, Styling Fonts, Styling Links, Styling Lists, Styling Tables, CSS Box Model, CSS Border, CSS Outline, CSS Margin, CSS Display, CSS Positioning, CSS Floating, CSS Navigation Bar, CSS Image Gallery, CSS Image Opacity, CSS, Align

JavaScript: Introduction, JavaScript Overview, JavaScript Syntax, Type of JavaScript, Embedding Script In HTML File, Variable Operators, Arithmetic, Logical, Comparison, Assignment, Conditional Statement & Looping Statement, If, If-Else, Switch, While, Do/while.

Unit 5:

(Network Utilities and devices tools/project)

1: Introduction to Computer Security Chapter

2: Networks and Internet (DoS Tools & Techniques – Tracert, Visual Route)

3: Cyber Stalking, Fraud, and Abuse

4: Denial of Service Attacks (Scanning – WireShark)

5: Malware Chapter 6: Techniques Used by Hackers.

FUNDAMENTALS OF ELECTRICAL & ELECTRONICS ENGINEERING

Unit – 1

Overview of Electrical Components

1.1 Concept of Passive Components: Resistors, Capacitors, Inductors.

1.2 Different types of signal waveforms: DC/AC, voltage/current, periodic/non-periodic

1.3 Voltage/current sources: Ideal/non-ideal, independent/dependent, source transformation,

1.4 Simple problems on source transformation.

Unit – 2

Electric and Magnetic Circuits

- 2.1 Concept of EMF, Current, Potential Difference, Power and Energy.
- 2.2 M.M.F, magnetic force, permeability, hysteresis loop, reluctance, leakage factor, BH curve and hysteresis loss, eddy current loss. Analogy between electric and magnetic circuits.
- 2.3 Electromagnetic induction; Faraday's laws, Lenz's law.
- 2.4 Dynamically induced emf; Statically induced emf; Principle of self and mutual inductance.
- 2.5 Energy stored in magnetic circuit.
- 2.6 Fleming's Left Hand Rule and Right Hand Rule.
- 2.7 Simple problems.

Unit – 3

A.C Circuits

- 3.1 Concept of Angular speed, Cycle, Frequency, Periodic time, Amplitude, RMS value, Average value, Form Factor, Peak Factor, impedance, phase and phase difference.
- 3.2 Representation of sinusoidal quantities in (i) exponential form (ii) complex form and (iii) polar form.
- 3.3 Expressions of voltage and current through pure resistance, inductance and capacitance with sinusoidal excitation and phasor representation.
- 3.4 Study of simple R-L, R-C, R-L-C series and parallel circuits with sinusoidal excitation and phasor representation.
- 3.5 Concept of impedance, impedance triangle, power factor, active, reactive and apparent power, power triangle.
- 3.6 Voltage and Current relationship in Star and Delta connections.
- 3.7 Simple problems.

Unit – 4

Transformer and Machines

- 4.1 Transformer and Machines
- 4.2 General construction and principle of different type of transformers
- 4.3 Emf equation and transformation ratio of transformers,
- 4.4 Auto transformers
- 4.5 Construction and Working principle of motors
- 4.6 Types of motors and applications
- 4.7 Simple problems on transformer and electrical machines.

Unit – 5

Overview of Basic Semiconductor Devices

- 5.1 Energy level diagrams of insulator, conductor & semi conductor.
- 5.2 Concept of Intrinsic & Extrinsic semiconductor, Idea of Doping concentration, Formation of P-Type and N-Type semiconductor and their properties.

- 5.3 Formation of P-N junction Diode and their properties.
- 5.4 Concept of PNP and NPN Transistor with their applications.
- Concept of FET, MOSFET & CMOS with their applications.

Unit – 6

Overview of Analog Circuits

- 6.1 Features of an ideal OPAMP, Pin configuration of 741C, Concept of Virtual Ground & Offset null adjustment.
- 6.2 Inverting and non-inverting mode and their gain calculation.
- 6.3 Applications of OPAMP: Amplifier, Adder, Integrator, & Differentiator circuit.

Unit – 7

Overview of Digital Electronics

- 7.1 Rules and laws of Boolean Algebra, Basic logic circuits-Logic gates (AND, OR, NOT, NAND, NOR, Ex-OR, Ex NOR and their truth tables), Universal Logic Gates.
- 7.2 De-Morgan's theorem, Min term (SOP), Max term(POS), Idea of Karnaugh-Map, Simplification of Logic circuits by Boolean Algebra and Karnaugh Map.
- 7.3 Basic concept of Flip Flops as a storage element.
- 7.4 Fundamental concept of Counters-Ripple and synchronous Up-Down Counter
- 7.5 Introduction to Digital IC Logic gates (TTL only).

ENGINEERING MECHANICS

Group A

Unit – I Basics of Mechanics and Force System

Concept of Engineering Mechanics – Statics & Dynamics; Space, time, mass, particle, flexible body and rigid body. Scalar Quantity and Vector Quantity; Addition & Subtraction of Vectors – Basic units – Derived Units – SI units.

Force: Definition, unit, representation as a vector and by Bow's notation, characteristics and effects of a force, Principle of transmissibility of force, Force systems and its classification with special emphasis of Co-planar Force System.

Co-planar Con-current Force System: Composition of Forces: Parallelogram Law, Triangle Law and Polygon Law of Forces. Determination of resultant by Analytical and Graphical method. Vector diagram. Resolution of Forces: Orthogonal components of a force. Simple problems on composition and resolution of forces.

Unit – II Moments and Couples

Moment: Definition of moment of a force about a point – Physical significance of moment – Resultant of a system of parallel and inclined forces – Varignon's Theorem. Simple Problems.

Couples: Definition of moment of a couple – Physical significance of Couples Equivalent couples – Resultant of any number of coplanar couples – Replacement of a

force about a point by an equal like parallel force together with a couple. Simple Problems.

Unit – III Condition of Equilibrium

Equilibrium of Co-planar Con-current Force System: Lami's Theorem, Triangle Law & Polygon Law of equilibrium Concept of Free Body diagram. Simple problems.

Equilibrium of Co-planar system of non-concurrent forces: Conditions of equilibrium of co-planar system of non-concurrent parallel forces (Like and Unlike) Simple problems excluding statically in-determinant.

Types of beams, supports (simple, hinged, roller and fixed) and loads. Beam reaction for simply supported beam with or without overhang subjected to combination of Point Load and Uniformly Distributed load. Simple Problems.

Unit – IV Friction

Friction and its relevance in engineering, Types and Laws of friction, Limiting Friction, Coefficient of friction, Angle of friction, Cone of Friction, Angle of Repose, Relation between coefficient of friction and angle of friction.

Equilibrium of bodies on horizontal surface subjected to force parallel and inclined to plane. Equilibrium of bodies on inclined plane subjected to force parallel and inclined to plane. Simple Problems.

Group B

Unit– V Centroid and Centre of Gravity

Concept & definition.

Determination of Centroid of Plane Figures like (i) Uniform triangular lamina, (ii) Uniform rectangular lamina, (iii) Uniform circular lamina, (iv) Uniform semi-circular lamina, and, (v) Uniform lamina of quadrant of a circle.

Determination of Centroid of Composite sections composed of not more than three plane figures like (i) T-section, (ii) equal and unequal angle-sections, (iii) equal and unequal I-sections, (iv) Channel-sections, (v) Z-sections, (vi) different cut-out sections, and, (vii) Built-up sections (Data of individual sections given). Simple Problems

Concept of Centre of Gravity of Simple Solids. Determination of Centre of Gravity of Simple Solids like (i) Cube (ii) Cuboid (iii) Cylinder (iv) Sphere. Determination of Centre of Gravity of composite solids composed of not more than two simple solids. No Problem.

Unit– VI Simple Lifting Machines

Definition of Lifting Machine, Applications and Advantages.

Load, Effort, Mechanical Advantage, Velocity Ratio, Efficiency of Machines and their relationships, Law of machine.

Ideal Machine, Friction in Machine, Maximum Mechanical Advantage and Efficiency, Reversible and non-reversible machines, conditions for reversibility. Simple Problems

Velocity ratios of (i) Simple and Differential Axle and Wheel (ii) Worm and worm wheel (iii) Single purchase and Double Purchase Crab Winch (v) Simple Screw Jack (vi) Simple Pulley Block, Simple numerical problems.

Group C

Unit– VI Motion in a Plane

Rectilinear Motion

Displacement- Time and Velocity-Time diagrams, Motion equations (with deduction). Newton's Second Law of linear motion $P = ma$ and momentum of a body, Conservation of momentum of a body, Simple numerical problems.

Curvilinear Motion

Concept and definition of Angular displacement, Angular velocity, relation between Linear & Angular velocity, Definition of Angular acceleration, Relation between linear & angular acceleration Concept, definition and deduction of expression for Centripetal and centrifugal force (numerical problems).

Work, Power, Energy

Concept, definition and mathematical expression of Work, Power and Energy, Discussion on their units in SI System.

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ENGINEERING MATHEMATICS - II

NOTE:

We have designed the complete ENGINEERING MATHEMATICS-II course material in accordance with new and changed syllabus under WBSCTE. Previously the subject was in 2nd semester. Hence we are providing relevant university solutions and some model questions & answers along with new university paper, so that students can get an idea about WBSCTE questions pattern.

Chapter 1: Determinants & Matrices

Chapter at a glance:

1.1 Definitions

A rectangular array of mn elements a_{ij} into m rows and n columns is called a matrix of order $m \times n$ over the field F . If F be the field of real numbers, the matrix is said to be a real matrix. Thus

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$

is a real matrix of order $m \times n$ and denoted by

$$A = (a_{ij})_{m \times n}, i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n.$$

If all the elements of a matrix be zero, then it is called a null matrix or a zero matrix and denoted by $O_{m \times n}$. If the elements are not all zero, then it is called non-null or non-zero matrix.

A matrix with single row is called a row matrix.

$$\text{e.g., } (a \ b \ c) \text{ is a row matrix.}$$

A matrix with single column is called a column matrix.

$$\text{e.g., } \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ is a column matrix.}$$

If in a matrix, number of rows be equal to the number of columns, then the matrix is called square matrix.

$$\text{e.g., } \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix} \text{ is a square matrix of order } n.$$

A square matrix whose non-diagonal elements are zero is called diagonal matrix.

$$\text{e.g., } \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \text{ is a diagonal matrix of order } n.$$

Thus for a diagonal matrix

$$(a_{ij})_{n \times n} \neq 0, i = j \\ = 0, i \neq j, i, j = 1, 2, \dots, n.$$

If all the elements of the diagonal matrix are equal, the matrix is called the scalar matrix.

$$\text{e.g., } \begin{pmatrix} a & 0 & \dots & 0 \\ 0 & a & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a \end{pmatrix} \text{ is a scalar matrix.}$$

Thus a scalar matrix

$$(a_{ij})_{n \times n} = a, i = j \\ = 0, i \neq j$$

If the elements of a diagonal matrix be all unity, the matrix is called the unit matrix or Identity matrix and it is denoted by I_n or simply I .

Therefore,

$$I_n = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

A square matrix A is called the singular matrix if $\det A = 0$.

$$\text{e.g., } \begin{pmatrix} 2 & 5 & 19 \\ 1 & -2 & -4 \\ -3 & 2 & 0 \end{pmatrix} \text{ is a singular matrix.}$$

If $\det A \neq 0$, then the square matrix is called non-singular matrix.

$$\text{e.g., } \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 25 \end{pmatrix} \text{ is a non-singular matrix.}$$

1.2 Operations on Matrices

1. Addition of two matrices:

Two matrices are conformable for addition if they are of the same order.

$$\text{If } A = (a_{ij})_{m \times n} \text{ and } B = (b_{ij})_{m \times n}, \text{ then}$$

$$A + B = (c_{ij})_{m \times n}, \quad c_{ij} = a_{ij} + b_{ij}.$$

The matrix addition is both commutative and associative.

$$\text{i.e., } A + B = B + A \quad [\text{Commutative}]$$

$$(A + B) + C = A + (B + C) \quad [\text{Associative}]$$

2. Subtraction of two matrices:

If $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$, then

$$A - B = (d_{ij})_{m \times n}, \quad d_{ij} = a_{ij} - b_{ij}.$$

3. Multiplication of a matrix by a scalar:

Multiplication of a matrix A by a scalar k means to multiply the every element of A by k .

e.g., If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$\text{then } kA = k \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}.$$

4. Multiplication of two matrices:

The product AB of two matrices A and B is possible only when the number of columns of A is equal to the number of rows of B .

If $A = (a_{ij})_{m \times n}$, $B = (b_{jk})_{n \times p}$

$$\text{then } AB = (c_{ik})_{m \times p} \quad \text{where } c_{ik} = \sum_{j=1}^n a_{ij} b_{jk}.$$

As an example let,

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \text{ and } B = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{pmatrix}$$

$$\text{Here, } AB = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{pmatrix} = \begin{pmatrix} a_1 A_1 + b_1 A_2 + c_1 A_3 & a_1 B_1 + b_1 B_2 + c_1 B_3 \\ a_2 A_1 + b_2 A_2 + c_2 A_3 & a_2 B_1 + b_2 B_2 + c_2 B_3 \end{pmatrix}$$

But

$$BA = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{pmatrix} \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} = \begin{pmatrix} A_1 a_1 + B_1 a_2 & A_1 b_1 + B_1 b_2 & A_1 c_1 + B_1 c_2 \\ A_2 a_1 + B_2 a_2 & A_2 b_1 + B_2 b_2 & A_2 c_1 + B_2 c_2 \\ A_3 a_1 + B_3 a_2 & A_3 b_1 + B_3 b_2 & A_3 c_1 + B_3 c_2 \end{pmatrix}$$

So it is seen that since A is of order 2×3 and B is of order 3×2 then the order of the product matrix AB is 2×2 while the order of the matrix BA is 3×3 .

In general, the matrix multiplication is not commutative. But matrix multiplication is associative.

Again if the product of two matrices is null matrix then we can not conclude that either of them is a null matrix.

e.g., If $A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 3 & 4 \end{pmatrix}$ then

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

but neither A nor B is a null matrix.

If A be a square matrix then A^2 is obtained by multiplication of A with A .

e.g., $A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$

$$\text{then } A^2 = A \cdot A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_1^2 + b_1 a_2 & a_1 b_1 + b_1 b_2 \\ a_2 a_1 + b_2 a_2 & a_2 b_1 + b_2^2 \end{pmatrix}$$

If A be a square matrix of order 3 then A is called the nilpotent matrix of index 2, if $A^2 = 0$ (null matrix).

e.g., $\begin{pmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ -4 & 4 & -4 \end{pmatrix}$ is a nilpotent matrix.

A is called involutory if $A^2 = I$

e.g., $\begin{pmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{pmatrix}$ is a involutory matrix.

A is called idempotent matrix if $A^2 = A$.

e.g., $\begin{pmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{pmatrix}$ is an idempotent matrix.

1.3 Equality of Two Matrices

Two matrices A and B are said to be equal iff they are of the same order and their corresponding elements are equal.

If $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$, then

$$A = B \quad \text{iff } a_{ij} = b_{ij}.$$

$$\text{e.g., } \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

iff $a = 1, b = 2, c = 3$ and $d = 4$.

1.4 Transpose of a Matrix

A matrix A^T of order $n \times m$ is obtained by interchanging rows and columns of a matrix A of order $m \times n$.

If $A = (a_{ij})_{m \times n}$ then $A^T = (a_{ji})_{n \times m}$

$$\text{e.g., } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

$$\text{then } A^T = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

We can easily verify that

$$(i) (A^T)^T = A$$

$$(ii) (A+B)^T = A^T + B^T$$

$$(iii) (AB)^T = B^T A^T$$

$$(iv) (kA)^T = kA^T, k \text{ is a scalar.}$$

1.5 Symmetric and Skew Symmetric Matrices

A matrix 'A' is called symmetric matrix if $A^T = A$.

$$\text{e.g., } \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix} \text{ is a symmetric matrix.}$$

A matrix 'A' is called skew symmetric matrix if $A^T = -A$.

$$\text{e.g., } \begin{pmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{pmatrix} \text{ is a skew symmetric matrix.}$$

Theorem:

Any square matrix A can be expressed as a sum of symmetric and skew symmetric matrices.

For any square matrix A, the symmetric matrix $= \frac{1}{2}(A + A^T)$ and skew symmetric

$$\text{matrix} = \frac{1}{2}(A - A^T).$$

1.6 Inverse of a Matrix

If A and B are square matrices of order n such that $AB = I = BA$, then the matrix B is called the inverse of the matrix A and vice-versa. We may write $B = A^{-1}$ or $A = B^{-1}$.

Thus

$$AA^{-1} = I = A^{-1}A$$

$$BB^{-1} = I = B^{-1}B$$

Two important properties of Inverses:

- A square matrix has an inverse if and only if it is non-singular.
- The inverse of a square matrix, if it exists, is unique.

1.7 Minors & Cofactors

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \text{ be a } n \times n$$

real matrix then the minor of any element a_{ij} of A is a determinant consisting all the elements of A except the row and column containing the element a_{ij} and it is denoted by

$$M_{ij}.$$

i.e.,

$$M_{ij} = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1,j-1} & a_{1,j+1} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2,j-1} & a_{2,j+1} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{i-1,1} & a_{i-1,2} & \dots & a_{i-1,j-1} & a_{i-1,j+1} & \dots & a_{i-1,n} \\ a_{i+1,1} & a_{i+1,2} & \dots & a_{i+1,j-1} & a_{i+1,j+1} & \dots & a_{i+1,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{n,j-1} & a_{n,j+1} & \dots & a_{nn} \end{vmatrix}$$

Cofactor of a_{ij} of A is defined as $C_{ij} = (-1)^{i+j} M_{ij}$

Adjoint matrix corresponding to the matrix A is denoted by $Adj(A)$ and defined by

$$Adj(A) = [C_{ji}]_{n \times n}$$

Inverse of the matrix A is defined by $A^{-1} = \frac{1}{\det(A)} Adj(A)$

1.8 Orthogonal Matrix

A square matrix 'A' is called the orthogonal matrix, if

$$A^T A = I = A A^T.$$

e.g., $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an orthogonal matrix, since

$$A^T A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Again $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is also an orthogonal matrix.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The value of $\begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ -2 & 3 & 1 \end{vmatrix}$ is - [WBSCTE 2004, 2006, 2008]
- (a) 0 (b) 13 (c) 19 (d) None

Answer: (c)

2. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$ where ω is a complex cube root of unity is - [WBSCTE 2004, 2007, 2017]
- (a) 0 (b) 1 (c) 2 (d) none of these

Answer: (a)

3. If $A = \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 0 & 4 \end{bmatrix}$ then A^T is - [WBSCTE 2004, 2005, 2007]
- (a) $\begin{bmatrix} 2 & 0 & 2 \\ -1 & 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}$ (c) none of these

Answer: (b)

4. The value of $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 9 \\ 5 & 6 & 7 \end{vmatrix}$ is - [WBSCTE 2005, 2007]
- (a) 0 (b) 13 (c) -1 (d) none of these

Answer: (c)

5. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 3 & 2 & 4 \end{bmatrix}$ is - [WBSCTE 2006, 2007, 2008]
- (a) a diagonal (b) an upper triangular
(c) a lower triangular (d) none of these

Answer: (c)

6. If $\Delta = \begin{vmatrix} 0 & 1 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$ then the minor of the element 4 is - [WBSCTE 2006, 2008, 2009]
- (a) 6 (b) 8 (c) 16 (d) none of these

Answer: (a)

7. The value of the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$ where ω is the complex cube root of 1 is - [WBSCTE 2009]
- (a) 0 (b) 3 (c) 1 (d) none of these

Answer: (b)

8. The matrix $\begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is - [WBSCTE 2009]
- (a) triangular (b) symmetric
(c) skew symmetric (d) no one is correct

Answer: (c)

9. Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is -

[WBSCTE 2009]

- (a) Unit matrix,
(c) diagonal matrix

- (b) scalar matrix
(d) none of these

Answer: (c)

10. Co-factor of the element 2 of the determinant $\begin{vmatrix} -1 & 3 & -2 \\ 5 & 4 & 0 \\ 2 & -3 & 5 \end{vmatrix}$ is

[WBSCTE 2009]

- (a) 8

- (b) -8

- (c) -1

- (d) none of these

Answer: (a)

11. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $I_3^2 + 2I_3$ equals to -

[WBSCTE 2010]

- (a) I_3

- (b) $2I_3$

- (c) $3I_3$

- (d) none is true

Answer: (c)

12. If $\Delta = \begin{vmatrix} -3 & 1 & 2 \\ 7 & 3 & -4 \\ -5 & 6 & -1 \end{vmatrix}$ then co-factor of element 7 is -

[WBSCTE 2010]

- (a) -13

- (b) 13

- (c) 21

- (d) none is correct

Answer: (b)

13. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then AB is -

[WBSCTE 2010]

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

- (d) none of these

Answer: (a)

14. If $A = \begin{bmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{bmatrix}$ then $\begin{bmatrix} a & b & c \\ b & c & a \\ 1 & 1 & 1 \end{bmatrix}$ is

[WBSCTE 2011]

- (a) 0

- (b) A

- (c) -A

- (d) none

Answer: (b)

15. The value of $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$ is

[WBSCTE 2011]

- (a) abc

- (b) 0

- (c) a+b+c

- (d) none is true

Answer: (b)

16. If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then the values of a, b, c and d respectively are -

[WBSCTE 2011]

- (a) (1, -2, 3, -4)

- (b) (1, 2, 3, 4)

- (c) (1, 2, -3, 4)

- (d) none is true

Answer: (b)

17. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ then A^2 is

[WBSCTE 2011]

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- (d) none of these

Answer: (d)

18. If $A = \begin{bmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{bmatrix}$ then $\begin{bmatrix} a & b & c \\ b & c & a \\ 1 & 1 & 1 \end{bmatrix}$ is

[WBSCTE 2012]

- (a) 0

- (b) A

- (c) -A

- (d) none

Answer: (b)

19. If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$; then the values of x and y are

[WBSCTE 2012]

- (a) (2, -2)

- (b) (-2, 2)

- (c) (3, 0)

- (d) none

Answer: (a)

20. The value of the determinant $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$ is

[WBSCTE 2012]

- (a) 0

- (b) 13

- (c) -13

- (d) none

Answer: (c)

21. Co-factor of element 6 of determinant $\begin{vmatrix} 2 & -2 & 3 \\ 1 & 5 & 6 \\ 2 & 7 & 13 \end{vmatrix}$ is [WBSCTE 2012]
- (a) 10 (b) 18 (c) -18 (d) none is correct

22. A square matrix is singular if - [WBSCTE 2013]
- (a) $\det A = 0$ (b) $\det A \neq 0$ (c) $\det A = 1$ (d) $\det A \neq 1$

23. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} =$ [WBSCTE 2014]
- (a) abc (b) 0 (c) $a + b + c$ (d) $-abc$

24. In the determinant $\begin{vmatrix} 1 & 3 & 4 \\ -1 & 2 & 0 \\ 5 & 6 & 9 \end{vmatrix}$ cofactor of 0 is - [WBSCTE 2014]
- (a) 0 (b) -9 (c) 9 (d) none of these

25. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $AB =$ [WBSCTE 2014]
- (a) $\begin{bmatrix} 1 & -2 \\ 6 & 12 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -2 \\ 11 & 16 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 6 \\ -2 & 12 \end{bmatrix}$ (d) none of these

26. The matrix $\begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -3 \\ -3 & 3 & 0 \end{bmatrix}$ is - [WBSCTE 2014]
- (a) diagonal (b) scalar (c) symmetric (d) skew-symmetric matrix

Answer: (d)

27. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$, $A^{-1} =$ [WBSCTE 2014]
- (a) $\frac{1}{3} \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ (b) $\frac{1}{3} \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$ (c) $\frac{1}{3} \begin{bmatrix} -1 & 1 \\ -2 & -1 \end{bmatrix}$ (d) $\frac{1}{3} \begin{bmatrix} -1 & -1 \\ 2 & -1 \end{bmatrix}$

Answer: (b)

28. The minor of the element 6 in the determinant $\begin{vmatrix} 1 & -2 & 3 \\ 0 & 4 & 6 \\ 1 & 2 & 5 \end{vmatrix}$ is - [WBSCTE 2015]
- (a) 3 (b) 4 (c) -4 (d) none of these

Answer: (b)

29. If A is a square matrix then [WBSCTE 2015]
- (a) $A + A^T$ is symmetric (b) $A + A^T$ is skew-symmetric
- (c) $A - A^T$ is symmetric (d) $A - A^T$ is skew-symmetric

Answer: (a) and (d)

30. If $3 \times \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$ then (p, q, r, s) is - [WBSCTE 2015]
- (a) (2, 3, 4, 3) (b) (2, 3, 3, 4) (c) (2, 0, 3, 4) (d) None of these

Answer: (b)

31. If A and B are two square matrices such that $A^2 = A$, $B^2 = B$ and $AB = BA$ then - [WBSCTE 2015]
- (a) $(AB)^2 = I$ (b) $(AB)^2 = AB$ (c) $(AB)^2 = BA$ (d) none of these

Answer: (b) and (c)

32. Given that matrix A has size 3×4 . Then $A^T B$ and BA^T are possible when B has size - [WBSCTE 2015]
- (a) 4×4 (b) 4×3 (c) 3×4 (d) 3×3

Answer: (c)

33. Given that matrix A has size 4×5 . Then $A^T B$ and BA^T are possible when B has size - [WBSCTE 2016]
- (a) 4×5 (b) 5×5 (c) 5×4 (d) 4×4

Answer: (a)

34. Minor and co-factor of -3 in the determinant $\begin{vmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}$ is [WBSCTE 2017]
- (a) 4 (b) -4 (c) 0 (d) none of these

Answer: (a)

35. The matrix $\begin{pmatrix} 0 & 5 & -8 \\ -5 & 0 & 11 \\ 8 & -11 & 0 \end{pmatrix}$ is known as - [WBSCTE 2017]
- (a) Symmetric matrix (b) Skew symmetric matrix
(c) Diagonal matrix (d) none of these

Answer: (b)

36. If $\begin{bmatrix} x+3 & x+2y \\ x-1 & 4t-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2t \end{bmatrix}$ then the value of x , y , z and t are respectively - [WBSCTE 2017]
- (a) 3, -2 , -4 , 3 (b) -3 , -2 , 4, 3
(c) -3 , 2, 4, -3 (d) none of these

Answer: (b)

37. If the matrix $\begin{pmatrix} x & 3 \\ 1 & 5 \end{pmatrix}$ is singular then $x =$ [WBSCTE 2017]
- (a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) 2

Answer: (b)

38. If $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $I_3^2 + 5I_3$ equals to [WBSCTE 2018]
- (a) $6I_3$ (b) $5I_3$ (c) $3I_3$ (d) I_3

Answer: (a)

39. If A and B be two square matrices then which one is correct - [WBSCTE 2018]
- (a) $\det(A.B) = \det(A).\det(B)$ (b) $\det(A+B) = \det(A) + \det(B)$
(c) $\det(A-b) = \det(A) - \det(B)$ (d) None of these

Answer: (a)

40. If $2 \begin{bmatrix} x & 2 \\ 7 & y+5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 15 & 14 \end{bmatrix}$, then find the value of (x, y) is - [WBSCTE 2018]
- (a) (0,0) (b) (1, 0) (c) (1, 1) (d) (0, 1)

Answer: (d)

41. Given that matrix A has size 2×3 . Then $A^T B$ and $B A^T$ - [WBSCTE 2018]
- (a) 3×3 (b) 2×2 (c) 3×2 (d) 2×3

Answer: (a)

42. If $\bar{X} = 3$ and $Y = 2X + 3$ then $\bar{Y}$ is equal to [WBSCTE 2019]
- (a) 6 (b) 12 (c) 15 (d) 9

Answer: (d)

43. If $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then $I_2^5 + 3I_2$ equals to [WBSCTE 2019]
- (a) $8I_2$ (b) $5I_2$ (c) $3I_2$ (d) $4I_2$

Answer: (d)

B. Answer the following questions:

44. If $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$; find A^T [WBSCTE 2009]

Answer:

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 4 & 6 & 3 \end{bmatrix} \text{ Ans.}$$

45. Find the value of x from the following

$$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

[WBSCTE 2009]

Answer:

$$2x - y = 6 \text{ and } y = -2$$

$$\therefore 2x - (-2) = 6 \text{ or, } x = 2 \text{ Ans.}$$

46. Find 'x' if $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

[WBSCTE 2009]

Answer:

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0 \Rightarrow 2x^2 = 8; x = \pm 2$$

47. Show that $(a+b+c)$ is a factor of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

[WBSCTE 2005, 2010]

Answer:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc - a^2) - b(b^2 - ac) + c(ab - c^2) \\ = 3abc - (a^2 + b^2 + c^2) = -(a^3 + b^3 + c^3 - 3abc) \\ = -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\ \therefore (a+b+c) \text{ is a factor.}$$

48. Evaluate $\begin{vmatrix} 1 & \cos 60^\circ & \cos 30^\circ \\ \frac{1}{2} & \sin 30^\circ & \sin 60^\circ \\ 1 & 0 & 2 \end{vmatrix}$

[WBSCTE 2010, 2015]

Answer:

$$\begin{vmatrix} 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & 2 \end{vmatrix} = \frac{1}{2} \times \frac{1}{2} \times \begin{vmatrix} 1 & 1 & \sqrt{3} \\ 1 & 1 & \sqrt{3} \\ 2 & 0 & 4 \end{vmatrix} \text{ [Taking } \frac{1}{2} \text{ common from } C_2 \text{ and } C_3]$$

$$= \frac{1}{4} \times \begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & 0 & 0 \\ 1 & 0 & 4 \end{vmatrix} [R_2' = R_2 - R_1] = \frac{1}{4} \times (-1) \times (-2 - 0) = \frac{1}{4} \times 2 = \frac{1}{2}$$

49. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -2 & 3 & 1 \end{bmatrix}$ find A^T

[WBSCTE 2010]

Answer: $A^T = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$ Ans.

50. If $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$, find AB . [WBSCTE 2011, 2016]

Answer: $AB = \begin{bmatrix} 1 & \sin 2\alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

51. Find the co-factor of element -3 of the determinant $\begin{vmatrix} 2 & 5 & 4 \\ -3 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix}$. [WBSCTE 2011]

Answer: $\begin{vmatrix} 2 & 5 & 4 \\ -3 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix} = (-1)^{2+1} \begin{vmatrix} 5 & 4 \\ 2 & 1 \end{vmatrix} = 1$

52. If $A = \begin{bmatrix} -1 & 1 & 0 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ write A^T .

[WBSCTE 2011]

Answer: $A^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 4 & 3 \end{bmatrix}$

53. Find minor corresponding to element 5 of the determinant $\begin{bmatrix} -6 & 2 & -1 \\ 1 & 5 & -2 \\ 3 & 4 & 1 \end{bmatrix}$.

[WBSCTE 2011, 2016]

Answer: $\begin{vmatrix} -6 & 2 & -1 \\ 1 & 5 & -2 \\ 3 & 4 & 1 \end{vmatrix} = \begin{vmatrix} -6 & -1 \\ 3 & 1 \end{vmatrix}$

54. Multiply $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

[WBSCTE 2012]

Answer: $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 9 & 14 \end{bmatrix}$.

55. Find minor of element 4 of determinant $\begin{vmatrix} -1 & 2 & 3 \\ 0 & -2 & -3 \\ 1 & 4 & 5 \end{vmatrix}$.

[WBSCTE 2012]

Answer: $\begin{vmatrix} -1 & 2 & 3 \\ 0 & -2 & -3 \\ 1 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 0 & -3 \end{vmatrix} = 3$

56. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ then write $A^T = ?$

[WBSCTE 2012]

Answer:

$A^T = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ -3 & 2 & 1 \end{bmatrix}$

57. If A is a symmetric matrix prove that $A.A^T$ is symmetric?

[WBSCTE 2013]

Answer:

$(A.A^T)^T = (A^T)^T.A^T = A.A^T$

So $A.A^T$ is a symmetric matrix.

58. Find the cofactor of the element '3' of the matrix $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 6 \\ 1 & 5 & 2 \end{bmatrix}$

[WBSCTE 2016]

Answer:

Cofactor of '3' is $(-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = (-1)^4 (0-2) = -2$

59. If $2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$, then find the value x and y.

[WBSCTE 2016]

Answer:

$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$

or, $\begin{bmatrix} 2x & 10 \\ 14 & 2y-3 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 14 & 12 \end{bmatrix}$

or, $\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 7 & 6 \end{bmatrix}$

$\therefore x=2, y-3=6 \Rightarrow y=9$

60. Find the value of the value of the determinant $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$.

[WBSCTE 2016]

Answer:

$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} = -a(0-bc) - b(ac-0) + abc - abc = 0$

61. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$.

[WBSCTE 2016]

Answer:

$|A| = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5$

$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} 5 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{bmatrix}$

62. Evaluate: $\begin{vmatrix} 41 & 42 & 43 \\ 47 & 48 & 49 \\ 44 & 45 & 46 \end{vmatrix}$.

[WBSCTE 2017]

Answer:

$\begin{vmatrix} 41 & 42 & 43 \\ 47 & 48 & 49 \\ 44 & 45 & 46 \end{vmatrix} = \begin{vmatrix} 41 & 1 & 2 \\ 47 & 1 & 2 \\ 44 & 1 & 2 \end{vmatrix} = 0 \quad \begin{matrix} C_2' = C_2 - C_1 \\ C_3' = C_3 - C_1 \end{matrix}$

63. Find the value of 'a' if $A = \begin{bmatrix} 2 & a \\ 3 & 6 \end{bmatrix}$ is singular.

[WBSCTE 2018]

Answer:

$$|A| = 0$$

$$\text{or, } \begin{vmatrix} 2 & a \\ 3 & 6 \end{vmatrix} = 0$$

$$\text{or, } 12 - 3a = 0 \quad \text{Ans. 4}$$

64. Find the value of the determinate $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{vmatrix}$.

[WBSCTE 2018, 2019]

Answer:

$$= 0 \begin{vmatrix} 0 & 5 \\ -5 & 0 \end{vmatrix} - 2 \begin{vmatrix} -2 & 5 \\ 3 & 0 \end{vmatrix} + (-3) \begin{vmatrix} -2 & 0 \\ 3 & -5 \end{vmatrix}$$

$$= -2(-15) - 3(10) = 30 - 30 = 0 \quad (\text{Ans.})$$

65. If $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 6 \\ 1 & 5 & 2 \end{bmatrix}$, then find A^T .

[WBSCTE 2018, 2019]

$$\text{Answer: } A^T = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 5 \\ 3 & 6 & 2 \end{bmatrix}$$

66. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$, then evaluate $2A - B$.

[WBSCTE 2018, 2019]

Answer:

$$2A - B = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 5 \end{bmatrix}$$

67. Find the inverse of the matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$.

[WBSCTE 2018]

Answer:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$$

68. Find minor corresponding to element '4' of the determinate $\begin{vmatrix} -6 & 2 & -1 \\ 1 & 5 & -2 \\ 3 & 4 & 1 \end{vmatrix}$.

[WBSCTE 2018]

Answer:

$$\begin{vmatrix} -6 & -1 \\ 1 & -2 \end{vmatrix} = 12 + 1 = 13$$

69. Find the cofactor of the element '5' of the matrix $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 6 \\ 1 & 5 & 2 \end{bmatrix}$.

[WBSCTE 2019]

Answer:

Cofactor of '5' is

$$(-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 0 & 6 \end{vmatrix} = - \begin{vmatrix} 1 & 3 \\ 0 & 6 \end{vmatrix} = -6$$

70. If $2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 14 & 14 \end{bmatrix}$, then find the value of x and y.

[WBSCTE 2019]

Answer:

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 14 & 14 \end{bmatrix}$$

$$\text{Here, } 2x = 7, \quad y - 3 = 14$$

$$\Rightarrow x = \frac{7}{2} = 3.5, \quad \Rightarrow y = 14 - 3 = 11$$

71. Find minor corresponding to element '-1' of the determinant $\begin{vmatrix} -6 & 2 & -1 \\ 1 & 5 & -2 \\ 3 & 4 & 1 \end{vmatrix}$.

[WBSCTE 2019]

Answer:

$$\text{Minor of '-1' is } \begin{vmatrix} 1 & 5 \\ 3 & 4 \end{vmatrix} = 4 - 15 = -11$$

72. If $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & -\sin \alpha \end{bmatrix}$ then find A^2 .

[WBSCTE 2019]

Answer:

$$\begin{aligned} A^2 &= A \cdot A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & -\sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & -\sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\because \sin^2 \alpha + \cos^2 \alpha = 1] \end{aligned}$$

73. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$.

[WBSCTE 2019]

Answer:

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{3} \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$$

74. Find the value of 'a' if $A = \begin{bmatrix} 2 & a \\ 1 & 5 \end{bmatrix}$ is singular.

[WBSCTE 2019]

Answer:

$$\begin{aligned} \begin{vmatrix} 2 & a \\ 1 & 5 \end{vmatrix} &= 1 \Rightarrow 10 - a = 1 \\ &\Rightarrow -a = 1 - 10 \\ &\therefore a = 9 \end{aligned}$$

$\therefore$ The value of A is 9.

75. In the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$ minor of "-1" is _____.

[WBSCTE 2022]

Answer: (-1)

76. Is the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 1 \end{bmatrix}$ non-singular? (Yes or No)

[WBSCTE 2022]

Answer: Yes

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 1 \end{vmatrix}$$

$$\begin{aligned} &= 1 \begin{vmatrix} 0 & 3 \\ -3 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} \\ &= (+6) - 5 + 10 \\ &= 11 \end{aligned}$$

$\therefore$ The matrix $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 1 \end{bmatrix}$ is non-singular. The correct answer is yes.

77. Is the matrix $\begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ orthogonal? (Yes/No)

[WBSCTE 2022]

Answer: Yes

$$A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{aligned} \therefore A^T \cdot A &= \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta + \sin \theta \cos \theta \\ \sin \theta \cos \theta + \cos \theta \sin \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$\therefore$ Yes, It is orthogonal matrix.

78. The value of the determinant $\begin{vmatrix} 0 & \omega & -\omega^2 \\ -\omega & 0 & \omega^3 \\ \omega^2 & -\omega^3 & 0 \end{vmatrix}$ is _____.

Answer: (-2)

[WBSCTE 2022]

79. The cofactor of 6 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is _____.

Answer: (32-35) = (-3).

[WBSCTE 2022]

80. Find the adjoin of the matrix $\begin{pmatrix} 5 & 0 \\ 1 & -2 \end{pmatrix}$.

Answer:

[WBSCTE 2022]

Adjoin of the matrix, $\text{adj } A = \begin{bmatrix} -2 & 0 \\ -1 & 5 \end{bmatrix}$

E. Pick up the correct answer:

81. Then matrix $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 4 \\ 2 & 4 & 3 \end{bmatrix}$ is a symmetric/skew-symmetric/scalar _____ matrix

* (Write the correct answer in the blank space)

Answer: Symmetric

[WBSCTE 2011]

82. The matrix $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ is symmetrical/skew symmetrical or scalar _____

matrix. (Write the name of matrix in the blank space).

Answer:

[WBSCTE 2012]

The matrix $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ is skew symmetrical.

Long Answer Type Questions

1. (a) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$, then show that $A+A^T$ is a symmetric matrix and $A-A^T$

is skew symmetric matrix.

[WBSCTE 2004, 2006, 2008]

(b) Evaluate the determinant $\begin{vmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 3 \\ 0 & 1 & 0 & 2 \\ -1 & -2 & -3 & 0 \end{vmatrix}$ by Chio's method.

[WBSCTE 2004, 2005, 2006, 2007, 2008, 2012]

Answer:

a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$$

$$\therefore A+A^T = \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix} \quad \therefore (A+A^T)^T = \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$$

$\therefore (A+A^T)$ is a symmetric matrix.

$$(A-A^T) = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \quad (A-A^T)^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$(A-A^T) = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix} = (A-A^T)^T$$

$(A-A^T)$ is skew symmetric. (Proved)

b) The determinant of the Matrix by Chio's method:

$$\begin{vmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 3 \\ 0 & 1 & 0 & 2 \\ -1 & -2 & -3 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 3 \\ 1 & 0 & 0 & 2 \\ -2 & -1 & -3 & 0 \end{vmatrix} \quad [\text{By interchanging } C_1 \text{ and } C_2]$$

$$\begin{aligned}
 &= (-1)^2 \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 3 \\ 1 & 0 & 0 & 2 \\ 2 & 1 & 3 & 0 \end{vmatrix} \quad [\text{Taking } (-1) \text{ common from } R_4] \\
 &= \frac{1}{(1)^{4-2}} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 3 \end{vmatrix} \quad (\text{According to the method}) \\
 &= \begin{vmatrix} -1 & -1 & 3 \\ 0 & 0 & 1 \\ 1 & 3 & -2 \end{vmatrix} = (-1)^{3-2} \begin{vmatrix} -1 & -1 & -1 & 3 \\ 0 & 0 & 1 \\ 1 & 3 & 1 & -2 \end{vmatrix} \\
 &= - \begin{vmatrix} 0 & -1 \\ -2 & -1 \end{vmatrix} = -(-2) = 2 \quad \text{Ans.}
 \end{aligned}$$

3. If $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $AA^T = I_3$ [WBSCTE 2005, 2007, 2008, 2012]

OR,

Show that the matrix $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal, i.e., $AA^T = A^T A = I_3$

[WBSCTE 2016]

Answer:

$$\begin{aligned}
 A &= \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \quad \therefore A^T = \frac{1}{3} \begin{bmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix} \\
 \therefore AA^T &= \frac{1}{9} \begin{bmatrix} (-1)(-1) + 2.2 + (-2)(-2) & (-1)(-2) + 2.1 + (-2)2 & (-1)2 + 2.2 + (-2)1 \\ (-2)(-1) + 1.2 + 2(-2) & (-2)(-2) + 1.1 + 2.2 & (-2)2 + 1.2 + 2.1 \\ 2(-1) - 2.2 + 1(-2) & 2(-2) + 2.1 + 1.2 & 2.2 + 2.2 + 1.1 \end{bmatrix}
 \end{aligned}$$

$$= \frac{1}{9} \begin{vmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1, \text{ Proved.}$$

3. If $x + y + z = 0$, then show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$ [WBSCTE 2007, 2010]

Answer:

$$\begin{aligned}
 \text{Let } \Delta &= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ x & x-y & x-z \\ x^3 & x^2-y^2 & x^2-z^2 \end{vmatrix} \begin{matrix} C'_1 = -C_2 + C_1 \\ C'_3 = -C_3 + C_1 \\ \text{from } C_1 \end{matrix} \\
 &= (x-y)(x-z) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^3 & x^2+xy+y^2 & x^2+zx+z^2 \end{vmatrix} \begin{matrix} \text{Taking } (x-y) \\ \text{and } (x-z) \text{ common} \\ \text{from } C_2 \text{ and } C_3 \text{ respectively} \end{matrix} \\
 &= (x-y)(x-z) \cdot 1 \{ (x^2+zx+z^2) - (x^2+xy+y^2) \} \\
 &= (x-y)(x-z) \{ x(z-y) + (z+y)(z-y) \} \\
 &= (x-y)(x-z)(z-y)(x+z+y) = (x+y+z)(x-y)(y-z)(z-x) \\
 \text{Since, } (x+y+z) &= 0 \quad \therefore \Delta = 0 \text{ Ans.}
 \end{aligned}$$

4. Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 4 & 6 \\ 1 & 1 & 5 & 8 \end{vmatrix}$ by Chio's method.

[WBSCTE 2009, 2019]

Answer:

$$\Delta = \frac{1}{(1)^{4-2}} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 4 & 6 \\ 1 & 1 & 5 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{vmatrix}$$

$$= \frac{1}{(1)^{1+2}} \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 5 \end{vmatrix} = \frac{3}{1} = 3 \quad \begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix} = 21 - 20 = 1 \text{ Ans.}$$

5. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$, find A^{-1}

[WBSCTE 2009]

Answer:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 9, A_{12} = \begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix} = -4, A_{13} = \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 8$$

$$A_{21} = \begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix} = -19, A_{22} = \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix} = 14, A_{23} = \begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix} = -3$$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix} = -4, A_{32} = \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = -1, A_{33} = \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} = 2$$

$$\text{Hence } \text{Adj. } A = \begin{bmatrix} 9 & -4 & 8 \\ -19 & 14 & -3 \\ -4 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -19 & -4 \\ -4 & 14 & -1 \\ 8 & -3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} + 2 \begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix} + 3 \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 9 + 2 \times (-4) + 3 \times 8 = 9 - 8 + 24 = 25 \neq 0.$$

$$\therefore A^{-1} = \frac{\text{Adj. } A}{|A|} = \frac{1}{25} \begin{bmatrix} 9 & -19 & -4 \\ -4 & 14 & -1 \\ 8 & -3 & 2 \end{bmatrix} \text{ Ans.}$$

6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ show that $A^2 - 5A - 2I = 0$ where I is the unit matrix of order 2×2 .

[WBSCTE 2009]

Answer:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 3 \times 2 & 2 \times 1 + 4 \times 2 \\ 3 \times 1 + 4 \times 3 & 2 \times 3 + 4 \times 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$2I = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^2 - 5A - 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \text{ [Proved]}$$

7. Evaluate $\begin{vmatrix} 0 & 1 & -1 & 2 \\ 3 & 2 & 2 & -2 \\ 4 & 2 & 2 & 1 \\ 1 & 3 & 5 & -3 \end{vmatrix}$ by Chio's method.

[WBSCTE 2010]

Answer:

$$\Delta = \begin{vmatrix} 0 & 1 & -1 & 2 \\ 3 & 2 & 2 & -2 \\ 4 & 2 & 2 & 1 \\ 1 & 3 & 5 & -3 \end{vmatrix} = (-1) \begin{vmatrix} 1 & 0 & -1 & 2 \\ 2 & 3 & 2 & -2 \\ 2 & 4 & 2 & 1 \\ 3 & 1 & 5 & -3 \end{vmatrix} \text{ [Interchanging 1st and 2nd columns]}$$

$$= (-1) \cdot \frac{1}{(1)^{4+2}} \begin{vmatrix} 1 & 0 & 1 & -1 & 2 \\ 2 & 3 & 2 & 2 & -2 \\ 2 & 4 & 2 & 2 & 1 \\ 3 & 1 & 3 & 5 & -3 \end{vmatrix} \text{ [by Chio's method]}$$

$$= (-1) \cdot 4 \cdot 4 \cdot (-3) \begin{vmatrix} 3 & 4 & -6 \\ 4 & 4 & -3 \\ 1 & 8 & -9 \end{vmatrix} \text{ [taking common factor 4 from 2nd column and -3 from 3rd column]}$$

$$= (-1) \times 4 \times (-3) \times \begin{vmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{12}{3^{3-2}} \begin{vmatrix} 3 & 1 & 3 & 2 \\ 4 & 1 & 4 & 1 \\ 3 & 1 & 3 & 2 \\ 1 & 2 & 1 & 3 \end{vmatrix} \quad [\text{by Chio's method}]$$

$$= \frac{12}{3} (-7 + 25) = 4 \times 18 = 72 \text{ Ans.}$$

8. (a) Find Adj A ; If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$ [WBSCTE 2010]

(b) Express $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ as a sum of two matrices such that one of them is symmetric and other skew symmetric matrix. [WBSCTE 2010, 2019]

Answer:

a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$

Now the cofactors of the elements 1, 2, 3, 2, -4, 5, 6, 1, 0 respectively -5, 30, 26, 3, -18, 11, 22, 1, -8.

Therefore adj A = $\begin{bmatrix} -5 & 3 & 22 \\ 30 & -18 & 1 \\ 26 & 11 & -8 \end{bmatrix}$ Ans.

b) $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

As, $\frac{1}{2}(A + A^T)$ is a symmetric matrix

and $\frac{1}{2}(A - A^T)$ is a skew symmetric matrix.

$$\therefore A + A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$$

$$\therefore A - A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \text{symmetric matrix} + \text{skew symmetric matrix.}$$

9. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$ [WBSCTE 2011]

Answer:

$$\text{L.H.S.} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \quad \begin{matrix} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{matrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

[Taking common (b-a) and (c-a) from R_2 and R_3 respectively]

$$= (b-a)(c-a)(c+a-a-b) = (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a) = \text{R.H.S. (proved).}$$

10. Show that $\begin{vmatrix} x & \beta & \gamma & 1 \\ \alpha & x & \gamma & 1 \\ \alpha & \beta & \gamma & 1 \\ \alpha & \beta & \gamma & 1 \end{vmatrix} = (x-\alpha)(x-\beta)(x-\gamma).$ [WBSCTE 2011, 2017]

Answer:

$$\text{L.H.S.} = \begin{vmatrix} x & \beta & \gamma & 1 \\ \alpha & x & \gamma & 1 \\ \alpha & \beta & \gamma & 1 \\ \alpha & \beta & \gamma & 1 \end{vmatrix} = D \quad (\text{Say})$$

If $x = \alpha$, then R_1 and R_4 are identical. So $(x-\alpha)$ is a factor of D .

If $x = \beta$ then R_2 and R_4 are identical. So $(x-\beta)$ is a factor of D .

If $x = \gamma$ then R_3 and R_4 are identical. So $(x-\gamma)$ is another factor of D .

$\therefore D = K(x - \alpha)(x - \beta)(x - \gamma)$, where K is a constant.

Here, L.H.S. and R.H.S. both are polynomial of x . Now equating the coefficient of x^3 from both sides we get $K = 1$

Hence, $D = (x - \alpha)(x - \beta)(x - \gamma)$.

11. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$ [WBSCTE 2011]

OR,

Show that the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$ and

hence find A^{-1} . [WBSCTE 2015]

OR,

Show that the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I_3 = \bar{0}$,

where $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\bar{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. [WBSCTE 2016]

Answer:

$$\text{Given } A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Thus, $A^2 - 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{proved}).$$

$$\therefore A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$\therefore \text{Adj}(A) = \begin{bmatrix} -3 & +2 & 2 \\ +2 & -3 & +2 \\ 2 & +2 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore |A| = 1(1 - 4) - 2(2 - 4) + 2(4 - 2) = -3 + 4 + 4 = 5$$

$$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad (\text{Ans.})$$

12. Given $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ obtain A^{-1} . [WBSCTE 2011]

Answer:

$$\text{Given, } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3 + 4) - 2(-3 + 4) = 3 - 2 = 1$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & +3 & -3 \end{bmatrix}$$

13. Show that $\begin{vmatrix} (1+a) & 1 & 1 \\ 1 & (1+b) & 1 \\ 1 & 1 & (1+c) \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right); a \neq b \neq c \neq 0$. [WBSCTE 2012]

Answer:

$$\text{L.H.S., } \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$$\begin{aligned}
 &= abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix} \quad \left[\text{Taking common } a, b, c \text{ from } c_1, c_2, c_3 \text{ respectively} \right] \\
 &= abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix} \quad [c'_1 = c_1 + c_2 + c_3] \\
 &= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & 1+\frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix} \\
 &= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \begin{bmatrix} R'_2 = R_2 - R_1 \\ R'_3 = R_3 - R_1 \end{bmatrix} \\
 &= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \right)
 \end{aligned}$$

14. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ [WBSCTE 2012]

Answer:

Let $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$; $\therefore |A| = \begin{vmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -1(1-3) + 3(3-6) = 3-9 = -6.$

$$\text{Adj. } (A) = \begin{bmatrix} -1 & 2 & 3 \\ 8 & -9 & 3 \\ -5 & +3 & -1 \end{bmatrix} \quad \therefore A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{-1}{6} \begin{bmatrix} -1 & 2 & 3 \\ 8 & -9 & 3 \\ -5 & 3 & -1 \end{bmatrix}$$

15. If A is symmetric matrix, then prove that $A+A^T$ is a symmetric and $A-A^T$ is skew-symmetric. [WBSCTE 2013]

Answer:

$$(A+A^T)^T = A^T + (A^T)^T = A^T + A = A+A^T$$

$$\text{and } (A-A^T)^T = A^T - (A^T)^T = A^T - A = -(A-A^T)$$

Hence $(A+A^T)$ is symmetric and $(A-A^T)$ is skew symmetric.

16. Define Orthogonal matrix. Prove that orthogonal matrix is non-singular. [WBSCTE 2013]

Answer:

If any matrix A satisfy $AA^T = I_n$, where I_n is the identity matrix of the same order that of A , then A is called are orthogonal matrix.

$$\text{Now } \det(AA^T) = \det(A)\det(A^T) = (\det(A))^2$$

If A is orthogonal then, $AA^T = I_n$

$$\text{or, } \det(AA^T) = 1 \quad \text{or, } (\det(A))^2 = 1 \quad \text{or, } \det(A) = \pm 1$$

So A is non-singular.

17. (a) Evaluate by Chio's method $\begin{vmatrix} 0 & -1 & 2 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{vmatrix}$ [WBSCTE 2014, 2015]

(b) Expression $A = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$ as a sum of two matrices such that one of them is

symmetric and the other is skew-symmetric. [WBSCTE 2014]

Answer:

a) $\Delta = \begin{vmatrix} 0 & -1 & 2 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{vmatrix}$

$$= (-1) \begin{vmatrix} -1 & 0 & 2 & 3 \\ 2 & 1 & -1 & 1 \\ 1 & 1 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{vmatrix} \quad [\text{Interchanging 1st and 2nd columns}]$$

$$= (-1) \cdot \frac{1}{(-1)^{4-2}} \begin{vmatrix} -1 & 0 & -1 & 2 \\ 2 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \\ -1 & 0 & -1 & 2 \end{vmatrix} \quad [\because \text{By Chio's method}]$$

$$= (-1) \cdot \begin{vmatrix} -1 & -3 & -7 \\ -1 & -4 & -6 \\ -3 & -5 & -6 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 7 \\ 1 & 4 & 6 \\ 3 & 5 & 6 \end{vmatrix}$$

$\therefore$ Again by applying Chio's method:

$$= \frac{1}{(1)^{3-2}} \begin{vmatrix} 1 & 3 & 1 & 7 \\ 1 & 4 & 1 & 6 \\ 1 & 3 & 1 & 7 \\ 3 & 5 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -4 & -15 \end{vmatrix} = -15 - 4 = -19 \quad (\text{Ans.})$$

b) We know,

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

As, $\frac{1}{2}(A + A^T)$ is a symmetric matrix

and, $\frac{1}{2}(A - A^T)$ is a skew symmetric matrix.

$$\therefore A + A^T = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 4 \\ 6 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 8 & 6 \\ 8 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix}$$

$$\therefore A - A^T = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ 6 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$$\therefore A = \frac{1}{2} \begin{bmatrix} 2 & 8 & 6 \\ 8 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow 18. \text{ Find Adj } A, \text{ if } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$$

[WBSCTE 2014]

Answer:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix} ; \text{ Adj}(A) = \begin{bmatrix} -5 & +30 & 26 \\ +3 & -18 & +11 \\ 22 & -22 & -8 \end{bmatrix}^T = \begin{bmatrix} -5 & 3 & 22 \\ 30 & -18 & -22 \\ 26 & 11 & -8 \end{bmatrix}$$

$$\Rightarrow 19. (a) \text{ Evaluate by Chio's method } \begin{vmatrix} 1 & 0 & -2 & 2 \\ 2 & 3 & 2 & -2 \\ 2 & 4 & 5 & 1 \\ 3 & 1 & 5 & 1 \end{vmatrix}$$

[WBSCTE 2015]

$$(b) \text{ Show that the matrix } A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \text{ satisfies } AA^T = A^T A = I_3$$

Answer:

$$a) \begin{vmatrix} 1 & 0 & -2 & 2 \\ 2 & 3 & 2 & -2 \\ 2 & 4 & 5 & 1 \\ 3 & 1 & 5 & 1 \end{vmatrix} = \frac{1}{(1)^{4-2}} \begin{vmatrix} 1 & 0 & 1 & -2 \\ 2 & 3 & 2 & 2 \\ 2 & 4 & 2 & 5 \\ 3 & 1 & 3 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 6 & -6 \\ 4 & 9 & -3 \\ 1 & 11 & -5 \end{vmatrix}$$

$$= 3(-45 + 33) - 6(-20 + 3) - 6(44 - 9) = \{-36 + 102 - 210\} = -144$$

$$b) A^T = \frac{1}{3} \begin{bmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$$

$$\therefore AA^T = \frac{1}{9} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = I_3$$

$$A^T A = \frac{1}{9} \begin{bmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = I_3$$

$$\therefore AA^T = A^T A = I_3$$

20. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$, then find inverse of A i.e., A^{-1} . [WBSCTE 2016]

Answer:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{vmatrix} = 1(0-5) - 2(0-30) + 3(2+4) = -5 + 60 + 78 = 133$$

$$\therefore A^{-1} = \frac{\text{Adj}(A)}{|A|} = \frac{1}{133} \begin{bmatrix} -5 & 3 & 22 \\ 30 & -18 & 1 \\ 26 & +11 & -8 \end{bmatrix}$$

21. If A is symmetric matrix then show that $P^T A P$ is symmetric. [WBSCTE 2016]

Answer:

Let A is symmetric

Therefore $A^T = A$

$$\text{Now } (P^T A P)^T = P^T A^T (P^T)^T$$

$$= P^T A^T P$$

$$= P^T A P \quad [\because A^T = A]$$

So $P^T A P$ is also symmetric matrix.

22. Find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$. [WBSCTE 2017]

Answer:

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$\therefore |A| = 2 \times 1 + 3 \times 4 + 4 \times (-1) = 2 + 12 - 4 = 10$$

$$\therefore \text{Adj}(A) = \begin{bmatrix} 1 & 8 & -3 \\ -4 & 8 & 2 \\ -1 & 2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 1 & 8 & -3 \\ -4 & 8 & 2 \\ -1 & 2 & -3 \end{bmatrix}$$

23. (a) Evaluate the determinate $\begin{vmatrix} 2 & 0 & 3 & 4 \\ 0 & 1 & 0 & 2 \\ 5 & -1 & 0 & 2 \\ 1 & 0 & 4 & 3 \end{vmatrix}$ by Chio's methods.

(b) Express $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ as sum of two matrices such that one of them is symmetric and other is skew-symmetric matrix. [WBSCTE 2018]

Answer:

a) We have $A = \begin{vmatrix} 2 & 0 & 3 & 4 \\ 0 & 1 & 0 & 2 \\ 5 & -1 & 0 & 2 \\ 1 & 0 & 4 & 3 \end{vmatrix} = \frac{1}{(2)^{4-1}} \begin{vmatrix} 2 & 0 \\ 0 & 1 \\ 5 & -1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 0 & 0 \\ 5 & 0 \\ 1 & 4 \end{vmatrix} \begin{vmatrix} 2 & 4 \\ 0 & 2 \\ 5 & 2 \\ 1 & 3 \end{vmatrix}$

$$= \frac{1}{2^2} \begin{vmatrix} 2 & 0 & 4 \\ -2 & -15 & -16 \\ 0 & 5 & 2 \end{vmatrix} = \frac{1}{4} \cdot \frac{1}{2^{3-1}} \begin{vmatrix} 2 & 0 \\ -2 & -15 \\ 2 & 0 \\ 0 & 5 \end{vmatrix} \begin{vmatrix} 2 & 4 \\ -2 & -16 \\ 2 & 4 \\ 0 & 2 \end{vmatrix}$$

$$= \frac{1}{4} \cdot \frac{1}{2} \begin{vmatrix} -30 & -24 \\ 10 & 4 \end{vmatrix} = \frac{1}{8} (-120 + 240) = 15.$$

b) Refer to Question No. 1(a) of Long Answer Type Questions.

24. (a) Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ca$ [WBSCTE 2018]

(b) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I_2 = \bar{0}$, where $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

$\bar{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and hence find A^{-1} . [WBSCTE 2018, 2019]

Answer:

a) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \begin{vmatrix} 1+a & 1 & 1 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} \begin{matrix} R'_2 = R_2 - R_1 \\ R'_3 = R_3 - R_1 \end{matrix}$
 $= (1+a) \begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} + a \begin{vmatrix} 1 & 1 \\ 0 & c \end{vmatrix} - a \begin{vmatrix} 1 & 1 \\ b & 0 \end{vmatrix}$
 $= (1+a)bc + ac + ab$
 $= abc + bc + ca + ab \quad (\text{Proved})$

b) Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$\therefore A^2 + 7I_2 - 4A = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - 4 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \bar{0} \quad (\text{Proved})$

Now, $A^2 - 4A + 7I_2 = 0$

or, $\frac{1}{7}(A^2 - 4A) = -I_2$

or $A\left(\frac{4}{7}I_2 - \frac{1}{7}A\right) = I_2$

$\therefore A^{-1} = \frac{4}{7}I_2 - \frac{1}{7}A = \frac{4}{7}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

25. If A and B be two square matrices and given that $AB = A$ and $BA = B$, then that $A^2 + B^2 = A + B$. [WBSCTE 2018]

Answer:

L.H.S. = $A^2 + B^2$

$= (AB)(AB) + (BA)(BA) \quad \left[\begin{matrix} \because A = AB \\ B = BA \end{matrix} \right]$

$= A(BA)B + B(AB)A \quad (\text{By associative law of matrix multiplication})$

$= ABB + BAA = (AB)B + (BA)A = AB + BA$

$= A + B \quad (\text{Proved})$

26. (a) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$. [WBSCTE 2019]

Answer:

$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ \alpha & \beta - \alpha & \gamma - \alpha \\ \alpha^2 & \beta^2 - \alpha^2 & \gamma^2 - \alpha^2 \end{vmatrix} \quad \begin{matrix} C'_2 = C_2 - C_1 \\ C'_3 = C_3 - C_1 \end{matrix}$

$= \begin{vmatrix} \beta - \alpha & \gamma - \alpha \\ \beta^2 - \alpha^2 & \gamma^2 - \alpha^2 \end{vmatrix} \quad \text{Expanding with respect to } R_1$

$= (\beta - \alpha)(\gamma - \alpha) \begin{vmatrix} 1 & 1 \\ \beta + \alpha & \gamma + \alpha \end{vmatrix}$

$= (\beta - \alpha)(\gamma - \alpha)(\gamma + \alpha - \beta - \alpha)$

$= (\beta - \alpha)(\gamma - \alpha)(\gamma - \beta)$

$= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

(b) Show that the matrix $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal, i.e., $AA^T = A^T A = I_3$,

hence find A^{-1} .

[WBSCTE 2019]

Answer:

1st Part: Refer to Question No. 2 of Long Answer Type Questions.

2nd Part:

An orthogonal matrix is a square matrix whose columns and rows are orthogonal unit vectors.

$\therefore AA^T = A^T A = I$, where I is the Identity matrix

This leads to the equivalent characterization: a matrix A is orthogonal if its transpose is equal to its inverse.

$$\therefore A^T = A^{-1}$$

27. Evaluate by Chio's method
$$\begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix}$$
 [WBSCTE 2022]

Answer:

$$\begin{aligned} \Delta &= \begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix} \\ &= (-1) \begin{vmatrix} 2 & 1 & -2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{vmatrix} \quad [\text{Interchange 1st \& 2nd columns}] \\ &= (-1) \frac{1}{2^{1+2}} \begin{vmatrix} 2 & 1 & 2 & -2 \\ 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} \quad [\text{By Chio's method}] \\ &= (-1) \frac{1}{4} \begin{vmatrix} -1 & 2 & 0 \\ -4 & 2 & 2 \\ 2 & -2 & 2 \end{vmatrix} \\ &= \left(-\frac{1}{4}\right) \times 2 \begin{vmatrix} -1 & 1 & 0 \\ -4 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \quad [\text{Taking common factor 2 from 2nd \& 3rd column}] \end{aligned}$$

$$\begin{aligned} &= \frac{-1}{2} \begin{vmatrix} -1 & 1 \\ -4 & 1 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ -4 & 1 \end{vmatrix} \\ &= \frac{-1}{2} \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} \quad [\text{By Chio's method}] \\ &= -\frac{1}{2} \begin{vmatrix} 3 & -1 \\ -1 & -1 \end{vmatrix} \\ &= -\frac{1}{2} (-3 - 1) \\ &= -\frac{1}{2} \times (-4) \\ &= 2 \end{aligned}$$

28. Solve the equations by cramer's rule $x+y+z=3$; $2x+z=5$; $x+2y=-1$. [WBSCTE 2022]

Answer:

$$x + y + z = 3$$

$$2x + z = 5$$

$$x + 2y = -1$$

$$\begin{aligned} \Delta &= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0-2) - 1(0-1) + 1(4-0) = (-2) + 1 + 4 = 3 \\ \Delta_x &= \begin{vmatrix} 3 & 1 & 1 \\ 5 & 0 & 1 \\ -1 & 2 & 0 \end{vmatrix} = 3(-2) - 1(1) + 1(10) = 3 \\ \therefore x &= \frac{\Delta_x}{\Delta} = \frac{3}{3} = 1 \\ \Delta_y &= \begin{vmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1(1) - 3(-1) + 1(-2-5) = 1 + 3 - 7 = (-3) \\ \therefore y &= \frac{\Delta_y}{\Delta} = \frac{-3}{3} = (-1) \\ \Delta_z &= \begin{vmatrix} 1 & 1 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & -1 \end{vmatrix} = 1(-10) - 1(-2-5) + 3(4-0) = -10 + 7 + 12 = 9 \end{aligned}$$

$$\therefore z = \frac{\Delta_z}{\Delta} = \frac{9}{3} = 3$$

So, the equations have been solved by using Cramer's rule, $x = 1, y = -1, z = 3$.

29. Find A^{-1} ; if $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and show that $AA^{-1} = I_3$. [WBSCTE 2022]

Answer:

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$|A| = 0(1+4) - 1(1) + (-1-0)$$

$$= 0 - 1 - 1 = (-2)$$

$$\text{Adj}(A) = \begin{bmatrix} 5 & 1 & -1 \\ -1 & 0 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 5 & 1 & -1 \\ -1 & 0 & 0 \\ -4 & 0 & 1 \end{bmatrix} \text{ or, } = -\frac{1}{2} \begin{bmatrix} 5 & 1 & -1 \\ -1 & 0 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 4 \\ 0 & -1 & 1 \end{bmatrix} \cdot \left(-\frac{1}{2}\right) \begin{bmatrix} 5 & 1 & -1 \\ -1 & 0 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$= \left(-\frac{1}{2}\right) \begin{bmatrix} 0+1+0 & 0+0+0 & 0+0+0 \\ 5-1-16 & 1+0+0 & -1+0+4 \\ 0+1-4 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \left(-\frac{1}{2}\right) \begin{bmatrix} 1 & 0 & 0 \\ -12 & 1 & 3 \\ -3 & 0 & 1 \end{bmatrix} = I_3 \quad [\text{Proved}]$$

30. Solve the following system of linear equations by Cramer's Rule or Matrix Inversion method [Model Question]

$$2x + 5y + 3z = 5$$

$$3x + y + 2z = 5$$

$$x + 2y - z = 0$$

Answer:

Given system of equations can be written in the following matrix form:

$$AX = B$$

$$\text{where, } A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$

Now by Cramer's rule we get,

$$x = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 5 & 5 & 3 \\ 5 & 1 & 2 \\ 0 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & -1 \end{vmatrix}} = \frac{-25 + 55}{-10 + 33 + 7} = \frac{30}{30} = 1$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 2 & 5 & 3 \\ 3 & 5 & 2 \\ 1 & 0 & -1 \end{vmatrix}}{30} = \frac{-5 + 5}{30} = 0$$

$$z = \frac{|A_3|}{|A|} = \frac{\begin{vmatrix} 2 & 5 & 5 \\ 3 & 1 & 5 \\ 1 & 2 & 0 \end{vmatrix}}{30} = \frac{25 + 5}{30} = \frac{30}{30} = 1$$

$$\therefore x = 1, y = 0, z = 1$$

Chapter 2: Co-ordinate Geometry

Chapter at a glance:

2.1 Co-ordinate Geometry definition:

Coordinate geometry (or analytic geometry) is defined as **the study of geometry using the coordinate points**. Using coordinate geometry, it is possible to find the distance between two points, dividing lines in m:n ratio, finding the mid-point of a line, calculating the area of a triangle in the Cartesian plane, etc.

2.2 Co-ordinate Plane:

A cartesian plane divides the plane space into two dimensions and is useful to easily locate the points. It is also referred to as the coordinate plane. The two axes of the coordinate plane are the horizontal x-axis and the vertical y-axis. These coordinate axes divide the plane into four quadrants, and the point of intersection of these axes is the origin (0, 0). Further, any point in the coordinate plane is referred to by a point (x, y), where the x value is the position of the point with reference to the x-axis, and the y value is the position of the point with reference to the y-axis.

The properties of the point represented in the four quadrants of the coordinate plane are:

- The origin O is the point of intersection of the x-axis and the y-axis and has the coordinates (0, 0).
- The x-axis to the right of the origin O is the positive x-axis and to the left of the origin, O is the negative x-axis. Also, the y-axis above the origin O is the positive y-axis, and below the origin O is the negative y-axis.
- The point represented in the first quadrant (x, y) has both positive values and is plotted with reference to the positive x-axis and the positive y-axis.
- The point represented in the second quadrant is (-x, y) is plotted with reference to the negative x-axis and positive y-axis.
- The point represented in the third quadrant (-x, -y) is plotted with reference to the negative x-axis and negative y-axis.
- The point represented in the fourth quadrant (x, -y) is plotted with reference to the positive x-axis and negative y-axis.

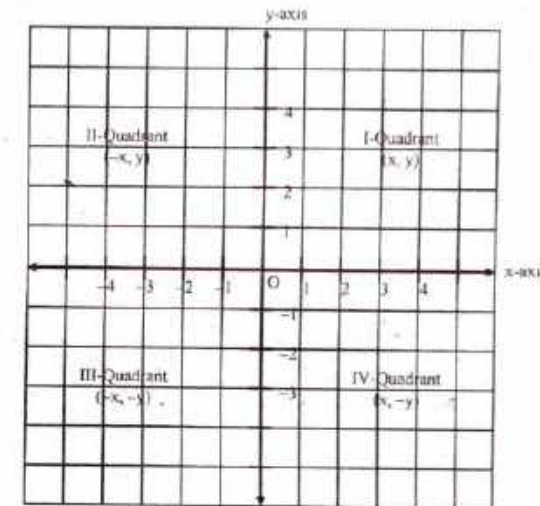


Fig: Co-ordinate plane

2.3 Co-ordinate Geometry Formulas:

1. Co-ordinate Geometry distance formula:

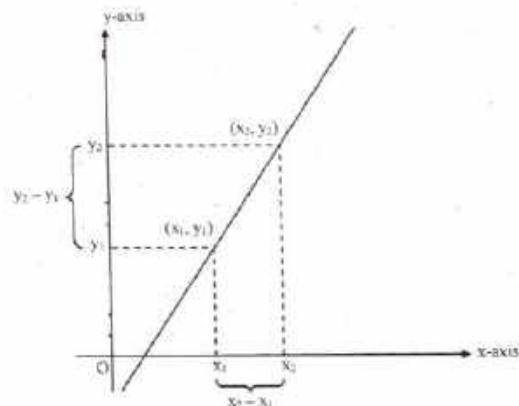
The distance between two points (x_1, y_1) and (x_2, y_2) is equal to the square root of the sum of the squares of the difference of the x-coordinates and the y-coordinates of the two given points. The formula for the distance between two points is as follows.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Slope formula:

The slope of a line is the inclination of the line. The slope can be calculated from the angle made by the line with the positive x-axis, or by taking any two points on the line. The slope of a line inclined at an angle θ with the positive x-axis is $m = \tan \theta$. The slope

of a line joining the two points (x_1, y_1) and (x_2, y_2) is equal to $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.



Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The curve $y = |x|$ is symmetric about
 (a) x-axis (b) y-axis (c) origin (d) none of these

Answer: (b)

2. If the equation of parabola is $3y^2 = 16x$ then directrix is
 (a) $3x + 4 = 0$ (b) $x + 2 = 0$ (c) $3x - 4 = 0$ (d) $4x + 3 = 0$

Answer: (c)

3. The eccentricity of the hyperbola $9x^2 - 16y^2 = 36$ is given by
 (a) $\frac{5}{4}$ (b) $\frac{-5}{4}$ (c) $\frac{-4}{5}$ (d) $\frac{4}{5}$

Answer: (a)

4. The gradient or slope of the curve $y = 3x^2 - 2x + 5$, when $x = 0$ is
 (a) 6 (b) 1 (c) 4 (d) -2

Answer: (d)

5. The eccentricity of ellipse $\frac{x^2}{12} + \frac{y^2}{4} = 1$ is

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Answer: (a)

6. The focus of the parabola $x^2 = 4by$ is

(a) (0, 0) (b) (0, b) (c) (b, 0) (d) none of these

Answer: (c)

7. The focus of $y^2 = 36x$ is

(a) (3, 0) (b) (9, 0) (c) (3, 9) (d) none of these

Answer: (b)

8. Parametric co-ordinates of the parabola $y^2 = 4ax$ is

(a) $(-at^2, -2at)$ (b) $(-at^2, 2at)$ (c) $(at^2, 2at)$ (d) none of these

Answer: (c)

9. The eccentricity of the ellipse $3x^2 + 4y^2 = 24$ is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Answer: (c)

10. The eccentricity of the ellipse $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{16} = 1$ is

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{-4}{5}$

Answer: (a)

11. The latus rectum of the parabola $y^2 = -16x$ is

(a) 4 (b) -4 (c) 16 (d) none of these

Answer: (d)

12. The curve $y = |x|$ is

[Model Question]

- (a) differentiable everywhere (b) discontinuous at $x=0$
(c) not derivable at $x=0$ (d) none of these

Answer: (d)

13. The directrix of the parabola $(y-3)^2 = 8a(x-5)$ is

[Model Question]

- (a) $x+a=5$ (b) $x-5=a$ (c) $x+2a=5$ (d) $x-5=2a$

Answer: (c)

14. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

[Model Question]

- (a) $\frac{4}{5}$ (b) $\frac{16}{25}$ (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

Answer: (a)

15. If e is the eccentricity of a hyperbola, then

[Model Question]

- (a) $e=0$ (b) $e=1$ (c) $e>1$ (d) $e<1$

Answer: (c)

16. The eccentricity of the rectangular hyperbola is:

[Model Question]

- (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) none of these

Answer: (c)

17. The length of latus rectum of the hyperbola $9x^2 - 25y^2 = 225$ is

[Model Question]

- (a) $\frac{9}{5}$ (b) $\frac{18}{5}$ (c) $\frac{10}{9}$ (d) $\frac{5}{9}$

Answer: (b)

18. If the straight line $y = mx + c$ be a tangent to the parabola $y^2 = 4ax$, then

[Model Question]

- (a) $c = m$ (b) $c^2 = m$ (c) $c = \frac{a}{m}$ (d) none of these

Answer: (c)

B. Answer the following questions:

19. The area of the region bounded by $y = 2x - x^2$ and $y = x$ is

[WBSCTE 2022]

Answer:

$$\begin{aligned} \text{Required area} &= \int_0^1 [(2x - x^2) - x] dx \\ &= \int_0^1 (x - x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{6} \text{ sq. unit.} \end{aligned}$$

20. Check the co-linearity of the points: (2,5), (0,0) and (-1,-2.5).

[WBSCTE 2022]

Answer:

(2,5) (0,0) (-1,-2.5)

$$AB = \sqrt{(0-2)^2 + (0-5)^2} = \sqrt{4+25} = \sqrt{29} = 2\sqrt{7}$$

$$BC = \sqrt{(-1-0)^2 + (-2.5-0)^2} = \sqrt{7.25}$$

$$AC = \sqrt{(-1-2)^2 + (-2.5-5)^2} = \sqrt{9+56.25} = \sqrt{65.25}$$

Since, $AB+BC = AC$

$$\therefore 2\sqrt{7} + \sqrt{7.25} = \sqrt{65.25}$$

$$\text{Or, } 7.98 (=8.00) = 8.07$$

$\therefore$ (2,5) (0,0) and (-1,-2.5) are co-linear.

21. Find the angle between the lines $x+2y = 5$ and $y = 2x$.

[WBSCTE 2022]

Answer:

$$x + 2y = 5$$

$$\Rightarrow 2y = -x + 5$$

$$\Rightarrow y = -\frac{x}{2} + \frac{5}{2} \dots (i)$$

$$y = 2x \dots (ii)$$

We know that equation of line is $y = mx + c \dots (iii)$ [Where m = slope of a line]

$$\text{Comparing (i) \& (ii) We get, } m = \left(-\frac{1}{2}\right)$$

$$\text{Comparing (ii) \& (iii) We get, } m = 2.$$

$$\therefore \text{ Multiplying the slopes of line 1 and 2, we get, } \left(-\frac{1}{2}\right) \times 2 = (-1)$$

$\therefore$ The angle of the given lines is 90° .

22. Find the length of the semi-latus rectum of hyperbola $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

[WBSCTE 2022]

Answer:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\therefore a = 5, b = 4$$

$$\begin{aligned} \therefore \text{Length of the semi-latus rectum of hyperbola} &= 2 \cdot \frac{b^2}{a} \\ &= 2 \cdot \frac{(4)^2}{5} \\ &= 2 \cdot \frac{16}{5} \\ &= \frac{32}{5} \end{aligned}$$

Long Answer Type Questions

1. A straight line forms a right a right angle triangle with the coordinate axis. If the area of triangle is 6 and the hypotenuse is 5, find the equation of the straight line.

[WBSCTE 2022]

Answer:

Here, $c = 5$ units.

$\therefore$ We know,

$$\text{Hypotenuse}^2 = \text{height}^2 + \text{base}^2$$

$$\Rightarrow c^2 = a^2 + b^2$$

$$\Rightarrow (5)^2 = a^2 + b^2$$

$$\boxed{\therefore a^2 + b^2 = 25} \quad \dots (i)$$

$$\frac{1}{2} \times \text{height} \times \text{base} = 2 \text{ square units.}$$

$$\Rightarrow \frac{1}{2} \times a \times b = 6$$

$$\boxed{\therefore a \times b = 12 \text{ units}} \quad \dots (ii)$$

On solving both equation, we get,

$$25 = \left[\frac{12}{b} \right]^2 + b^2$$

$$\Rightarrow 25 = \frac{144}{b^2} + b^2$$

$$\Rightarrow 25b^2 = 144 + b^4$$

$$\Rightarrow b^4 - 25b^2 + 144 = 0$$

$$\therefore \Rightarrow b^2 = \frac{25 \pm 13}{2}$$

$$\therefore b = \frac{25+13}{2} \text{ and } b = \frac{25-13}{2}$$

$$= 19 \text{ and } = 6$$

$\therefore$ The value of $b = 19, 6$.

Which means when b is 6 then a will be 19.

Hence, equation of line AB will be

$$\frac{x}{b} + \frac{y}{a} = 1$$

$$\Rightarrow \frac{x}{6} + \frac{y}{19} = 1$$

$$\boxed{\Rightarrow 19x + 6y = 114} \text{ This is the required equation of straight line.}$$

2. Find the volume of the solid generated by revolving; the area bounded by the curve $y = \sin x$, $x = 0$, $x = \frac{\pi}{2}$ about the x-axis.

[WBSCTE 2022]

Answer:

Area of a curve, $A = \pi \sin^2 x$

$$\text{Volume, } V = \int_0^{\frac{\pi}{2}} \pi \sin^2 x dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx = \pi \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} dx - \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{\pi}{2} x - \frac{\pi}{4} I.$$

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \cos 2x dx \quad [\text{Let, } u = 2x]$$

$$= \int \cos u \frac{du}{2} \quad \Rightarrow du = 2dx$$

$$= \frac{1}{2} \int \cos u du \quad \Rightarrow dx = \frac{du}{2}$$

$$\begin{aligned}
 &= \frac{1}{2} \sin u \\
 &= \frac{1}{2} \sin 2x \\
 \therefore V &= \frac{\pi}{2} x - \frac{\pi}{2} \left(\frac{1}{2} \sin 2x \right) \\
 &= \left[\frac{\pi}{4} (2x - \sin 2x) \right]_0^{\pi} \\
 &= \frac{\pi}{2} (2\pi + \sin \pi - \pi - \sin 2\pi) \\
 &= \frac{\pi}{4} (\pi - 0 + 0) = \frac{\pi^2}{4}
 \end{aligned}$$

3. A circle has radius 4 unit and its center lies on $x + y = 0$ and it passes through (1,1). Find its equation. [WBSCTE 2022]

Answer:

$$A(1,1) \text{ lies on the circle } \therefore r = |AC| = 2 = \sqrt{(h-1)^2 + (k-1)^2}$$

$$\Rightarrow 16 = (h-1)^2 + (k-1)^2$$

$$\therefore h^2 + k^2 - 2h - 2k - 14 = 0 \quad \dots (i)$$

Center $C(h,k)$ lie on the given line,

$$k = 3h + 7$$

Putting value of k in (i), we get

$$h^2 + (3h+7)^2 - 2h - 2(3h+7) - 14 = 0$$

$$\Rightarrow 10h^2 + 34h + 21 = 0$$

$$\therefore h = \frac{-34 \pm \sqrt{34^2 - 4(10)(21)}}{20}$$

$$= \frac{-34 \pm \sqrt{316}}{20}$$

$$= \frac{-17 \pm \sqrt{79}}{10}$$

$$\therefore k = 3 \left(\frac{-17 \pm \sqrt{79}}{10} \right) + 7$$

$$= \frac{-51 \pm 3\sqrt{79}}{10} + 7$$

$$= \frac{19 + 3\sqrt{79}}{10}$$

Now equation of circle with center at

$$\left(\frac{-17 + \sqrt{79}}{10}, \frac{19 + 3\sqrt{79}}{10} \right)$$

and radius 4 is

$$\left(x + \frac{17 - \sqrt{79}}{10} \right)^2 + \left(y - \frac{19 + 3\sqrt{79}}{10} \right)^2 = 16$$

And equation of circle with center at

$$\left(\frac{-17 - \sqrt{79}}{10}, \frac{19 - 3\sqrt{79}}{10} \right)$$

and radius 4 is

$$\left(x + \frac{17 + \sqrt{79}}{10} \right)^2 + \left(y - \frac{19 - 3\sqrt{79}}{10} \right)^2 = 16$$

4. If the straight line $3x + 4y = 1$ is a tangent to the parabola $y^2 = 4ax$, find the value of a . [Model Question]

Answer:

If $3x + 4y = 1$ is a straight line tangent with $y^2 = 4ax$ then the coordinate of the point of tangency will satisfy both equations. Substituting $y = \frac{1-3x}{4}$ in $y^2 = 4ax$ we get -

$$\frac{(1-3x)^2}{4} = 4ax \quad \text{or, } 1 + 9x^2 - 6x = 64ax \quad \text{or, } 9x^2 - (6 + 64a)x + 1 = 0$$

It is a quadratic equation of x . Here equation has two roots. If it is a tangent, then roots are equal. Condition of roots being equal are, $b^2 - 4ac = 0$

Here $b = -(6 + 64a)$, $a = 9$, $c = 1$

$$\therefore [-(6 + 64a)]^2 - 4 \times 9 \times 1 = 0$$

$$\text{or, } [-(6 + 64a)]^2 = 6^2 \quad \text{or, } -(6 + 64a) = 6$$

$$\text{or, } -64a = +12 \quad \text{or, } a = \frac{12}{64} = \frac{3}{16}$$

[Alternatively, condition of an equation becoming tangent of a parabola is $c = \frac{a}{m}$. Here

$$m = -\frac{3}{4}, c = \frac{1}{4}$$

$$\therefore \frac{1}{4} = \frac{a}{-3/4} \quad \text{or, } 4a = -\frac{3}{4} \quad \text{or, } a = -\frac{3}{16}$$

5. Find the equation of an ellipse for which the principal axes are along the coordinate axis, the length of latus rectum is 4 unit and eccentricity is $\frac{1}{3}$.

Answer:

[Model Question]

$$\text{Length of latus rectum} = \frac{2b^2}{a} = 4 \quad \dots (1)$$

$$\text{Eccentricity } (e) = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{1}{3} \quad \dots (2)$$

$$\text{From (1), } a = \frac{2b^2}{4} = \frac{b^2}{2}. \text{ Substitute in (2)}$$

$$\sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{3}$$

$$\text{or, } \frac{b^2}{a^2} = 1 - \frac{1}{9} = \frac{8}{9} \quad \text{or, } a^2 = \frac{8}{9b^2}$$

$$\text{or, } \frac{b^4}{4} = \frac{8}{9b^2} \quad \text{or, } 9b^6 = 32$$

$$\text{or, } b^4 = \frac{32}{9}$$

$$\therefore b^2 = \pm \sqrt{\frac{32}{9}} = \pm \frac{4\sqrt{2}}{3}$$

$$\therefore a^2 = \frac{8}{9 \left(\frac{4\sqrt{2}}{3} \right)} = \frac{8}{12\sqrt{2}} = \frac{2}{3\sqrt{2}}$$

$\therefore$ Equation of ellipse is -

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{or, } \frac{x^2}{\frac{2}{3\sqrt{2}}} + \frac{y^2}{\frac{4\sqrt{2}}{3}} = 1 \quad \text{or, } \frac{3\sqrt{2}x^2}{2} + \frac{3y^2}{4\sqrt{2}} = 1$$

$$\text{or, } \frac{3x^2}{\sqrt{2}} + \frac{3y^2}{4\sqrt{2}} = 1 \quad \text{or, } 12x^2 + 3y^2 = 4\sqrt{2}$$

6. The curve $y = Ax + Bx^2$ passes through the point $(1, 0)$ and its gradient at the point is $\frac{1}{2}$. Find the values of A and B .

[Model Question]

Answer:

$$y = Ax + Bx^2$$

$$\therefore \text{Slope} = \frac{dy}{dx} = A + 2Bx = \frac{1}{2}$$

$$\text{or, } A + 2B(1) = \frac{1}{2}$$

$$\text{or, } 2A + 4B = 1 \quad \dots (1)$$

Point $(1, 0)$ is on the curve. So coordinates of this point will satisfy equation.

$$y = A(1) + B(1)^2$$

$$\text{or, } A + B = 0 \quad \dots (2)$$

Substituting $A = -B$ in equation (1)

$$2A + 4B = 1$$

$$\text{or, } 2(-B) + 4B = 1$$

$$\text{or, } 2B = 1$$

$$\text{or, } B = \frac{1}{2}$$

$$\therefore A = -\frac{1}{2}$$

7. Find the equation of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the axis of x and the straight line $\frac{x}{3} + \frac{y}{5} = 1$ on the axis of y and whose axis lie along the axes of co-ordinates. Determine the eccentricity and the co-ordinates of the foci of the ellipse.

[Model Question]

Answer:

Since axes of ellipse lie along the axes of coordinates, the equation of ellipse will be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Further ellipse is meeting straight line $\frac{x}{7} + \frac{y}{2} = 1$ at x axis where $y = 0$.

$$\text{So, } \frac{x}{7} = 1 \quad \text{or, } x = 7.$$

Substituting $(7, 0)$ in the equation of ellipse -

$$\frac{7^2}{a^2} + \frac{0}{b^2} = 1$$

or, $a^2 = 49$ also ellipse is meeting $\frac{x}{3} + \frac{y}{5} = 1$ line at y axis where $x = 0$. So, $y = 5$.

Substituting (0,5) in ellipse equation we get,

$$\frac{0}{a^2} + \frac{5^2}{b^2} = 1$$

$$\text{or, } b^2 = 25$$

$$\therefore \text{Eccentricity } = e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{49}} = \sqrt{\frac{24}{49}} = \frac{2\sqrt{6}}{7}$$

$$\text{Foci} = (\pm ae, 0) = \left(\pm 7 \times \frac{2\sqrt{6}}{7}, 0 \right) = (\pm 2\sqrt{6}, 0)$$

8. Find the equation of the normal to the parabola $y^2 = 3x$ which is perpendicular to $y = 2x + 4$. [Model Question]

Answer:

Here normal is perpendicular to $y = 2x + 4$ which must be tangent with parabola. Let the coordinate of the tangency point is (x_1, y_1) and 'm' is the slope of normal curve where

$$m = -\frac{y_1}{2a}$$

Here $m = 2$ [Since equation of perpendicular is $y = 2x + 4$]

and $a = \frac{3}{4}$ [Since equation of parabola is $y^2 = 3x$, therefore $4a = 3$]

$$\therefore m = -\frac{y_1}{2a}$$

$$\text{or, } -\frac{y_1}{3/4} = 2 \quad \text{or, } -y_1 = 2 \times \frac{3}{4} \quad \text{or, } y_1 = -\frac{3}{2}$$

Substituting $y_1 = -\frac{3}{2}$ in the equation $y = 2x + 4$

$$x = \frac{y-4}{2} = \frac{-\frac{3}{2}-4}{2} = -\frac{11}{2} \times \frac{1}{2} = -\frac{11}{4} = x_1$$

$\therefore$ Equation of normal is -

$$(y - y_1) = m(x - x_1)$$

$$\text{or, } \left(y + \frac{3}{2} \right) = 2 \left(x + \frac{11}{4} \right) \quad \text{or, } y + \frac{3}{2} = 2x + \frac{11}{2}$$

$$\text{or, } y = 2x + \frac{8}{2}$$

$$\text{or, } y = 2x + 4.$$

9. Find the co-ordinates of the vertex, length of the latus rectum and the equation of the directrix of the parabola $x^2 + 4x + 2y - 11 = 0$. [Model Question]

Answer:

$$x^2 + 4x + 2y - 11 = 0$$

$$\text{or, } (x^2 - 2(x)(-2) + (-2)^2) + 2y - 15 = 0$$

$$\text{or, } (x - 2)^2 = (-2y + 15)$$

$$\text{or, } (x - 2)^2 = -2 \left(y - \frac{15}{2} \right)$$

$$\text{or, } (x - 2)^2 = 4 \left(-\frac{1}{2} \right) \left(y - \frac{15}{2} \right)$$

It is a parabola equation of

$$(x - \alpha)^2 = -4a(y - \beta) \text{ type}$$

$$\therefore \alpha = 2, \beta = \frac{15}{2} \text{ and } a = \frac{1}{2}$$

$$\therefore \text{vertex } (\alpha, \beta) = 2, \frac{15}{2}$$

$$\text{Length Latus rectum} = 4a = 4 \times \frac{1}{2} = 2$$

$$\text{Equation of directrix} \Rightarrow y - a = \beta \quad \text{or, } y - \frac{1}{2} = \frac{15}{2} \quad \text{or, } y = 8$$

10. If the area of the circle increases at a uniform rate, show that the rate of increase of the circumference of the circle varies inversely as the radius. [Model Question]

Answer:

$$\text{Area of circle} = \pi r^2 \quad r = \text{radius}$$

$$\text{Change in area of circle} = \frac{d\pi r^2}{dr} = 2\pi r = a(\text{constant}) \quad \dots (1)$$

$$\text{Change in circumference} = \frac{d(2\pi r)}{dr} = 2\pi = \frac{a}{r} \text{ (from (1))} \quad [\text{Circumference} = 2\pi r]$$

Equation (2) indicates that as area of a circle increases at a uniform rate 'a', circumference will increase at $\frac{a}{r}$. So rate of change in circumference is inversely related with 'r'.

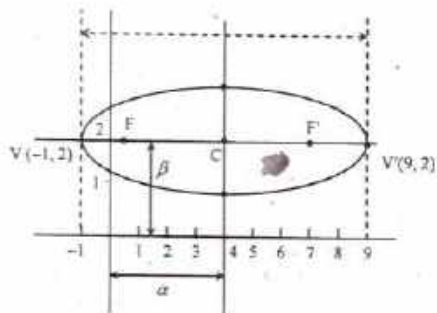
11. The vertices of an ellipse are $(-1, 2)$ and $(9, 2)$. If the distance between its foci be 8, find the equation of the ellipse. [Model Question]

Answer:

Since coordinates of two vertices are $(-1, 2)$ and $(9, 2)$, therefore coordinates of centre 'c' will be,

$$\left(\frac{-1+9}{2}, \frac{2+2}{2} \right) = (4, 2)$$

If coordinate of centre 'c' is (α, β) then, $\alpha = a, \beta = 2$



Distance between two vertex is major axis. It is $2a$.

$$\therefore 2a = 10 \text{ or, } a = 5$$

Further, distance between Foci is $2ae$.

$$\therefore 2ae = 8 \text{ or, } ae = 4 \text{ or, } e = \frac{4}{5}$$

Since eccentricity $(e) = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$

$$\therefore \sqrt{1 - \frac{b^2}{a^2}} = \frac{4}{5}$$

$$\text{or, } \frac{b^2}{25} = 1 - \frac{16}{25} = \frac{9}{25} \therefore b = 3$$

$\therefore$ Equation of ellipse is -

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$$

$$\text{or, } \frac{(x-4)^2}{5^2} + \frac{(y-2)^2}{3^2} = 1$$

$$\text{or, } \frac{x^2 - 8x + 16}{25} + \frac{y^2 - 4y + 4}{9} = 1$$

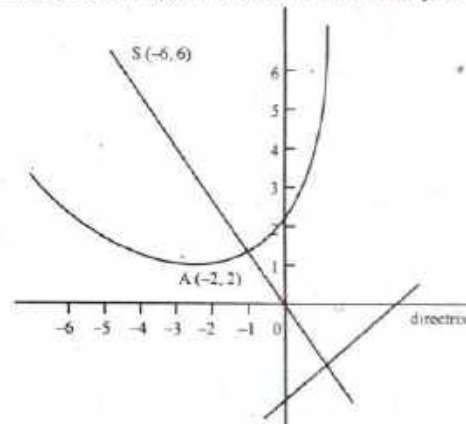
$$\text{or, } 9x^2 - 72x + 144 + 25y^2 - 100y + 100 = 225$$

$$\text{or, } 9x^2 - 72x + 25y^2 - 100y = -19$$

12. Find the equation of the parabola whose vertex is $(-2, 2)$ and focus is $(-6, 6)$. [Model Question]

Answer:

Consider above diagram. Focus is point S $(-6, 6)$ and vertex is point A $(-2, 2)$. Line XA is extended further and a point Z (α, β) is taken on the extended part where $AZ = AS$.



Since A is the mid point of line ZS, $-2 = \frac{-6 + \alpha}{2}$ and $2 = \frac{6 + \beta}{2}$

Therefore, $\alpha = 6 - 4 = 2$ and $\beta = -6 + 4 = -2$

Equation passing through point $(2, -2)$ is -

$$(y - \beta) = m(x - \alpha)$$

$$\text{or, } (y + 2) = m(x - 2)$$

Line of this equation is directrix when it is perpendicular at Z. Here slope,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{2 - (-2)} = \frac{-4}{4} = -1$$

Since directrix is perpendicular, its slope will be -

$$m \times (-1) = -1 \text{ i.e. } m = 1$$

Therefore equation of the directrix is -

$$(y + 2) = (x - 2)$$

$$\text{or } x - y = 4$$

Now we consider a point $P(x, y)$ on parabola. Then equation of the parabola will be,

$$(x-\alpha)^2 + (y-\beta)^2 = \left\{ \frac{Ax+By+C}{\sqrt{A^2+B^2}} \right\}^2$$

Here $Ax+By+C=0$ is the equation of directrix. Therefore
 $A=1, B=-1, C=-4, \alpha=2$ and $\beta=-2$.

$$\text{Therefore, } (x-2)^2 + (y-(-2))^2 = \left\{ \frac{x-y-4}{\sqrt{1^2+(-1)^2}} \right\}^2$$

$$\text{or } x^2 - 4x + 4 + y^2 + 4y + 4 = \frac{(x-y-4)^2}{2}$$

$$\text{or } 2x^2 - 8x + 8 + 2y^2 + 8y + 8 = x^2 - xy - 4x - xy + y^2 + 4y - 4x + 4y + 16$$

$$\text{or } x^2 + y^2 + 2xy = 0 \quad \text{equation of parabola.}$$

13. Find the area above the x-axis bounded by $x-2y+4=0$, $x=1$ and $x=9$.

[Model Question]

Answer:

$$x-2y+4=0$$

$$\text{or, } 2y = x+4$$

$$\text{or, } y = \frac{x}{2} + 2$$

Area above x axis bounded by equation (1), $x=1$ and $x=9$ is

$$\begin{aligned} \int_1^9 \left(\frac{x}{2} + 2 \right) dx &= \int_1^9 \left(\frac{x}{2} \right) dx + \int_1^9 (2) dx = \frac{1}{2} \int_1^9 (x) dx + 2 \int_1^9 (1) dx \\ &= \frac{1}{2} \left(\frac{x^2}{2} \right)_1^9 + 2(x)_1^9 = \frac{1}{2} \left(\frac{9^2}{2} - \frac{1^2}{2} \right) + 2(9-1) \\ &= 20 + 16 = 36 \text{ square unit.} \end{aligned}$$

14. Find the focus, vertex and directrix of the parabola $y^2=4y-4x$.

[Model Question]

Answer:

Consider the equation of parabola-

$$y^2 = 4y - 4x$$

$$\text{or, } (y^2 - 4y + 4) = -4x + 4$$

$$\text{or, } (y-2)^2 = -4(x-1)$$

From the above equation we get-

$$\alpha=1, \beta=2 \text{ and } a=-1$$

Therefore,

$$\text{Focus } (a+\alpha, \beta) = [(-1+1), 2] = [0, 2]$$

$$\text{Vertex } (a, \beta) = (1, 2)$$

$$\text{Directrix, } x+a=\alpha$$

$$x-1=1 \quad \therefore x=2$$

15. Find the equation of the hyperbola whose vertices are (2, 3), (8, 3) and the distance between foci is 12. [Model Question]

Answer:

When centre of hyperbola is not (0, 0) the equation is,

$$\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1, \text{ where } (\alpha, \beta) \text{ is the coordinate of the centre. Since vertex are}$$

(2, 3) and (8, 3), therefore coordinates of the centre is $(2+8)/2$, 3 i.e., (5, 3).

Therefore, $\alpha=5, \beta=3$.

Distance between centre and vertex is the value of 'a'. Therefore $a=8-5=3$.

Distance between foci is $2ae=12$

$$\therefore 2 \times 3 \times e = 12$$

$$\therefore e=2$$

$$\text{Now, eccentricity } e^2 = 1 + \frac{b^2}{a^2}$$

$$\therefore 1 + \frac{b^2}{3^2} = 2^2$$

$$\therefore b^2 = (4-1) \times 9 = 27$$

$$\text{Therefore, equation of hyperbola is, } \frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1 \text{ or, } \frac{(x-5)^2}{9} - \frac{(y-3)^2}{27} = 1$$

$$\therefore 3x^2 - y^2 - 30x + 6y + 39 = 0$$

16. Find the points on the curve $y = x + \frac{1}{x}$ at which the tangents to the curve are parallel to x-axis. [Model Question]

Answer:

$$\text{We have, } y = x + \frac{1}{x}$$

$$\text{or, } \frac{dy}{dx} = 1 - \frac{1}{x^2} \quad \dots (1)$$

Since $\frac{dy}{dx}$ represents the slope of the tangent at any point (x, y) . Now the tangent will be parallel to x-axis if $\frac{dy}{dx} = 0$.

Therefore, $\frac{dy}{dx} = 0$ gives, $1 - \frac{1}{x^2} = 0$

or, $x = \pm 1$.

Now at $x = 1, y = 2$ and at $x = -1, y = -2$

So at $(1, 2)$ and $(-1, -2)$ the tangents of the curve $y = x + \frac{1}{x}$ are parallel to x-axis.

17. Find the coordinates of the foci of the ellipse $3x^2 + 4y^2 = 12$. [Model Question]

Answer:

(a) We have $3x^2 + 4y^2 = 12$

$$\text{or, } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\text{or, } \frac{x^2}{(2)^2} + \frac{y^2}{(\sqrt{3})^2} = 1 \quad \dots (1)$$

Hence coordinates of the foci of the given ellipse are $(2, 0)$ and $(-2, 0)$.

18. Find the equation of the ellipse whose latus rectum is 5 and whose eccentricity is $\frac{2}{3}$, the axes of the ellipse being the coordinate axes.

[Model Question]

Answer:

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (1)

So according to given conditions:

$$(i) \quad \frac{2b^2}{a} = 5$$

$$\text{or, } b^2 = \frac{5a}{2} \quad \dots (2)$$

$$\text{and (ii) } \sqrt{1 - \frac{b^2}{a^2}} = \frac{2}{3}$$

$$\text{or, } \sqrt{1 - \frac{5a}{2a^2}} = \frac{2}{3} \quad [\text{using (2)}]$$

$$\text{or, } 1 - \frac{5}{2a} = \frac{4}{9} \quad \text{or, } \frac{5}{2a} = 1 - \frac{4}{9}$$

$$\text{or, } \frac{5}{2a} = \frac{5}{9} \quad \text{or, } a = \frac{9}{2}$$

$\therefore$ From equation (2),

$$b^2 = \frac{5}{2} \times \frac{9}{2} \quad \text{or, } b^2 = \frac{45}{4}$$

Hence required equation of the ellipse is

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1 \quad \text{or, } \frac{x^2}{9} + \frac{y^2}{5} = \frac{9}{4}$$

19. Find the latus rectum and the co-ordinates of the foci of the equilateral hyperbola $x^2 - y^2 = 25$. [Model Question]

Answer:

We have $x^2 - y^2 = 25$

$$\text{or, } \frac{x^2}{5^2} - \frac{y^2}{5^2} = 1$$

So the length of the latus rectum of the given hyperbola is $\frac{2 \times 5^2}{5} = 10$.

Coordinates of the foci are $\left(\pm 5\sqrt{1 + \frac{5^2}{5^2}}, 0 \right) = (\pm 5\sqrt{2}, 0)$.

20. Find the equation of the parabola whose focus is at the point $(3, -2)$ and directrix is the line $2x - y + 3 = 0$. [Model Question]

Answer:

Let us assume a point P with coordinates (x, y) on parabola. Its focus is $(3, -2)$ at point p

$$\therefore \text{distance } PS = \sqrt{(x-3)^2 + (y+2)^2} \dots (1)$$

Now equation of Directrix is $2x - y + 3 = 0$.

$$\text{So } A = 2, B = -1 \quad \dots (1)$$

$$\frac{Ax^2 + By + c}{\sqrt{A^2 + B^2}} = \frac{2x - y + 3}{\sqrt{2^2 + (-1)^2}} \dots (2)$$

Since in parabola distance (1) and distance (2) are same, therefore,

$$\sqrt{(x-3)^2 + (y+2)^2} = \frac{2x - y + 3}{\sqrt{4+1}}$$

$$\text{or, } (x-3)^2 + (y+2)^2 = \frac{(2x-y+3)^2}{5}$$

$$\text{or, } 5[x^2 - 6x + 9 + y^2 + 4y + 4] = [4x^2 - 2xy + 6x - 2xy + y^2 - 3y + 6x - 3y + 9]$$

$$\text{or, } 5x^2 - 30x + 45 + 5y^2 + 20y + 20 = 4x^2 - 2xy + 6x - 2xy + y^2 - 3y + 6x - 3y + 9$$

$$\text{or, } x^2 - 42x + 4y^2 + 26y + 56 + 4xy = 0$$

$$\text{or, } (x^2 + 4xy + 4y^2) - 42x + 26y + 56 = 0$$

$$\text{or, } (x+2y)^2 = 42x - 26y - 56$$

21. Prove that the straight line $3x+5y=12$ is not a tangent to the ellipse $3x^2+4y^2=16$. [Model Question]

Answer:

We know that the condition of tangency for the line $y=mx+c$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{is } c = \pm \sqrt{a^2 m^2 + b^2}.$$

$$\text{Here, } a^2 = \frac{16}{3}, b^2 = \frac{16}{4}, m = -\frac{3}{5}, c = \frac{12}{5}.$$

Now according to the condition:

$$c = \pm \sqrt{\frac{16}{3} \times \frac{9}{25} + \frac{16}{4}} = \pm \sqrt{\frac{48}{25} + 4} = \pm \frac{\sqrt{148}}{5} \neq \frac{12}{5}.$$

Hence the given line will not be a tangent to the ellipse.

22. Find the equation of the parabola whose focus is (2, 1) and directrix is $3x-y+1=0$. [Model Question]

Answer:

Let $P(\alpha, \beta)$ be any point on the required parabola.

We know, for the parabola; $PS = PM$, where S denotes the focus and M is the Foot of $\perp$ from P to the directrix.

$$\text{or, } (\alpha-2)^2 + (\beta-1)^2 = \left(\frac{3\alpha-\beta+1}{\sqrt{9+1}} \right)^2$$

$$\text{or, } 10[\alpha^2 - 4\alpha + \beta^2 - 2\beta + 5] = (3\alpha - \beta + 1)^2$$

$$\text{or, } \alpha^2 + 9\beta^2 - 6\alpha\beta - 46\alpha - 18\beta + 49 = 0$$

Hence the equation of the parabola is $x^2 - 6xy + 9y^2 - 46x - 18y + 49 = 0$

23. Find the directrix of the parabola whose focus is (3, 4) and vertex (0, 0). [Model Question]

Answer:

Let the equation of the parabola whose vertex is at (0, 0) be

$$y^2 = 4ax \dots (1)$$

Since the parabola passing through the point (3, 4) therefore,

$$16 = (4a).3$$

$$\text{or, } 16 = 12a$$

$$\text{or, } a = \frac{16}{12} = \frac{4}{3}$$

So the equation of the parabola is $y^2 = 3x$

Therefore, the equation of the directrix is $x + \frac{4}{3} = 0$ i.e. $3x + 4 = 0$

24. Find the equation of the normal to the curve $y = 2x^3 - 3x + 5$ at the point (1, 4). [Model Question]

Answer:

We have, $y = 2x^3 - 3x + 5$

$$\therefore \frac{dy}{dx} = 6x^2 - 3$$

$$\therefore \frac{dy}{dx} \Big|_{(1,4)} = 6 - 3 = 3$$

So the equation of the normal, at (1, 4) is $(y-4) = -\frac{1}{3}(x-1)$

$$\text{or, } 3(y-4) + x - 1 = 0$$

$$\text{or, } x + 3y = 13$$

25. a) If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through (-3, 2) and its eccentricity be $\sqrt{\frac{3}{5}}$ then find the length of latus rectum. [Model Question]

Answer:

If ellipse passes through (-3, 2) then the coordinate of the point will satisfy the equation of ellipse. So substituting $x = -3$ and $y = 2$ in the equation of ellipse,

$$\text{Now eccentricity } (e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{3}{5}}$$

$$\therefore 1 - \frac{b^2}{a^2} = \frac{3}{5}$$

$$\text{or, } \frac{b^2}{a^2} = 1 - \frac{3}{5} = \frac{2}{5} \dots (1)$$

Further ellipse is passing through $(-3, 2)$. Therefore equation of ellipse is satisfied by the co-ordinates of this point.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{or, } \frac{(-3)^2}{a^2} + \frac{(-2)^2}{b^2} = 1$$

$$\text{or, } \frac{9}{a^2} + \frac{4}{b^2} = 1 \quad \dots(2)$$

Substituting $b^2 = \frac{2a^2}{5}$ of equation (1) and (2),

$$\frac{9}{a^2} + \frac{4}{\frac{2a^2}{5}} = 1 \quad \text{or, } \frac{9}{a^2} + \frac{20}{2a^2} = 1 \quad \text{or, } \frac{9}{a^2} + \frac{10}{a^2} = 1$$

$$\text{or, } \frac{19}{a^2} = 1 \quad \text{or, } a^2 = 19$$

$$\therefore b^2 = \frac{2a^2}{5} = \frac{2 \times 19}{5} = \frac{38}{5}$$

$$\therefore \text{Length of latus rectum} = \frac{2b^4}{a} = 2 \times \frac{38}{9} \times \frac{1}{\sqrt{19}} = \frac{4\sqrt{19}}{5}$$

26. b) If the centre, vertex and eccentricity of a hyperbola be $(2, 4)$, $(6, 4)$ and $\sqrt{5}$, then find its equation. [Model Question]

Answer:

When centre of the ellipse is not $(0, 0)$, its equation is $\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$ where

(α, β) is the coordinate of hyperbola.

Since coordinate of centre is $(2, 4)$, therefore $\alpha = 2$ and $\beta = 4$

Further vertex is $(6, 4)$. It means distance between centre and vertex is 4. So $a = 6 - 4$

Finally eccentrically, $e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{5}$

$$\therefore \frac{a^2 + b^2}{a^2} = 5$$

$$\text{or, } 1 + \frac{b^2}{a^2} = 5$$

$$\text{or, } \frac{b^2}{a^2} = 4$$

$$\therefore b^2 = 4(4)^2 = 64$$

Therefore, equation of ellipse is $\frac{(x-2)^2}{4^2} - \frac{(y-4)^2}{64} = 1$

$$\text{or, } 4(x^2 - 4x + 4) - (y^2 - 8y + 16) = 64$$

$$\text{or, } 4x^2 - 16x + 16 - y^2 + 8y - 16 = 64$$

$$\text{or, } 4x^2 - 16x - 8y - y^2 - 64 = 0$$

26. a) The equation of directrix of a hyperbola is $x - y + 3 = 0$. Its focus is $(-1, 1)$ and eccentricity is 3. Find the equation of the hyperbola. [Model Question]

Answer:

Let us assume P is a point on the hyperbola with coordinate (x, y) . Distance of focus $(-1, 1)$ from $P(x, y)$ is

$$= \sqrt{(x+1)^2 + (y-1)^2} = \sqrt{x^2 + 2x + 1 + y^2 - 2y + 1}$$

$$= \sqrt{x^2 + y^2 + 2x - 2y + 2} \quad \dots(1)$$

Also distance from directrix $x - y + 3$ and point $P(x, y)$ is -

$$= \frac{x - y + 3}{\sqrt{1^2 + 1^2}}$$

$$= \frac{x - y + 3}{\sqrt{2}} \quad \dots(2)$$

Since, $\frac{\text{Distance 1}}{\text{Distance 2}} = e = 3$

$$\therefore \frac{\sqrt{x^2 + y^2 + 2x - 2y + 2}}{\frac{x - y + 3}{\sqrt{2}}} = 3$$

$$\text{or, } \frac{\sqrt{x^2 + y^2 + 2x - 2y + 2}}{x - y + 3} = \frac{3}{\sqrt{2}}$$

$$\text{or, } \frac{x^2 + y^2 + 2x - 2y + 2}{x^2 - xy + 3x - xy + y^2 - 3y + 3x - 3y + 9} = \frac{9}{2}$$

$$\text{or, } 2(x^2 + y^2 + 2x - 2y + 2) = 9(x^2 + y^2 - 2xy + 6x - 6y + 9)$$

$$\text{or, } 7x^2 + 7y^2 - 18xy + 50x - 50y - 77 = 0 \quad \text{Equation of hyperbola}$$

b) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = x\phi\left(\frac{y}{x}\right)$ [Model Question]

Answer:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(x\phi\left(\frac{y}{x}\right) \right) + \frac{\partial}{\partial x} \left(\psi\left(\frac{y}{x}\right) \right) \cdot x + \frac{\partial}{\partial x} \psi\left(\frac{y}{x}\right)$$

$$= \phi\left(\frac{y}{x}\right) + x\phi'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + \psi'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right)$$

$$\begin{aligned} \therefore \frac{\partial u}{\partial y} &= x\phi'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) + \psi'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) \\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} &= x\phi\left(\frac{y}{x}\right) + x^2\phi'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + x\psi'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + x\phi'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) + y\cdot\frac{1}{x}\psi'\left(\frac{y}{x}\right) \\ &= x\phi\left(\frac{y}{x}\right) - y\phi'\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2\psi'\left(\frac{y}{x}\right) + y\phi'\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\psi'\left(\frac{y}{x}\right) \\ &= x\phi\left(\frac{y}{x}\right) \text{ proved} \end{aligned}$$

27. In the hyperbola $9x^2 - 4y^2 + 54x + 16y + 29 = 0$, find (i) co-ordinates of centre, (ii) length of latus rectum, (iii) eccentricity, (iv) co-ordinates of foci.

[Model Question]

Answer:

$$9x^2 - 4y^2 + 54x + 16y + 29 = 0$$

$$\text{or, } (9x^2 + 2 \cdot 9 \cdot 3x + 81) - (4y^2 - 2 \cdot 4 \cdot 2y + 16) - 36 = 0$$

$$\text{or, } (3x + 9)^2 - (2y - 4)^2 = 36$$

$$\text{or, } 9(x + 3)^2 - 4(y - 2)^2 = 36$$

$$\text{or, } \frac{(x+3)^2}{\frac{36}{9}} - \frac{(y-2)^2}{\frac{36}{4}} = 1 \quad \text{or, } \frac{(x+3)^2}{4} - \frac{(y-2)^2}{9} = 1$$

Let us assume $x + 3 = X$ and $y - 2 = Y$ where X and Y are axes of hyperbola, then coordinates of centre will be $(-3, 2)$.

Now equation of hyperbola is -

$$\frac{X^2}{4} - \frac{Y^2}{9} = 1$$

$$\text{Here } a^2 = 4, b^2 = 9$$

$$\therefore \text{Eccentricity } e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{4 + 9}{4}} = \sqrt{\frac{13}{4}}$$

$\therefore$ Co-ordinate of Focus (considering X and Y as axes)

$$= (\pm ae, 0) = \left(\pm 2 \cdot \sqrt{\frac{13}{4}}, 0 \right) = (\pm \sqrt{13}, 0)$$

$\therefore$ Co-ordinate of Focus (considering x and y as axes)

$$(-3 \pm \sqrt{13}, 0)$$

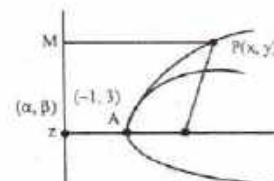
$$\begin{aligned} X &= \pm \sqrt{13} \\ \text{or, } x + 3 &= \pm \sqrt{13} \\ \therefore x &= -3 \pm \sqrt{13} \end{aligned}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \cdot 9}{2} = 9$$

28. a) Find the equation of the parabola whose vertex is $(-1, 3)$ and the focus is $(3, -1)$.

[Model Question]

Answer:



Here $A(-1, 3)$ is vertex and $S(3, -1)$ is focus. As line is extended upto ZA , so that $ZA = AS$. Let coordinate of point Z is (α, β) .

Here A is the mid point of the line ZS .

$$\therefore \frac{\alpha + 3}{2} = -1 \quad \therefore \alpha = -1 \times 2 - 3 = -5$$

$$\frac{\beta - 1}{2} = 3 \quad \therefore \beta = 3 \times 2 + 1 = 7$$

Now equation of a straight line passing through point Z is

$$(y - \beta) - m(x - \alpha)$$

$$\text{or, } (y - 7) = m(x + 5) \quad \dots (1)$$

Here equation (1) will be directrix only where it is perpendicular at point 'Z' with line AZ .

$$\text{Here slope of } AZ \text{ line is } = \frac{3 - \alpha}{-1 - \beta} = \frac{3 - (-5)}{-1 - 7} = \frac{2}{-8} = -\frac{1}{4}$$

$$\therefore (\text{Slope of directrix}) (\text{slope of } AZ \text{ line}) = -1$$

$$\text{or, } m \times -\frac{1}{4} = -1 \quad \text{or, } m = 4$$

$$\therefore \text{Equation of directrix is } (y - 7) = 4(x + 5)$$

$$\text{or, } y = 4x + 27$$

Now consider a point $P(x, y)$ as the parabola. Then $PS^2 = PM^2$

$$(x-3)^2 + (y+1)^2 = \left(\frac{4x-y+27}{\sqrt{2}}\right)^2$$

$$2(x^2 - 6x + 9 + y^2 + 2y + 1) = (16x^2 - 4xy + 108x - 4xy + y^2 - 27y + 108x - 27y + 729)$$

$$\text{or, } 2x^2 - 12x + 18 + 2y^2 + 4y + 2 = 16x^2 - 8xy + 216x + y^2 - 54y + 729$$

$$\text{or, } 14x^2 - 8xy + 228x - y^2 - 58y + 709 = 0 \quad \text{Equation of parabola}$$

- ☉ b) Taking the major and minor axes as the axes of the co-ordinates, find the equation of the ellipse whose length of latus rectum is $\frac{32}{5}$ units and co-ordinates of one focus are $(3, 0)$.

[Model Question]

Answer:

$$(1) \text{ Latus rectum} = \frac{2b^2}{a} = \frac{32}{5}$$

$$\text{or, } \frac{b^2}{a} = \frac{16}{5}$$

$$\therefore 5b^2 = 16a$$

$$(2) \text{ Focus } (ae, 0) = (3, 0)$$

$$\text{Here } ae = 3$$

$$\text{Since } e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\therefore a\sqrt{1 + \frac{b^2}{a^2}} = 3$$

$$\text{or, } a \times \frac{a^2 + b^2}{a^2} = 9$$

$$\therefore a^2 + b^2 = 9$$

.... (ii)

$$\text{Substituting } b^2 = \frac{16a}{5} \text{ in (ii)}$$

$$a^2 - \frac{16a}{5} = 9$$

$$\text{or, } 5a^2 - 16a = 45$$

$$\text{or, } 5a^2 - 25a + 9a - 45 = 0$$

$$\text{or, } 5a^2 - 16a - 45 = 0$$

$$\text{or, } 5a(a-5) + 9(a-5) = 0$$

$$\therefore a = 5 \text{ or, } -\frac{9}{5}$$

$$\text{Substituting } a = 5 \text{ in equation (ii)}$$

$$a^2 - b^2 = 9$$

$$\text{or, } 25 - b^2 = 9$$

$$\text{or, } b^2 = 16$$

$$\text{or, } b = \pm 4$$

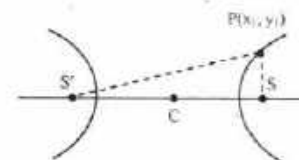
$\therefore$ Equation of ellipse is -

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or, } \frac{x^2}{25} + \frac{y^2}{16} = 1.$$

- ☉ c) If S and S' are the foci and P be any point on the hyperbola $x^2 - y^2 = a^2$, prove that $SP \cdot S'P = CP^2$, where C is the centre of the hyperbola.

[Model Question]

Answer:



Let us consider a hyperbola with Foci S, S' and C centre of hyperbola. P is a point as the hyperbola with coordinate (x_1, y_1) .

We have to prove that $PS \times PS' = CP^2$

Co-ordinate of point $S = (ea, 0)$

Co-ordinate of point $S' = (-ea, 0)$ where $e = \sqrt{1 + \frac{b^2}{a^2}}$

Since hyperbola is $x^2 - y^2 = a^2$

$$\therefore \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$\therefore b^2 = a^2$$

$$\therefore e = \sqrt{1 + \frac{a^2}{a^2}} = \sqrt{2}$$

$$\therefore \text{ point } S = (\sqrt{2}a, 0); \text{ point } S' = (-\sqrt{2}a, 0)$$

We know that $SP = ex_1 - a = \sqrt{2}x_1 - a$ and $S'P = ex_1 + a = \sqrt{2}x_1 + a$

$$\therefore SP \times S'P = (\sqrt{2}x_1 - a)(\sqrt{2}x_1 + a) = 2x_1^2 - a^2$$

Now C being the centre of hyperbola has coordinate $(0, 0)$

$$\therefore CP = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

$$\therefore CP^2 = x_1^2 + y_1^2 = x_1^2 + x_1^2 - a^2 = 2x_1^2 - a^2 = SP \cdot S'P \quad (\text{Proved}).$$

29. a) Find the equations of the tangent and normal to the curve $y = x^2 - 4x + 2$ at the point (5, 3). [Model Question]

Answer:

Given equation of the curve $y = x^2 - 4x + 2$

Derivating it with respect to x

$$\frac{dy}{dx} = 2x - 4$$

At $x=5$, $\frac{dy}{dx} = 2 \times 5 - 4 = 6 = m_1$. It is the slope at point (5, 3).

Therefore equation of tangent is,

$$(y - y_1) = m_1(x - x_1)$$

$$\text{or, } (y - 3) = 6(x - 5) \quad \text{or, } y - 6x + 27 = 0$$

Let slope of normal at point (5, 3) is m_2 . Then -

$$m_1 m_2 = -1 \quad \text{or, } 6m_2 = -1$$

$$\therefore m_2 = -\frac{1}{6}$$

Therefore, equation of normal is -

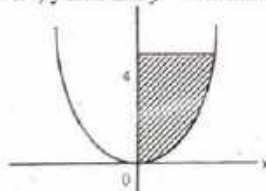
$$(y - y_1) = m_2(x - x_1) \quad \text{or, } (y - 3) = -\frac{1}{6}(x - 5)$$

$$\therefore 6y + x - 23 = 0$$

- b) Find the area of the region bounded by $y = x^2$, y -axis and the straight line $y = 4$. [Model Question]

Answer:

Area bounded by parabola $y = x^2$, y axis and $y = 4$ is shown in the diagram below-



Consider the diagram. Area is the result of following integration.

$$\int_0^4 x dy = \int_0^4 \sqrt{y} dy = \left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^4 = \frac{2}{3} \left[y^{\frac{3}{2}} \right]_0^4 = \frac{2}{3} \left[4^{\frac{3}{2}} \right] = \frac{2}{3} \times 8 = \frac{16}{3} \text{ Sq. units}$$

30. a) Find the equation of the parabola where vertex is at the origin, the axis on the y -axis and which passes through the point (6, -3). [Model Question]

Answer:

Let the equation of the parabola be $(x - 0)^2 = 4a(y - 0)$ (1)

Since the parabola passes through the point (6, -3), therefore

$$(6 - 0)^2 = 4a(-3 - 0)$$

$$\text{or, } 36 = 36a \quad \text{or, } a = 1.$$

Hence the required equation of the parabola is $x^2 = 4y$.

- b) Find the area bounded by the ellipse $4x^2 + 9y^2 = 36$ and the x -axis. [Model Question]

Answer:

We have $4x^2 + 9y^2 = 36$

$$\text{or, } y = \frac{1}{3} \pm \sqrt{36 - 4x^2}$$

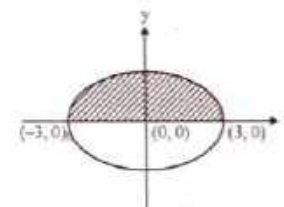
So the area bounded by the ellipse and x -axis is

$$2 \times \frac{1}{3} \int_0^3 \sqrt{36 - 4x^2} dx = \frac{2}{3} \int_0^3 4\sqrt{9 - x^2} dx$$

$$\text{subs: } x = 3 \sin \theta$$

$$\text{or, } dx = 3 \cos \theta d\theta = 4 \times \frac{2}{3} \int_0^{\pi/2} 3 \cos \theta \cdot 3 \cos \theta d\theta$$

$$= 24 \int_0^{\pi/2} \cos^2 \theta d\theta = 12 \int_0^{\pi/2} (1 + \cos 2\theta) d\theta = 12 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} = 6\pi$$



31. a) Taking y -axis as major axis and x -axis as minor axis, find the equation of the ellipse whose sum of the squares of the lengths of the major and minor axes is 20 and eccentricity is $\frac{1}{\sqrt{3}}$. [Model Question]

Answer:

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ be the equation of the ellipse.

$$\therefore 4(a^2 + b^2) = 20$$

$$\text{or, } a^2 + b^2 = 5 \quad \dots (1)$$

$$\text{and } 1 - \frac{b^2}{a^2} = \frac{1}{3}$$

$$\text{or, } a^2 - b^2 = \frac{a^2}{3} \quad \dots (2)$$

Adding (1) and (2) we get, $2a^2 = 5 + \frac{a^2}{3}$

$$\text{or, } \frac{5a^2}{3} = 5$$

$$\text{or, } a^2 = 3$$

$$\therefore b^2 = 2$$

So the equation of the ellipse is $\frac{x^2}{3} + \frac{y^2}{2} = 1$

➤ b) Find the point on the parabola $y = x^2$ at which the tangent is parallel to the line $y = 4x - 5$.

[Model Question]

Answer:

Let (α, β) be the point on the parabola

$$y = x^2 \quad \dots (1)$$

at which the tangent is parallel to the line $y = 4x - 5$ (2)

Now equation of the tangent of $y = x^2$ at (α, β) is

$$y - \beta = 2\alpha x$$

$$\text{or, } y = 2\alpha x + \beta \quad \dots (3)$$

Since equations (2) and (3) are parallel

$$2\alpha = 4$$

$$\text{or, } \alpha = 2$$

Since (α, β) lies on $y = x^2$ i.e. $\beta = 4$ at $\alpha = 2$

So the required point is (2, 4).

Chapter 3: Integral Calculus

Chapter at a glance:

3.1 Integration

If $\frac{d}{dx}[F(x) + c] = f(x)$ or $d[F(x) + c] = f(x)dx$, then $F(x)$ is called the integral of $f(x)dx$ and is denoted by, $\int f(x)dx = F(x) + c$

Some important formulae:

$$(1) \int A \cdot f(x)dx = A \int f(x)dx$$

$$(2) \int [f_1(x) \pm f_2(x) \pm \dots \pm f_n(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx \pm \dots \pm \int f_n(x)dx$$

$$(3) \int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$(4) \int dx = x + c$$

$$(5) \int e^{ax} dx = \frac{e^{ax}}{a} + c$$

$$(6) \int e^x dx = e^x + c$$

$$(7) \int a^{mx} dx = \frac{a^{mx}}{m \log_e a} + c$$

$$(8) \int a^x dx = \frac{a^x}{\log_e a} + c$$

$$(9) \int \frac{1}{x} dx = \log|x| + c$$

$$(10) \int \sin mx dx = -\frac{\cos mx}{m} + c$$

$$(11) \int \sin x dx = -\cos x + c$$

$$(12) \int \cos mx dx = \frac{\sin mx}{m} + c$$

$$(13) \int \cos x dx = \sin x + c$$

$$(14) \int \sec^2 mx dx = \frac{\tan mx}{m} + c$$

- (15) $\int \sec^2 x dx = \tan x + c$
- (16) $\int \cos ec^2 mx dx = -\frac{\cot mx}{m} + c$
- (17) $\int \cos ec^2 x dx = -\cot x + c$
- (18) $\int \sec mx \tan mx dx = \frac{\sec mx}{m} + c$
- (19) $\int \sec x \tan x dx = \sec x + c$
- (20) $\int \cos ec mx \cot mx dx = \frac{\cos ec mx}{m} + c$
- (21) $\int \cos ecx \cot x dx = -\cos ecx + c$, c is the arbitrary constant/integrating constant.

3.2 Integrals by Substitution:

- (1) If $n \neq 1$, then $\int \{f(x)\}^n f'(x) dx = \frac{1}{n+1} \{f(x)\}^{n+1} + c$
- (2) $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$
- (3) $\int \tan x dx = \log |\sec x| + c$
- (4) $\int \cot x dx = \log |\sin x| + c$
- (5) $\int \sec x dx = \log |\sec x + \tan x| + c = \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + c$
- (6) $\int \cos ecx dx = \log |\cos ecx - \cot x| + c = \log \left| \tan \frac{x}{2} \right| + c$
- (7) $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

Some important formulas:

- (1) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$; $\int \frac{dx}{x^2 - 1} = \tan^{-1} x + c$
- (2) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$ [$|x| > |a|$]
- (3) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$ [$|x| < |a|$]
- (4) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left| (x + \sqrt{x^2 \pm a^2}) \right| + c$

$$(5) \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c; \int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + c$$

3.3 Integration by-parts:

$$(1) \int uv dx = u \cdot \int v dx - \int \left[\frac{du}{dx} \cdot \int v dx \right] dx$$

i.e., the integral of the product of two functions
= first function $\times$ integral of second function
- integral of [derivative of first function $\times$ integral of second function].

- (2) $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + k = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \frac{b}{a} \right) + k$
- (3) $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + k$
 $= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left(bx - \tan^{-1} \frac{b}{a} \right) + k$
- (4) $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + c$
- (5) $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c$
- (6) $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

3.4 Area Under the Curve

- (1) The area bounded by the curve $y = f(x)$, the x -axis, the ordinate $x = a$ and the ordinate $x = b$ is $\int_a^b f(x) dx = \int_a^b y dx$
- (2) The area bounded by the curve $x = \phi(y)$, the y axis and the two horizontals $y = c$ and $y = d$ is $\int_c^d dy = \int_c^d \phi(y) dy$.

Remember that.....

- First draw a rough sketch of the given function
- Next indicate that region of the graph whose area is to be evaluated; this region is usually shaded by straight lines.
- Now, set up the definite integral which represents the area shown by shaded lines; the limiters of integrations are so chosen that the independent variable varies throughout the shaded region.
- Finally, the required area is obtained by evaluating the definite integral.

Short Answer Type Questions

1. If $\frac{d}{dx}[F'(x)] = f(x)$, then $\int f(x) dx$ is- [WBSCTE 2003, 2007]

(a) $f(x) + c$ (b) $F'(x) + c$ (c) $F'(x) + c$ (d) none of these

Answer: (b)

2. The value of $\int_0^{\pi} \cos 2x dx$ is equal to: [WBSCTE 2003, 2004, 2009]

(a) 0 (b) 1 (c) 2 (d) none of these

Answer: (a)

3. The value of $\int_0^{\pi} \tan^2 x dx$ is equal to: [WBSCTE 2005, 2006, 2009, 2011, 2017]

(a) $\frac{\pi}{4}$ (b) $1 + \frac{\pi}{4}$ (c) $1 - \frac{\pi}{4}$ (d) none of these

Answer: (c)

4. The area bounded by straight line $y=2x$, above x -axis from $x=1$ to $x=4$ is: [WBSCTE 2007, 2016]

(a) 16 sq. unit (b) 15 sq. unit (c) 7.5 sq. unit (d) none of these

Answer: (b)

5. $\int \cos^2 x dx$ is: [WBSCTE 2008, 2010]

(a) $-\sin 2x + c$ (b) $\frac{1}{4}(2x + \sin 2x) + c$
(c) $\frac{1}{2}(x - \sin 2x) + c$ (d) $\frac{1}{4}(2x - \sin 2x) + c$

Answer: (b)

6. $\int_0^{\pi} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is: [WBSCTE 2008, 2015]

(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{4}$ (d) none of these

Answer: (b)

7. $\int x e^{x^2} dx$ is: [WBSCTE 2009, 2011]

(a) $x e^{x^2} + c$ (b) $-e^{x^2} + c$ (c) $\frac{1}{2} e^{x^2} + c$ (d) none of these

Answer: (c)

8. $\int \frac{dx}{1+x^2}$ is equal to- [WBSCTE 2009]

(a) $\tan^{-1} x$ (b) $\cot^{-1} x$ (c) $-\tan^{-1} x$ (d) $-\cot^{-1} x$

Answer: (a)

9. The value of $\int_1^1 |x| dx$ is: [WBSCTE 2009, 2019]

(a) 1 (b) 1/2 (c) -1 (d) none of these

Answer: (a)

10. $\int_0^1 \frac{1-x}{1+x} dx$ is equal to: [WBSCTE 2009]

(a) $\log 2$ (b) $2 \log 2 - 1$ (c) $2 \log 2 + 1$ (d) none of these

Answer: (b)

11. $\int \sqrt{1 + \sin x} dx$ is: [WBSCTE 2010]

(a) $\frac{1}{2} \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right) + c$ (b) $2 \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right) + c$
(c) $\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) + c$ (d) none of these

Answer: (b)

12. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is: [WBSCTE 2010]

(a) $\frac{1}{2} (\sin^{-1} x)^2 + c$ (b) $(\sin^{-1} x)^2 + c$ (c) $2 (\sin^{-1} x) + c$ (d) none of these

Answer: (a)

13. $\int \frac{dx}{x^2 + a^2}$ is: [WBSCTE 2010, 2015]

(a) $\tan^{-1} \frac{x}{a} + c$ (b) $a \tan^{-1} \frac{x}{a} + c$ (c) $\frac{1}{a} \tan^{-1} \frac{x}{a} + c$ (d) none of these

Answer: (c)

14. $\int \frac{dx}{1+2x}$ is - [WBSCTE 2010]
 (a) $\log(1+2x)$ (b) $2 \log(1+2x)$ (c) $1/2 \log(1+2x)$ (d) none of these

Answer: (c)

15. $\int xe^x dx$ is - [WBSCTE 2010]
 (a) $e^x(x-1)+c$ (b) $e^x(x+1)+c$ (c) $e^x(1-x)+c$ (d) none of these

Answer: (a)

16. $\int \frac{dx}{1+\sin x}$ is : [WBSCTE 2010]
 (a) $\tan x - \sec x + c$ (b) $\tan x + \sec x + c$
 (c) $\sec x - \tan x + c$ (d) none of these

Answer: (a)

17. $\int \cos(ax+b) dx$ is : [WBSCTE 2011]
 (a) $-a \sin(ax+b) + c$ (b) $\frac{-\sin(ax+b)}{a} + c$
 (c) $\frac{\sin(ax+b)}{a} + c$ (d) none of these

Answer: (c)

18. $\int_0^{\pi/4} \sec^2 x dx$ is - [WBSCTE 2011]
 (a) 1 (b) $\frac{\pi}{4}$ (c) 0 (d) none of these

Answer: (a)

19. $\int_0^1 \frac{dx}{1+x^2}$ is - [WBSCTE 2011, 2016]
 (a) 1 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) none of these

Answer: (b)

20. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ [WBSCTE 2014]
 (a) $-\cos \sqrt{x} + c$ (b) $\frac{\cos \sqrt{x}}{2} + c$ (c) $-2 \cos \sqrt{x} + c$ (d) $-\frac{\cos \sqrt{x}}{2} + c$

Answer: (c)

21. $\int \tan^2 x dx =$ [WBSCTE 2014]
 (a) $\cot x + c$ (b) $\tan x + c$ (c) $\tan x - x + c$ (d) $\tan x + x + c$
 Answer: (c)

22. The value of $\int_{-1}^1 |x| dx$ is - [WBSCTE 2014]
 (a) 1 (b) 2 (c) 0 (d) -1
 Answer: (a)

23. The value of $\int_{-\pi}^{\pi} \frac{x}{1+\cos^2 x} dx$ is - [WBSCTE 2014]
 (a) 1 (b) 2 (c) 0 (d) -1
 Answer: (c)

24. Area bounded by the straight line $y = 2x$, above x-axis from $x = 1$ to $x = 4$ is - [WBSCTE 2014]
 (a) 16 sq. unit (b) 15 sq. unit (c) 7.5 sq. unit (d) None of these
 Answer: (b)

25. The volume of revolution of the area enclosed by $y = \sqrt{\cos x}$, x-axis and the lines $x = 0$, $x = \pi/4$ about x-axis is - [WBSCTE 2014]
 (a) π (b) $\frac{\pi}{\sqrt{2}}$ (c) $\frac{\pi^2}{\sqrt{2}}$ (d) $\pi\sqrt{2}$
 Answer: (b)

26. The integrating factor of the linear equation $\frac{dy}{dx} - \frac{y}{x} = x$ is - [WBSCTE 2014]
 (a) e^x (b) e^{-x} (c) $\frac{1}{x}$ (d) $-\frac{1}{x}$
 Answer: (c)

27. The value of $\int_1^e e^{1/x} dx$ is [WBSCTE 2015]
 (a) $e-1$ (b) $2(e-1)$ (c) $e+1$ (d) none of these
 Answer: (b)

28. The area of the triangle enclosed by the line $y = -x$, the x-axis and $x = 4$ is - [WBSCTE 2015]
 (a) 4 (b) 6 (c) 10 (d) 8
 Answer: (d)

29. The area bounded by parabola $y^2 = x$, the line $x = 0$ and the straight line $y = 3$ is –

[WBSCTE 2015]

- (a) 4 (b) 5 (c) 9 (d) none of these

Answer: (c)

30. The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x dx$ is

[WBSCTE 2015]

- (a) 0 (b) 1 (c) π (d) none of these

Answer: (a)

31. The value of $\int_{-1}^1 x dx$ is

[WBSCTE 2016]

- (a) 1 (b) 2 (c) 0 (d) 0.5

Answer: (a)

32. The area bounded by the curve $y = \tan x$, x-axis and the ordinate $x = \frac{\pi}{6}$ is

[WBSCTE 2017]

- (a) $\log 2$ (b) $\log \frac{2}{\sqrt{3}}$ (c) $\log \frac{1}{\sqrt{2}}$ (d) $\log \sqrt{2}$

Answer: (b)

33. Find the value of $\int_{-\pi/2}^{\pi/2} \sin^3 x dx$

[WBSCTE 2017]

Answer:

Since, $\sin^3 x$ is an odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^3 x dx = 0$$

34. The area bounded by the curve $y = x^2$, x-axis and the straight line $x = 2$ is –

[WBSCTE 2017]

- (a) $\frac{1}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{3}$ (d) none of these

Answer: (b)

35. The value of $\int \frac{dx}{x^2 + 4}$ is

[WBSCTE 2018]

- (a) $\tan^{-1} \frac{x}{2}$ (b) $\frac{1}{2} \tan^{-1} \frac{x}{2}$ (c) $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$ (d) $\frac{1}{2} \tan^{-1} x + C$

Answer: (c)

36. In Trapezoidal Rule $f(x)$ can be replaced by –

[WBSCTE 2018]

- (a) Parabolic Curve (b) Straight line
(c) Cubic Curve (d) None of these

Answer: (b)

37. The value of $\int_0^a x^3 dx$ is –

[WBSCTE 2018]

- (a) 1 (b) 2 (c) a (d) 0

Answer: (d)

38. The value of $\int \operatorname{cosec}^2 2x dx$ is

[Model Question]

- (a) $-\frac{1}{2} \cot 2x$ (b) $\cot 2x$ (c) $\frac{\operatorname{cosec} 2x}{2}$ (d) $-\cot 2x$

Answer: (a)

39. The value of $\int_1^2 dx$ is equal to

[Model Question]

- (a) 1 (b) 2 (c) 3 (d) 0

Answer: (a)

40. The value of $\int (\cos^2 x - \sin^2 x) dx$ is

[Model Question]

- (a) $\sec x \tan x$ (b) $\sin 2x$ (c) $\frac{1}{2} \sin 2x$ (d) $-\frac{1}{2} \sin 2x$

Answer: (c)

41. The value of $\int \frac{dx}{x \log x}$ is

[Model Question]

- (a) $\log|x| + c$ (b) $\log|\log x| + c$ (c) $x \log|x| + c$ (d) none of these

Answer: (b)

42. The value of $\int \operatorname{cosec}^2 2x \, dx$ is

[Model Question]

- (a) $-\frac{1}{2} \cot 2x$ (b) $\cot 2x$ (c) $\frac{(\operatorname{cosec} 2x)}{2}$ (d) $-\cot 2x$

Answer: (a)

B. Answer the following questions:

43. If $u = \tan^{-1} \frac{y}{x}$, then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.

[WBSCTE 2018]

Answer:

$u = \tan^{-1} \frac{y}{x}$ or, $\tan u = \frac{y}{x} \rightarrow$ homogeneous function of degree 0.

$\therefore x \frac{\partial}{\partial x} (\tan u) + y \frac{\partial}{\partial y} (\tan u) = 0$ function

or, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

44. $\int_{-2}^{+2} x^3 \, dx = ?$

[WBSCTE 2022]

Answer:

$$\int_{-2}^{+2} x^3 \, dx$$

$$= \int -x^3 \, dx$$

$$= -\frac{x^4}{4} + C$$

45. $2 \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx = ?$

[WBSCTE 2022]

Answer:

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx \quad \dots (1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} \, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx \quad \dots (2)$$

(1) + (2), we get

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\Rightarrow 2I = \left[x \right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2} - 0$$

$$\therefore I = \frac{\pi}{4}$$

$\therefore$ The required solution, $2 \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$

46. Evaluate: $\int \log_e x \, dx$.

[WBSCTE 2022]

Answer:

$$\int \log_e x \, dx$$

$$= \int \log x \cdot x - \int \frac{1}{x} \cdot x \, dx$$

$$= [x \log x - x]_1^e$$

$$= [x(\log x - 1)]_1^e$$

$$= e(\log e - 1) + 1$$

$$= 1$$

47. Find the integrating factor (I.F) of $\frac{dy}{dx} + 2xy = 9x$.

[WBSCTE 2022]

Answer:

$$\frac{dy}{dx} + 2xy = 9x$$

Compare $\left[\frac{dy}{dx} + Py = Q \right]$ equation & get, $P=2x$, $Q=9x$.

$$\therefore \text{I.F} = e^{\int P \, dx} = e^{\int 2x \, dx} = e^{x^2}$$

$$Y.I.F = \int Q.I.F dx + L$$

$$\Rightarrow Y.e^{x^2} = \int 9x.e^{x^2} dx + c$$

$$\Rightarrow Y.e^{x^2} = 9 \cdot \frac{x^2}{2} \cdot \frac{1}{2} e^{x^2} + c$$

$$\therefore Y = \frac{e^{-x^2} + c}{2} + \frac{9}{2}$$

Long Answer Type Questions

1. Integrate: $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

[WBSCTE 2003, 2008, 2010, 2015]

Answer:

$$\begin{aligned} \int \frac{dx}{\sin(x-a)\sin(x-b)} &= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-(x-b)\} dx}{\sin(x-a)\sin(x-b)} \\ &= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)} dx \\ &= \frac{1}{\sin(b-a)} \left[\int \cot(x-b) dx - \int \cot(x-a) dx \right] = \frac{1}{\sin(b-a)} [\log \sin(x-b) - \log \sin(x-a)] + c \\ &= \frac{1}{\sin(b-a)} \log \frac{\sin(x-b)}{\sin(x-a)} + c \quad \text{where } c \text{ is an arbitrary constant} \end{aligned}$$

2. Integrate: $\int \frac{dx}{e^x + e^{-x}}$

[WBSCTE 2003, 2006, 2009]

Answer:

$$\begin{aligned} \int \frac{dx}{e^x + e^{-x}} &\text{ Let } z = e^x \quad [dz = e^x dx] \\ &= \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{e^x dx}{e^{2x} + 1} = \int \frac{dz}{z^2 + 1} = \tan^{-1} z + c = \tan^{-1}(e^x) + c \quad [\text{where } c \text{ is an arbitrary constant}] \end{aligned}$$

3. Integrate: $\int e^x \left(\frac{x-1}{x^2} \right) dx$

[WBSCTE 2004, 2009]

Answer:

$$\int e^x \left(\frac{x-1}{x^2} \right) dx = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx \quad \left[\text{Let, } e^x \cdot \frac{1}{x} = z \Rightarrow \left\{ e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) \right\} dx = dz \right]$$

$$\therefore \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx = \int dz = z + c = \frac{e^x}{x} + c \quad [c = \text{constant}]$$

4. Integrate: $\int \frac{xe^x}{(1+x)^2} dx$

[WBSCTE 2006, 2008, 2009, 2010, 2014]

Answer:

$$\begin{aligned} \int \frac{xe^x}{(1+x)^2} dx &= \int \frac{\{(x+1)-1\}e^x}{(x+1)^2} dx = \int \frac{e^x}{(x+1)} dx - \int \frac{e^x}{(x+1)^2} dx \\ &= \frac{1}{(x+1)} \int e^x dx - \int \left\{ \frac{d}{dx} \left(\frac{1}{1+x} \right) e^x dx \right\} - \int \frac{e^x}{(x+1)^2} dx \\ &= \frac{e^x}{x+1} + \int \frac{e^x}{(x+1)^2} dx - \int \frac{e^x}{(x+1)^2} dx + c = \frac{e^x}{x+1} + c \quad [c = \text{Integration constant}] \end{aligned}$$

5. Find the area bounded by the circle $x^2 + y^2 = 4$ and the straight line $y = x$, in the first quadrant.

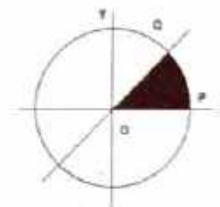
[WBSCTE 2006, 2014, 2016]

Answer:

The circle $x^2 + y^2 = 4$ and the straight line $x = y$ bounded by the area given by OPQO.

$$\text{Area OPQO} = \frac{1}{2} \int_0^2 y dx \quad \text{where } x^2 + y^2 = 2^2 \text{ so radius} = 2$$

$$\begin{aligned} &\frac{1}{2} \int_0^2 \sqrt{4-x^2} dx \quad [y = \sqrt{4-x^2}] \\ &= \frac{1}{2} \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 = \frac{1}{2} \left[2 \times \frac{\pi}{2} \right] = \frac{\pi}{2} \end{aligned}$$



6. Compute $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$ $b > a$.

[WBSCTE 2007, 2008]

Answer:

$$\text{Let } x-a = z^2 \text{ or, } dx = 2z dz$$

$$x = z^2 + a \quad \therefore b-x = b-z^2-a$$

when $x=a$, $z=0$

and when $x=b$, $z=\sqrt{b-a}$

$$\int_0^{\sqrt{b-a}} \frac{2zdz}{\sqrt{z^2((b-a)-z^2)}} = \int_0^{\sqrt{b-a}} \frac{2zdz}{z\sqrt{((b-a)-z^2)}} = 2 \left[\sin^{-1} \frac{z}{\sqrt{b-a}} \right]_0^{\sqrt{b-a}} + c$$

$$= 2 \left(\frac{\pi}{2} - 0 \right) + c = \pi + c \quad \text{where } c \text{ is an arbitrary constant}$$

7. $\int \frac{\log x dx}{(1+x)^2}$

[WBSCTE 2007, 2011]

Answer:

$$\int \frac{\log x}{(1+x)^2} dx \quad 1+x=z, dx=dz$$

$$I = \int \frac{\log(z-1) dz}{z^2} = \log(z-1) \int \frac{dz}{z^2} - \int \frac{1}{1-z} \cdot \frac{z^{-1}}{(-1)} dz$$

$$= -\frac{\log(z-1)}{z} + \int \frac{1}{(z-1)z} dz = -\frac{\log(z-1)}{z} + \int \frac{1}{(z-1)} dz + \int \frac{1}{z} dz$$

$$= -\frac{\log(z-1)}{z} + \log|z-1| - \log|z| + c = -\frac{\log x}{1+x} + \log \left| \frac{x}{x+1} \right| + c$$

8. Evaluate: $\int \frac{dx}{1+\cos x}$

[WBSCTE 2009]

Answer:

$$\int \frac{dx}{1+\cos x} = \int \frac{dx}{2\cos^2 \frac{x}{2}} = \frac{1}{2} \int \sec^2 \frac{x}{2} dx, \quad \text{Let } \frac{x}{2} = z \quad \text{or, } \frac{1}{2} dx = dz$$

$$\therefore I = \int \sec^2 z dz = \tan z + c = \tan \frac{x}{2} + c$$

9. Evaluate: $\int \frac{dx}{5+4\cos x}$

[WBSCTE 2009, 2011]

Answer:

$$\int \frac{dx}{5+4\cos x} = \int \frac{dx}{5\left(\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}\right) + 4\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)} = \int \frac{dx}{\sin^2 \frac{x}{2} + 9\cos^2 \frac{x}{2}} = \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = z$ or, $\frac{1}{2} \sec^2 \frac{x}{2} = dz$ or, $\sec^2 \frac{x}{2} = 2dz$ and $a=3$

$$\therefore I = \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx = 2 \int \frac{dz}{a^2 + z^2} = 2 \frac{1}{a} \tan^{-1} \frac{z}{a} + c = \frac{2}{3} \tan^{-1} \tan \left(\frac{x/2}{3} \right) + c = \frac{2}{3} \times \frac{x}{6} + c = \frac{x}{9} + c$$

10. $\int \frac{dx}{\sqrt{x(a^2-x)}}$

[WBSCTE 2009]

Answer:

$$\int \frac{dx}{\sqrt{x(a^2-x)}} = I$$

Let $x = a^2 \sin^2 \theta$ or, $\sin \theta = \frac{\sqrt{x}}{a}$, $\theta = \sin^{-1} \frac{\sqrt{x}}{a}$

$$dx = a^2 \cdot 2 \sin \theta \cos \theta d\theta$$

$$a^2 - x = a^2 (1 - \sin^2 \theta) = a^2 \cos^2 \theta$$

$$\therefore I = 2a^2 \int \frac{\sin \theta \cos \theta d\theta}{a \sin \theta a \cos \theta} = 2 \int d\theta = 2\theta + c = 2 \sin^{-1} \frac{\sqrt{x}}{a} + c$$

11. $\int (\log x)^2 dx$

[WBSCTE 2009]

Answer:

$$\int (\log x)^2 dx = (\log x)^2 \int dx - \left[\left\{ \int \frac{d}{dx} (\log x)^2 \cdot \int dx \right\} dx \right] = x(\log x)^2 - \left[\int \left\{ 2 \frac{\log x}{x} \cdot x \right\} dx \right]$$

$$= x(\log x)^2 - 2 \int \log x dx = x(\log x)^2 - 2 \left[\log x \int dx - \int \left\{ \frac{d}{dx} (\log x) \cdot \int dx \right\} dx \right]$$

$$= x(\log x)^2 - 2x \log x + 2 \int \frac{1}{x} dx = x(\log x)^2 - 2x \log x + 2x + c$$

12. $\int \sin^{-1} \sqrt{\frac{x}{x+a}} dx$

[WBSCTE 2009]

Answer:

$$\int \sin^{-1} \sqrt{\frac{x}{x+a}} dx$$

$$\text{Let } x = a \tan^2 \theta \Rightarrow x + a = a(1 + \tan^2 \theta) = a \sec^2 \theta$$

Differentiating both sides of $x = a \tan^2 \theta$

$$dx = 2a \tan \theta \sec^2 \theta d\theta$$

$$I = 2a \int \sin^{-1} \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}} \sec^2 \theta \tan \theta d\theta = 2a \int \sin^{-1} \left(\frac{\tan \theta}{\sec \theta} \right) \sec^2 \theta \tan \theta d\theta$$

$$= 2a \int \sin^{-1} \left(\frac{\sin \theta}{\cos \theta} \right) \sec^2 \theta \tan \theta d\theta = 2a \int \sin^{-1} (\sin \theta) \sec^2 \theta \tan \theta d\theta$$

$$= 2a \int \theta \sec^2 \theta \tan \theta d\theta = 2a \left[\theta \int \sec^2 \theta \tan \theta d\theta - \int \left\{ \frac{d}{d\theta} \theta \right\} \sec^2 \theta \tan \theta d\theta \right]$$

$$\text{Let } I' = \int \sec^2 \theta \tan \theta d\theta = \tan \theta \int \sec^2 \theta d\theta - \int \left\{ \frac{d}{d\theta} (\tan \theta) \right\} \sec^2 \theta \tan \theta d\theta$$

$$= \tan \theta \tan \theta - \int \sec^2 \theta \tan \theta d\theta = \tan^2 \theta - I'$$

$$\therefore 2I' = \tan^2 \theta \Rightarrow I' = \frac{\tan^2 \theta}{2}$$

$$\therefore I = 2a \left[\theta \frac{\tan^2 \theta}{2} - \int \frac{\tan^2 \theta}{2} d\theta \right] = a \left[\theta \tan^2 \theta - \int \tan^2 \theta d\theta \right] = a \left[\theta \tan^2 \theta - \int (\sec^2 \theta - 1) d\theta \right]$$

$$= a \left[\theta \tan^2 \theta - (\tan \theta - \theta) + c \right] = a \left[\theta \tan^2 \theta - \tan \theta + \theta + c \right] = a \left[\theta (1 + \tan^2 \theta) - \tan \theta \right] + c$$

$$= a \left[\theta (1 + \tan^2 \theta) - \tan \theta \right] + c = a \left[\tan^{-1} \sqrt{\frac{x}{a}} \left(\frac{x+a}{a} \right) - \sqrt{\frac{x}{a}} \right] + c = (x+a) \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + c$$

13. Evaluate: $I = \int_0^{1/2} \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$

[WBSCTE 2009, 2017]

Answer:

$$I = \int_0^{1/2} \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$$

$$\text{Let } x = \sin \theta, dx = \cos \theta d\theta$$

$$\therefore \sqrt{1-x^2} = \sqrt{1-\sin^2 \theta} = \cos \theta, \text{ and } 1-2x^2 = 1-2\sin^2 \theta = \cos 2\theta$$

$$\text{When } x=0, \sin \theta=0 \Rightarrow \theta=0 \text{ and } x=\frac{1}{2}, \sin \theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$$

$$\therefore I = \int_0^{\pi/6} \frac{\cos \theta d\theta}{\cos 2\theta \cdot \cos \theta} = \int_0^{\pi/6} \sec 2\theta d\theta = \frac{1}{2} [\log(\sec 2\theta + \tan 2\theta)]_0^{\pi/6}$$

$$= \frac{1}{2} \left[\log \left(\sec \frac{\pi}{3} + \tan \frac{\pi}{3} \right) \right] - \frac{1}{2} \log(\sec 0 + \tan 0)$$

$$= \frac{1}{2} \log(2 + \sqrt{3}) - \frac{1}{2} \log 1 = \frac{1}{2} \log(2 + \sqrt{3}) \log 1 = 0 \quad = \frac{1}{2} \log(2 + \sqrt{3})$$

14. Compute $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$

[WBSCTE 2009]

Answer:

$$\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}} = \int_2^3 \frac{dx}{\sqrt{2^2 - (x-3)^2}}$$

$$x-3=z \Rightarrow dx=dz \quad \Rightarrow a=2$$

$$\therefore \text{when } x \rightarrow 2, z \rightarrow -1 \text{ and when } x \rightarrow 0, z \rightarrow 0$$

$$\therefore I = \int_{-1}^0 \frac{dz}{\sqrt{a^2 - z^2}} = \left[\sin^{-1} \left(\frac{z}{a} \right) \right]_{-1}^0 = 0 - \sin^{-1} \left(-\frac{1}{2} \right) = -\sin^{-1} \left(\sin \left(-\frac{\pi}{6} \right) \right) = \frac{\pi}{6}$$

15. $\int_0^{\pi/4} \log(1 + \tan x) dx$

[WBSCTE 2009, 2014]

Answer:

$$\int_0^{\pi/4} \log(1 + \tan x) dx = \log(1 + \tan x) \int_0^{\pi/4} dx - \int_0^{\pi/4} \left\{ \frac{d}{dx} (\log(1 + \tan x)) \right\} \left[x \right]_0^{\pi/4} dx$$

$$= \left[x \right]_0^{\pi/4} \log(1 + \tan x) - \int_0^{\pi/4} \frac{\sec^2 x}{(1 + \tan x)} dx \left[x \right]_0^{\pi/4} = \frac{\pi}{4} \log(1 + \tan x) - \frac{\pi}{4} \int_0^{\pi/4} \frac{\sec^2 x}{(1 + \tan x)} dx$$

$$\text{when } x=0, z=1$$

$$\text{and } x=\frac{\pi}{4}, z=2$$

$$\int_0^{\pi/4} \frac{\sec^2 x}{(1 + \tan x)} dx = \int_1^2 \frac{dz}{z} = [\log z]_1^2 = \log 2 - \log 1 = \log 2$$

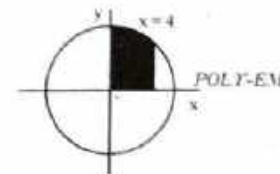
$$\therefore I = \frac{\pi}{4} \log(1 + \tan x) - \frac{\pi}{4} \log 2 = \frac{\pi}{4} \{ \log(1 + \tan x) - \log 2 \}$$

16. Find the area of the region bounded by $x^2 + y^2 = 25, y=0, x=0, x=4$.

[WBSCTE 2009]

Answer:

$$x^2 + y^2 = 5^2 \Rightarrow y^2 = 25 - x^2 \Rightarrow y = \pm \sqrt{25 - x^2}$$



$$y = \int_0^4 \sqrt{5^2 - x^2} dx = \left[\frac{x\sqrt{5^2 - x^2}}{2} + \frac{5^2}{2} \sin^{-1} \frac{x}{5} \right]_0^4$$

$$= \left(\frac{4 \cdot 3}{2} + \frac{25}{2} \sin^{-1} \frac{4}{5} \right) - \left(0 + \frac{25}{2} \sin^{-1} (0) \right)$$

$$= 6 + \frac{25}{2} \sin^{-1} \frac{4}{5} - \frac{25}{2} \sin^{-1} (0) = 6 + \frac{25}{2} \sin^{-1} \frac{4}{5}$$

17. Find the area bounded by the curves $4y = 3x^2$ and the line $3x - 2y + 12 = 0$. [WBSCTE 2009]

Answer:

$$4y = 3x^2 \text{ or, } y = \frac{3}{4}x^2 \dots\dots\dots (1)$$

$$2y = 3x + 12 \text{ or, } y = \frac{3}{2}x + 6 \dots\dots\dots (2)$$

Putting the value of y in equation (2) get,

$$\frac{3}{4}x^2 - \frac{3}{2}x - 6 = 0 \text{ or, } 3x^2 - 6x - 24 = 0$$

$$\text{or, } x^2 - 2x - 8 = 0 \text{ or, } x^2 - 4x + 2x - 8 = 0 \text{ or, } (x+2)(x-4) = 0$$

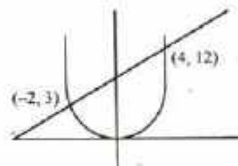
$$\therefore x+2=0 \text{ or, } x=-2 \text{ and } x-4=0 \text{ or, } x=4 \text{ when } x=-2, y=3$$

and when $x=4, y=12$. So the points of intersection of the curve $y = \frac{3}{4}x^2$

and the straight line $2y = 3x + 12$ are $(-2, 3)$ and $(4, 12)$.

So the area bounded by the curve and the line is

$$\int_{-2}^4 y dx = \frac{3}{4} \int_{-2}^4 x^2 dx = \frac{3}{4} \left[\frac{x^3}{3} \right]_{-2}^4 = \frac{1}{4} [64 - (-8)] = \frac{72}{4} = 18 \text{ square units}$$



18. Evaluate: $I = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)}$

[WBSCTE 2009, 2016]

Answer:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)}$$

Let $\sin x = z$; $\cos x dx = dz$ when $x \rightarrow \frac{\pi}{2}, z = 1$ and when $x = 0, z = 0$.

$$= \int_0^1 \frac{dz}{(1+z)(2+z)} = \int_0^1 \frac{(2+z) - (1+z) dz}{(1+z)(2+z)}$$

$$= \int_0^1 \frac{(2+z) dz}{(1+z)(2+z)} - \int_0^1 \frac{(1+z) dz}{(1+z)(2+z)} = \int_0^1 \frac{dz}{(1+z)} - \int_0^1 \frac{dz}{(2+z)}$$

$$= [\log|1+z|]_0^1 - [\log|2+z|]_0^1$$

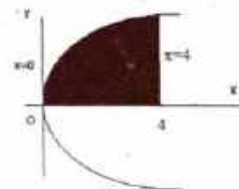
$$= \log_e 2 - \log_e 1 - \log_e 3 + \log_e 2 = 2 \log_e 2 - \log_e 3$$

19. Find the area bounded by the parabola $y^2 = 4x$, the x-axis and the ordinates $x=0$ and $x=4$. [WBSCTE 2010]

Answer:

$$\text{Area} = \int_0^4 y dx = \int_0^4 \sqrt{4x} dx$$

$$= 2 \int_0^4 (x)^{\frac{1}{2}} dx = 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 = 2 \left[\frac{2}{3} (4)^{\frac{3}{2}} - 0 \right] = \frac{32}{3} \text{ sq. unit}$$



20. Compute the area above the x-axis which is bounded by semicubical parabola $y^2 = x^3$ and the st. line $y=x$. [WBSCTE 2010]

Answer:

$$y = x, \text{ the eq. of st. line } \dots (1)$$

$$\text{Semi cubical eq. of parabola } y^2 = x^3 \dots (2)$$

$$x^2 = x^3 \quad [\because y = x]$$

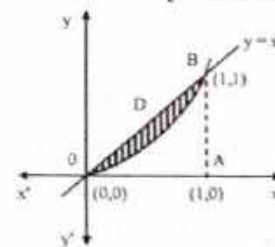
Putting value of $y=x$ of eq. (1) into (2)

$$\therefore x^3 - x^2 = 0, \text{ or, } x^2 (x-1) = 0$$

$$\therefore x = 0, 1$$

$$\text{The defined Region area is} = \int_0^1 y_1 dx - \int_0^1 y_2 dx = \int_0^1 x dx - \int_0^1 x^{\frac{2}{3}} dx \quad [\because y^2 = x^3 \Rightarrow y = x^{\frac{3}{2}}]$$

$$= \left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right]_0^1 = \left(\frac{1}{2} - 0 \right) - \left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right)_0^1 = \frac{1}{2} - \left[\frac{1}{\frac{5}{2}} \right] = \frac{1}{2} - \frac{2}{5} = 0.10 \text{ sq. unit}$$



21. Integrate $I = \int \frac{x \sin^{-1} x dx}{\sqrt{1-x^2}}$

[WBSCTE 2010]

Answer:

$$I = \int \frac{x \sin^{-1} x dx}{\sqrt{1-x^2}}$$

Let, $\sin^{-1} x = z$ or, $x = \sin z \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$

$$I = \int z \sin z dz = z \int \sin z dz - \int \left\{ \frac{d}{dz}(z) \int \sin z dz \right\} dz = -z \cos z + \int \cos z dz + c$$

$$= \sin z - z \cos z + c_1 = x - \sqrt{1-x^2} \sin^{-1} x + c_1 \left[\because \cos z = \sqrt{1-\sin^2 z} = \sqrt{1-x^2} \right]$$

22. Evaluate $I = \int \frac{dx}{2 + \cos x}$

[WBSCTE 2010]

Answer:

$$I = \int \frac{dx}{2 + \cos x} = \int \frac{dx}{2 \left(\cos \frac{x}{2} + \sin^2 \frac{x}{2} \right) + \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right)}$$

$$= \int \frac{\sec^2 \frac{x}{2} dx}{2 \left(1 + \tan^2 \frac{x}{2} \right) + \left(1 - \tan^2 \frac{x}{2} \right)} = \int \frac{\sec^2 \frac{x}{2} dx}{3 + \tan \frac{x}{2}}$$

Let, $\tan \frac{x}{2} = z$

Differentiating both sides, $\frac{1}{2} \sec^2 \frac{x}{2} dx = dz \Rightarrow \sec^2 \frac{x}{2} dx = 2 dz$

$$\therefore I = \int \frac{2 dz}{z^2 + (\sqrt{3})^2} = 2 \int \frac{dz}{z^2 + (\sqrt{3})^2} = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{z}{\sqrt{3}} \right) + c, \text{ where } c \text{ is an arbitrary constant}$$

23. Integrate $\int \frac{x + \sin x}{1 + \cos x} dx$

[WBSCTE 2010]

Answer:

$$\int \frac{x + \sin x}{1 + \cos x} dx = \int \frac{x}{1 + \cos x} dx + \int \frac{\sin x}{1 + \cos x} dx = \int \frac{x(1 - \cos x)}{\sin^2 x} dx + \int \frac{\sin x(1 - \cos x)}{\sin^2 x} dx$$

$$= \int x(\sec^2 x - \sec x \cot x) dx + \int (\sec x - \cot x) dx$$

$$= x \int (\sec^2 x - \sec x \cot x) dx - x \int \left(\frac{d}{dx}(x) \right) (\sec^2 x - \sec x \cot x) dx$$

$$+ \int (\sec x - \cot x) dx$$

$$= x[-\cot x + \sec x] - \left[\int (\sec x - \cot x) dx \right] + [(\sec x - \cot x)] dx = x + (\sec x - \cot x) + c$$

24. $\int \sqrt{\frac{2+x}{2-x}} dx$

[WBSCTE 2010]

Answer:

$$\int \sqrt{\frac{2+x}{2-x}} dx = \int \frac{\sqrt{2+x} \cdot \sqrt{2+x}}{\sqrt{2-x} \cdot \sqrt{2+x}} dx = \int \frac{2+x}{\sqrt{4-x^2}} dx = 2 \int \frac{dx}{\sqrt{(2)^2 - x^2}} + \int \frac{xdx}{\sqrt{4-x^2}}$$

$$= 2 \sin^{-1} \left(\frac{x}{2} \right) + \int \frac{-\theta d\theta}{\theta} \quad [4-x^2 = \theta^2 \Rightarrow -2x dx = 2\theta d\theta]$$

$$= 2 \sin^{-1} \left(\frac{x}{2} \right) - \int d\theta = 2 \sin^{-1} \left(\frac{x}{2} \right) - \theta + c = 2 \sin^{-1} \left(\frac{x}{2} \right) - \sqrt{4-x^2} + c$$

25. $\int \frac{e^x(1 - \sin x) dx}{1 - \cos x}$

[WBSCTE 2010]

Answer:

$$\int \frac{e^x(1 - \sin x) dx}{(1 - \cos x)} = \int \frac{e^x(1 - \sin x) dx}{2 \sin^2 \frac{x}{2}} = \int \frac{e^x \left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} \right) dx}{2 \sin^2 \frac{x}{2}}$$

$$= \int e^x \left[\frac{1}{2} \cos^2 \frac{x}{2} + \frac{1}{2} - \cot \frac{x}{2} \right] dx = \int e^x \cdot \frac{1}{2} \cot^2 \frac{x}{2} dx + \int \frac{e^x}{2} dx - \int e^x \cdot \cot \frac{x}{2} dx$$

$$= \int e^x \cdot \frac{1}{2} \left[\sec^2 \frac{x}{2} - 1 \right] dx + \int \frac{e^x}{2} dx - \int e^x \cdot \cot \frac{x}{2} dx$$

$$= \int e^x \cdot \frac{1}{2} \sec^2 \frac{x}{2} dx - \int \frac{e^x}{2} dx + \int \frac{e^x}{2} dx - \int e^x \cdot \cot \frac{x}{2} dx$$

$$= - \int e^x \cdot \cot \frac{x}{2} dx + \int \frac{e^x}{2} \sec^2 \frac{x}{2} dx$$

$$= - \left[\cot \frac{x}{2} \int e^x dx + \int \frac{1}{2} \sec^2 \frac{x}{2} \cdot e^x dx \right] + \int \frac{e^x}{2} \cdot \sec^2 \frac{x}{2} dx$$

$$= - \cot \frac{x}{2} \cdot e^x + \int \frac{e^x}{2} \cdot \sec^2 \frac{x}{2} dx - \int \frac{e^x}{2} \cdot \sec^2 \frac{x}{2} dx = - \frac{e^x}{\theta} \cdot \cot \frac{x}{2} + c$$

26. $\int \frac{\sqrt{\tan x}}{\cos x \sin x} dx$

[WBSCTE 2010]

Answer:

$$\int \frac{\sqrt{\tan x}}{\cos x \sin x} dx$$

Let $\theta = \sqrt{\tan x} \Rightarrow \theta^2 = \tan x \Rightarrow 2\theta d\theta = \sec^2 x dx \Rightarrow dx = \frac{2\theta d\theta}{\sec^2 x}$

$$= \int \theta \frac{2\theta d\theta}{\sec^2 x} \frac{1}{\cos x \sin x} = \int \frac{2\theta^2 d\theta}{\sin x \sec x}$$

$$= \int \frac{2\theta^2 d\theta}{\sin x} = \int \frac{2\theta^2 d\theta}{\tan x} = \int \frac{2\theta^2 d\theta}{\theta^2} = 2 \int d\theta + c = 2\theta + c = 2\sqrt{\tan x} + c$$

27. Evaluate: $\int_0^{\pi} \frac{x \sin x dx}{1 + \cos^2 x}$

[WBSCTE 2010, 2015]

Answer:

$$\int_0^{\pi} \frac{x \sin x dx}{1 + \cos^2 x} = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x) dx}{1 + \cos^2(\pi - x)} = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} - \int_0^{\pi} \frac{x \sin x dx}{1 + \cos^2 x}$$

$$\therefore I = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} - \int_0^{\pi} \frac{x \sin x dx}{1 + \cos^2 x} \Rightarrow I = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} - I \Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x}$$

$$\therefore I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} \quad \left[\begin{array}{l} -\cos x = \theta \\ \sin x dx = d\theta \end{array} \right]$$

When $x \rightarrow 0$ then $z \rightarrow -1$ and When $x \rightarrow \pi$ then $z \rightarrow 1$

$$I = \pi \int_{-1}^1 \frac{d\theta}{1 + \theta^2} = \pi [\tan^{-1} \theta]_{-1}^1 = \pi [\tan^{-1} 1 + \tan^{-1} (-1)] = \pi \left[\frac{\pi}{4} + \frac{\pi}{4} \right] = \frac{\pi^2}{2}$$

28. $\int_0^1 \frac{dx}{x(1 + \log x)^2}$

[WBSCTE 2010]

Answer:

Let, $1 + \log x = z \Rightarrow \frac{1}{x} dx = dz$

When $x = 0$ then $z = \infty$ and when $x = 1$ then $z = 1$

$$\Rightarrow \int_{\infty}^1 \frac{dz}{z^2} = \int_{\infty}^1 z^{-2} dz = \left[\frac{z^{-1}}{(-1)} \right]_{\infty}^1 = - \left[\frac{1}{(1 + \log x)} \right]_0^1$$

$$= - \left[\frac{1}{(1 + \log x)} \right]_0^1 = \left[\frac{1}{1 + \alpha} - \frac{1}{1 + 0} \right] = [0 - 1] = -1 \quad [\because \log 0 = -\alpha \text{ \& } \log 1 = 0]$$

29. Evaluate: $\int \frac{dx}{\sqrt{(x-2)(3-x)}}$

[WBSCTE 2011]

Answer:

$$\int \frac{dx}{\sqrt{(x-2)(3-x)}} = \int \frac{dx}{\sqrt{3x - 6 - x^2 + 2x}}$$

$$= \int \frac{dx}{\sqrt{5x - x^2 - 6}} = \int \frac{dx}{\sqrt{-(x^2 - 5x + 6)}} = \int \frac{dx}{\sqrt{-(x^2 - 2x \cdot \frac{5}{2} + \frac{25}{4} - \frac{25}{4} + 6)}}$$

$$= \int \frac{dx}{\sqrt{-\left[\left(x - \frac{5}{2}\right)^2 + 6 - \frac{25}{4}\right]}} = \int \frac{dx}{\sqrt{-\left[\left(x - \frac{5}{2}\right)^2 - \frac{1}{4}\right]}} = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}}$$

$$= \sin^{-1} \frac{\left(x - \frac{5}{2}\right)}{\left(\frac{1}{2}\right)} + c = \sin^{-1} (2x - 5) + c$$

30. Integrate: $\int \frac{dx}{x\sqrt{x^4 - 1}}$

[WBSCTE 2014]

Answer:

$$\int \frac{dx}{x\sqrt{x^4 - 1}}$$

Let, $\sqrt{x^4 - 1} = z$

$$\therefore x^4 - 1 = z^2$$

$$\therefore 4x^3 dx = 2z dz$$

$$\therefore \int \frac{x^3 \cdot z dz}{x^4 \cdot z \cdot 2x^3} = \int \frac{dz}{z^2 + 1} = \tan^{-1} z + C = \tan^{-1} \sqrt{x^4 - 1} + C$$

31. (a) Integrate: $\int \frac{dx}{2 - x - x^2}$

(b) Integrate: $\int \frac{dx}{(x-1)^2(x-2)}$

[WBSCTE 2014]

Answer:

$$a) \int \frac{dx}{2-x-x^2} = \int \frac{dx}{\left(\frac{3}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2} = \frac{1}{2 \cdot \frac{3}{2}} \log \left| \frac{\frac{3}{2} + x}{\frac{3}{2} - x} \right| + C = \frac{1}{3} \log \left| \frac{3+2x}{3-2x} \right| + C$$

$$b) \int \frac{dx}{(x-1)^2(x-2)} = \int \frac{(x-1) - (x-2)}{(x-1)^2(x-2)} dx = \int \frac{1}{(x-1)(x-2)} dx - \int \frac{1}{(x-1)^2} dx$$

$$= \int \left[\frac{1}{(x-1)} - \frac{1}{x-2} \right] dx - \int \frac{1}{(x-1)^2} dx = \ln(x-1) - \ln(x-2) + \frac{1}{x-1} + c$$

32. Integrate $\int e^x \frac{x-1}{(x+1)^3} dx$

[WBSCTE 2015, 2016]

Answer:

$$\int e^x \frac{1-x}{(1+x)^3} dx = \int e^x \frac{1+x-2x}{(1+x)^3} dx = \int \frac{e^x}{(1+x)^2} dx - 2 \int \frac{xe^x}{(1+x)^3} dx$$

$$= \left[\frac{e^x}{(1+x)^2} + 2 \int \frac{xe^x}{(1+x)^3} dx \right] - 2 \int \frac{xe^x}{(1+x)^3} dx + c = \frac{e^x}{(1+x)^2} + c$$

33. Integrate $\int \frac{(x-1)}{(x-3)(x+2)} dx$

[WBSCTE 2015, 2016]

Answer:

$$\int \frac{x-1}{(x-3)(x+2)} dx = \frac{1}{2} \int \frac{2x-1}{(x-3)(x+2)} dx - \frac{1}{2} \int \frac{dx}{(x-3)(x+2)}$$

$$= \frac{1}{2} \ln[(x-3)(x+2)] - \frac{1}{2} \int \left[\frac{1}{x-3} - \frac{1}{x+2} \right] dx$$

$$= \frac{1}{2} \ln((x-3)(x+2)) - \frac{1}{2} \ln \frac{x-3}{x+2} + c = \frac{1}{2} \ln(x+2)^2 + c = \ln(x+2) + c$$

34. a) Find by integration the area enclosed between the parabola $y^2 = 2x$, and the two ordinates $x = 1$ and $x = 4$

b) The line segment $x + y = 1$, $0 \leq y \leq 1$ is revolved about the y-axis to generate a cone. Find the volume of the cone.

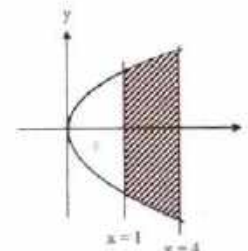
[WBSCTE 2015]

Answer:

$$a) \text{Area} = 2 \int_1^4 y dx = 2 \int_1^4 \sqrt{2x} dx = 2\sqrt{2} \left[\frac{x^{3/2}}{3/2} \right]_{x=1}^{x=4}$$

$$= 2\sqrt{2} \left[\frac{2}{3} (2\sqrt{2} - 1) \right]$$

$$= \frac{4\sqrt{2}}{3} (2\sqrt{2} - 1)$$

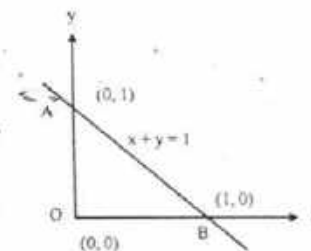


b) Area of the triangle OAB is

$$\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2} \text{ Sq. units}$$

So the volume of the generated cone is $\int_{r=0}^1 \frac{1}{2} \times 2\pi r dr$

$$= \pi \left[\frac{r^2}{2} \right]_0^1 = \frac{\pi}{2}$$



35. Evaluate $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ (substitute $z = xe^x$)

[WBSCTE 2016]

Answer:

$$\text{Let, } I = \int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$$

$$\text{subs: } xe^x = z \Rightarrow (x+1)e^x dx = dz$$

$$\therefore I = \int \frac{dz}{\cos^2 z} = \int \sec^2 z dz = \tan z + c = \tan(xe^x) + c$$

where c is the integrating constant.

36. a) Evaluate: $\int e^x \frac{1+x^2}{(1+x)^2} dx$

b) Evaluate: $\int \frac{\cos x}{(\sin x - a)(\sin x - b)} dx$

[WBSCTE 2017]

Answer:

a) $\int e^x \frac{1+x^2}{(1+x)^2} dx$

$$= \int e^x \frac{(1+x)^2}{(1+x)^2} dx - \int e^x \frac{2x}{(1+x)^2} dx$$

$$= \int e^x dx - \int e^x \frac{2x}{(1+x)^2} dx$$

$$= \int e^x dx - \int e^{z-1} \frac{2(z-1)}{z^2} dz \quad \text{Subs for 2nd integral } 1+x=z$$

$$\Rightarrow dx = dz$$

$$= e^x - e^{-1} \int \frac{2e^z}{z} dz + 2e^{-1} \int \frac{e^z}{z^2} dz$$

$$= e^x - e^{-1} \cdot \frac{2e^z}{z} - e^{-1} \cdot 2 \int \frac{e^z}{z^2} dz + 2e^{-1} \int \frac{e^z}{z^2} dz$$

$$= e^x - 2 \frac{e^{x-1}}{z} + c \quad \text{where } c \text{ is the integrating constant.}$$

$$= e^x - \frac{2e^x}{1+x} + c$$

b) $\int \frac{\cos x}{(\sin x - a)(\sin x - b)} dx$

$$= \int \frac{dz}{(z-a)(z-b)}$$

$$\text{Subs: } \sin x = z$$

$$\Rightarrow \cos x dx = dz$$

$$= \frac{1}{b-a} \left[\int \left(\frac{1}{z-b} - \frac{1}{z-a} \right) dz \right]$$

$$= \frac{1}{b-a} [\ln|z-b| - \ln|z-a|] + c$$

$$= \frac{1}{b-a} [\ln|\sin x - b| - \ln|\sin x - a|] + c$$

37. Evaluate:

$$\int_1^3 2-x dx$$

[WBSCTE 2017]

Answer:

$$\int_1^3 2-x dx$$

$$= \int_1^3 2-x dx + \int_3^5 2-x dx$$

$$= \int_1^3 (2-x) dx + \int_3^5 (x-2) dx$$

$$= \left[2x - \frac{x^2}{2} \right]_{x=1}^3 + \left[\frac{x^2}{2} - 2x \right]_{x=3}^5$$

$$= 2 - \frac{1}{4}(4-1) + \frac{1}{2}(9-4) - 2 = \frac{5}{2} - \frac{3}{4} = \frac{7}{4}$$

38. (a) Evaluate: $\int_0^{\pi/2} \frac{dx}{4+5\cos x}$

[WBSCTE 2017]

(b) Shade the area enclosed by the parabola $y^2 = 4x$ and $x^2 = 4y$. Use method of integration to find this area.

Answer:

a) $\int_0^{\pi/2} \frac{dx}{4+5\cos x}$

$$= \int_0^{\pi/2} \frac{dx}{4\left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}\right) + 5\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)}$$

$$= \int_0^{\pi/2} \frac{dx}{9\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$$

$$= \int_0^{\pi/2} \frac{\sec^2 \frac{x}{2} dx}{9 - \tan^2 \frac{x}{2}}$$

$$\text{Subs: } \tan \frac{x}{2} = z \Rightarrow \sec^2 \frac{x}{2} dx = 2 dz$$

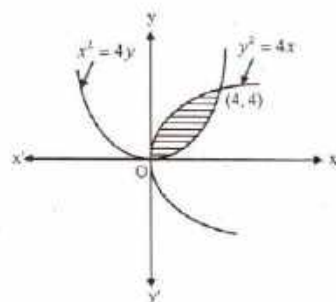
$$= 2 \int_0^1 \frac{dz}{9-z^2} \quad \left[\because \int \frac{dx}{a^2-x^2} = \sin^{-1} \frac{x}{a} \right]$$

$$= 2 \left[\sin^{-1} \frac{z}{3} \right]_0^1$$

$$= 2 \sin^{-1} \left(\frac{1}{3} \right)$$

b) Area of the shaded region

$$\begin{aligned} &= \int_0^4 \sqrt{4x} dx - \int_0^4 \frac{1}{4} x^2 dx \\ &= 2 \left[\frac{x^{3/2}}{3/2} \right]_0^4 - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^4 \\ &= \frac{4}{3} (4)^{3/2} - \frac{1}{12} (4)^3 \\ &= \frac{256}{3} - \frac{16}{3} = \frac{240}{3} = 80 \text{ sq. units} \end{aligned}$$



$$\begin{aligned} \text{or, } y^2 &= 4x \\ \text{or, } y^4 &= 16x^2 \\ \text{or, } y^4 &= 16 \cdot 4y \\ \text{or, } y(y^3 - 64) &= 0 \\ \text{or, } y &= 0, 4 \end{aligned}$$

39.(a) Integrate (i) $\int \frac{10^x (\log_e 10) + 10x^9}{10^x + x^{10}} dx$ (substitute $z = 10^x$); [WBSCTE 2018]

$$(ii) \int \frac{x-1}{(x-3)(x+2)} dx$$

(b) (i) Evaluate $\int_0^{\pi/2} |1-x| dx$ (ii) Show that $\int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x} = \frac{\pi}{4}$

Answer:

$$a) (i) \int \frac{10^x (\log_e 10) + 10x^9}{10^x + x^{10}} dx$$

$$\text{Subs: } z = 10^x \quad (\text{For } 1^{st})$$

$$\therefore dz = 10^x \log_e 10 dx$$

$$= \int \frac{10^x (\log_e 10) + 10x^9}{10^x + x^{10}} dx = \int \frac{10x^9}{10^x + x^{10}} dx$$

$$\text{Subs: } x^{10} = y$$

$$\therefore 10x^9 dx = dy \quad (\text{For } 2^{nd})$$

$$= \int \frac{dz}{z + \log_{10} z} + \int \frac{dy}{y + \dots}$$

(ii) Refer to Question No. 33 of Long Answer Type Questions.

$$b) (i) \int_0^2 |1-x| dx = \int_0^1 |1-x| dx + \int_1^2 |1-x| dx$$

$$\begin{aligned} &= \int_0^1 (1-x) dx + \int_1^2 (x-1) dx \\ &= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^2 = 1 - \frac{1}{2} + \frac{4}{2} - 2 - \frac{1}{2} + 1 = 1 \end{aligned}$$

$$\begin{aligned} (ii) \text{ Let } I &= \int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x} = \int_0^{\pi/2} \frac{\sin(-x + \frac{\pi}{2})}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx \quad [\because \int_0^{\pi/2} f(x) dx \\ &= \int_0^{\pi/2} f(\frac{\pi}{2} - x) dx] \end{aligned}$$

$$= \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

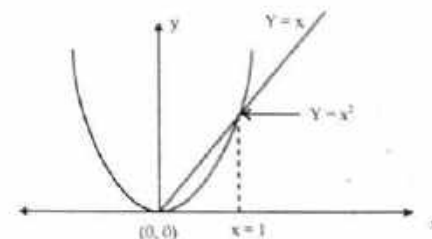
$$\therefore I = \frac{\pi}{4}$$

40. Show that the area bounded by the curves $y = x^2$ and $y = x$ is $\frac{1}{6}$.

[WBSCTE 2018]

Answer:

$$\begin{aligned} \text{Area} &= \int_{x=0}^1 (x - x^2) dx \\ &= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \end{aligned}$$



41. Integrate $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

[WBSCTE 2018]

Answer:

Refer to Question No. 3 of Long Answer Type Questions.

42. (a) If $\int_0^{\pi/2} (A \sin x) dx + \int_0^t (2x + C) dx = \frac{\pi}{2} + Bt^2 + 2t$, then find A , B and C .

[WBSCTE 2019]

Answer:

$$\begin{aligned} & \int_0^{\pi/2} (A \sin x) dx + \int_0^t (2x + C) dx \\ &= A[-\cos x]_{x=0}^{\pi/2} + [x^2 + Cx]_{x=0}^t \\ &= A[0+1] + [t^2 + Ct] \\ &= A + t^2 + Ct \end{aligned}$$

Now comparing LHS to RHS in the given relation we obtain;

$$A = \frac{\pi}{2}; B = 1; C = 2$$

- (b) Integrate: (i) $\int \frac{2^x (\log_e 2) + 10x^9}{2^x + x^{10}} dx$ (substitute $z = 2^x$); (ii) $\int \frac{x-1}{(x-3)(x+2)} dx$.

[WBSCTE 2019]

Answer:

$$\begin{aligned} \text{(i)} \quad & \int \frac{2^x (\log_e 2) + 10x^9}{2^x + x^{10}} dx \quad (\text{substitute } z = 2^x); \\ &= \int \frac{2^x (\log_e 2)}{2^x + x^{10}} dx + \int \frac{10x^9}{2^x + x^{10}} dx \end{aligned}$$

$$\text{Subs: } z = 2^x \quad (\text{For 1st})$$

$$\therefore dz = 2^x \log_e 2 dx$$

$$\text{Subs: } x^{10} = y \quad (\text{For 2nd})$$

$$\therefore 10x^9 dx = dy$$

Therefore,

$$\int \frac{dz}{z + \log^2 10} + \int \frac{dy}{z + y}$$

(ii) Refer to Question No. 39(a)(ii) of Long Answer Type Questions.

43. Evaluate: $\int \frac{x dx}{(x-1)(2x+1)}$.

[WBSCTE 2022]

Answer:

$$\begin{aligned} & \int \frac{x dx}{(x-1)(2x+1)} \\ &= \int \frac{(2x-x) dx}{(x-1)(2x+1)} \\ &= \int \frac{(2x+1-1-x) dx}{(x-1)(2x+1)} \\ &= \int \frac{(2x+1)-(x-1)}{(x-1)(2x+1)} dx \\ &= \int \frac{(2x+1) dx}{(x-1)(2x+1)} - \int \frac{(x-1) dx}{(x-1)(2x+1)} \\ &= \log|x-1| - \frac{1}{2} \log|2x+1| \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a} \log|ax+b| = c \right] \\ &= \log|x-1| - \frac{1}{2} \log|\sqrt{2x+1}| + c \\ &= \log \left| \frac{x-1}{\sqrt{2x+1}} \right| + c \end{aligned}$$

44. Evaluate: $\int e^x \cos x dx$

[WBSCTE 2022]

Answer:

$$\begin{aligned} & \int e^x \cos x dx = I \quad (\text{Let}) \\ \therefore I &= \cos x \int e^x dx - \int [(\cos x) \cdot \int e^x dx] \\ \Rightarrow I &= \cos x e^x + \int \sin x e^x dx \\ \Rightarrow I &= \cos x e^x + \sin x \int e^x dx - \int (\sin x) \int e^x dx \\ \Rightarrow I &= \cos x e^x + \sin x e^x - \int \cos x e^x dx \\ \Rightarrow I &= e^x (\cos x + \sin x) - I \\ \Rightarrow 2I &= e^x (\cos x + \sin x) \\ \Rightarrow I &= \frac{e^x}{2} (\cos x + \sin x) \\ \therefore \int e^x \cos x dx &= \frac{e^x}{2} (\cos x + \sin x) \end{aligned}$$

➤ 45. Integrate: $\int x e^x dx$.

Answer:

$$\begin{aligned}\int x e^x dx &= x \int e^x dx - \int \left[\frac{d}{dx}(x) \int e^x dx \right] dx \\ &= x e^x - \int 1(e)^x dx = x e^x - e^x + c = (x-1)e^x + c\end{aligned}$$

➤ 46. Evaluate $\int \frac{\cos x dx}{(2 + \sin x)(3 + \sin x)}$.

Answer:

$$\begin{aligned}&\int \frac{\cos x dx}{(2 + \sin x)(3 + \sin x)} \\ \text{put, } \sin x &= z \\ \cos x dx &= dz \\ &= \int \frac{dz}{(2+z)(3+z)} \\ &= \int \left[\frac{1}{2+z} - \frac{1}{3+z} \right] dz \\ &= \log|2+z| - \log|3+z| + c \\ &= \log \left| \frac{2+z}{3+z} \right| + c \\ &= \log \left| \frac{2 + \sin x}{3 + \sin x} \right| + c\end{aligned}$$

➤ 47. Evaluate $\int \tan^{-1} \left(\frac{2x}{1+x^2} \right) dx$

Answer:

$$\begin{aligned}&\int \tan^{-1} \left(\frac{2x}{1+x^2} \right) dx \\ &= \int 2 \tan^{-1} x dx \\ &= 2 \int \tan^{-1} x \cdot 1 dx\end{aligned}$$

[Model Question]

[Model Question]

[Model Question]

$$\begin{aligned}&= 2 \left[\tan^{-1} x \int 1 dx - \int \left\{ \frac{d}{dx}(\tan^{-1} x) \int 1 dx \right\} dx \right] \\ &= 2 \left[x \tan^{-1} x - \int \frac{1}{1+x^2} \cdot x dx \right] \\ &= 2 \left[x \tan^{-1} x - \frac{1}{2} \log|1+x^2| \right] + c\end{aligned}$$

➤ 48. Evaluate $\int \frac{\sin 2x dx}{a \cos^2 x + b \sin^2 x}$

[Model Question]

Answer:

$$\begin{aligned}&\int \frac{\sin 2x dx}{a \cos^2 x + b \sin^2 x} \\ \text{put, } a \cos^2 x + b \sin^2 x &= z \\ 2(b-a) \sin x \cos x dx &= dz \\ (b-a) \sin 2x dx &= dz \\ &= \frac{1}{b-a} \int \frac{dz}{z} \\ &= \frac{1}{b-a} \log|z| + c \\ &= \frac{1}{b-a} \log|a \cos^2 x + b \sin^2 x| + c\end{aligned}$$

➤ 49. Evaluate $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$.

[Model Question]

Answer:

$$\begin{aligned}&\int \frac{dx}{(1+x)\sqrt{1-x^2}} \\ \text{put } 1+x &= \frac{1}{z} \quad dx = -\frac{1}{z^2} dz \\ &= \int \frac{-\frac{1}{z^2} dz}{\frac{1}{z} \sqrt{1 - \left(\frac{1}{z} - 1\right)^2}} = - \int \frac{\frac{1}{z} dz}{\sqrt{z^2 - (1-z)^2}} = - \int \frac{dz}{\sqrt{2z-1}} \\ \text{Put, } 2z-1 &= m^2 \\ 2dz &= 2m dm\end{aligned}$$

50. Evaluate $\int \frac{dx}{3+2\sin x}$

Answer:

$$\int \frac{dx}{3+2\sin x} = \int \frac{dx}{3+2\frac{2\tan \frac{x}{2}}{1+\tan^2 \frac{x}{2}}} = \int \frac{\sec^2 \frac{x}{2} dx}{3\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 3}$$

put

$$\tan \frac{x}{2} = z$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dz$$

$$= 2 \int \frac{dz}{3z^2 + 4z + 3} = \frac{2}{3} \int \frac{dz}{z^2 + \frac{4}{3}z + 1}$$

$$= \frac{2}{3} \int \frac{dz}{\left(z + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} = \frac{2}{3} \cdot \frac{3}{\sqrt{5}} \tan^{-1} \left(\frac{z + \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right) + c = \frac{2}{\sqrt{5}} \tan^{-1} \left(\frac{3\tan \frac{x}{2} + 2}{\sqrt{5}} \right) + c$$

51. Evaluate $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$

Answer:

$$\int \frac{1}{x^2} e^{\frac{1}{x}} dx$$

put, $\frac{1}{x} = z$

$$\text{then } -\frac{1}{x^2} dx = dz \quad \Rightarrow -\int e^z dz = -e^z + c$$

$$= -e^{\frac{1}{x}} + c, \text{ where } c \text{ is integrating constant.}$$

52. Integrate $\int x e^x / (x+1)^2 dx$

Answer:

$$\int \frac{x e^x}{(x+1)^2} dx = \int \frac{(x+1-1) e^x}{(x+1)^2} dx$$

$$= \int \frac{1}{x+1} e^x dx - \int \frac{1}{(x+1)^2} e^x dx$$

[Model Question]

[Model Question]

[Model Question]

$$= \frac{1}{x+1} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{x+1} \right) \right] \int e^x dx \cdot dx - \int \frac{e^x}{(x+1)^2} dx$$

$$= \frac{e^x}{x+1} + \int \frac{1}{(x+1)^2} e^x dx - \int \frac{e^x}{(x+1)^2} dx$$

$$= \frac{e^x}{x+1} + c, \text{ where } c \text{ is integrating constant}$$

53. Evaluate $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$

[Model Question]

Answer:

$$\int \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{1}{b^2-a^2} \int \frac{(x^2+b^2)-(x^2+a^2)}{(x^2+a^2)(x^2+b^2)} dx = \frac{1}{b^2-a^2} \left[\int \frac{dx}{x^2+a^2} - \int \frac{dx}{x^2+b^2} \right]$$

$$= \frac{1}{b^2-a^2} \left[\frac{1}{a} \tan^{-1} \frac{x}{a} - \frac{1}{b} \tan^{-1} \frac{x}{b} \right] + c,$$

where c is an integrating constant.

54. Evaluate the integral $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

[Model Question]

Answer:

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{1}{2} \int \frac{(\cos x + \sin x) - (\cos x - \sin x)}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int dx - \frac{1}{2} \int \frac{\cos x - \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int dx - \frac{1}{2} \int \frac{dz}{z} \text{ where, } \sin x + \cos x = z, (\cos x - \sin x) dx = dz$$

$$= \frac{1}{2} x - \frac{1}{2} \log|z| + c = \frac{1}{2} x - \frac{1}{2} \log|\sin x + \cos x| + c$$

55. Evaluate: $\int \frac{x-1}{(x-2)(x-3)} dx$

[Model Question]

Answer:

$$\int \frac{x-1}{(x-2)(x-3)} dx = \int \frac{(x-2)+1}{(x-2)(x-3)} dx$$

$$\begin{aligned}
 &= \int \frac{dx}{x-3} + \int \frac{1}{(x-2)(x-3)} dx = \ln(x-3) + \int \left[\frac{1}{x-3} - \frac{1}{x-2} \right] dx \\
 &= \ln(x-3) + \ln(x-3) - \ln(x-2) + C \quad [C \text{ is the integrating constant}] \\
 &= \ln \frac{(x-3)^2}{(x-2)} + C
 \end{aligned}$$

Chapter 4: Ordinary Differential Equation

Chapter at a glance:

4.1 Definition

A differential equation (DE) is an equation involving an unknown function y of one or more independent variables $x, t, \dots$ and its derivatives.

Differential equations are of two types – “ordinary and partial” depending on the number of independent variables appearing in the equation.

Standard Methods for the solution of first order first degree DE

a) Separation of variables

An equation of the form $f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0 \quad \dots(1)$

is called an equation with variables separable, because the variables x and y can be easily separated. We can write equation (1) as

$$\frac{f_1(x)}{f_2(x)} dx + \frac{g_2(y)}{g_1(y)} dy = 0 \quad \dots(2)$$

$$f_2(x) \neq 0, g_1(y) \neq 0.$$

$$\Rightarrow F(x)dx + G(y)dy = 0 \quad \dots(3)$$

where $F(x) = \frac{f_1(x)}{f_2(x)}$ and $G(y) = \frac{g_2(y)}{g_1(y)}$

The equation (3) can be integrated to get the solution as

$$\int F(x)dx + \int G(y)dy = c, \text{ where } c \text{ is an arbitrary constant.}$$

b) Homogeneous Equation

If M and N of the equation $Mdx + Ndy = 0$ are both of the same degree in x and y and are homogeneous, the DE is said to be homogeneous. Such an equation can be put in the

form $y' = \frac{dy}{dx} = f\left(\frac{y}{x}\right) = g\left(\frac{y}{x}\right) \quad \dots(1)$

Every homogeneous equation of type (1) can be easily solved by putting $y = vx \quad \dots(2)$, where v is a function of x . Then

$$\frac{dy}{dx} = \frac{dv}{dx}x + v \quad \dots(3) \text{ and (1) reduces to}$$

$$v + x \frac{dv}{dx} = g(v)$$

$$\Rightarrow \frac{dy}{g(v) + v} = \frac{dy}{dx} \quad \dots(4)$$

and the variables are separated.

c) Exact Differential Equation

If M and N are both functions of x and y, then the DE

$Mdx + Ndy = 0$(1) is said to be *exact*, if there is a function u of x and y such that

$Mdx + Ndy = du$, where du is a perfect differential and du can be written as

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy = M(x, y) dx + N(x, y) dy$$

$$\therefore M(x, y) = \frac{\partial u}{\partial x} \dots\dots\dots(2) \text{ and}$$

$$N(x, y) = \frac{\partial u}{\partial y} \dots\dots\dots(3)$$

so that $Mdx + Ndy = 0$ gives $du = 0$ and hence $u = c$, an arbitrary constant of integration.

From (2) and (3), we can write $\frac{\partial M}{\partial y} = \frac{\partial^2 u}{\partial y \partial x}$ and $\frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$

Assuming that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$, we get the necessary condition for DE (1) to be an exact DE

$$\text{is } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \dots\dots\dots(4)$$

To solve an *exact* DE of type (1), follow this rule:

- Integrate the terms in Mdx as if y is constant
- Then integrate the terms in Ndy considering x as constant and rejecting the terms already obtained in (i).
- Finally, equate the sum of (i) and (ii) to a constant.

Before solving a DE, try to put it in the form of equation (1) and verify (4). Then follow the rules.

d) Reduction of Non-exact DE to Exact DE by using Integrating Factors

Consider the DE $M(x, y)dx + N(x, y)dy = 0$ (1) which is *not exact*. But if there exists a function $f(x, y)$ such that $f(x, y)[M(x, y)dx + N(x, y)dy] = 0$ becomes exact, then $f(x, y)$ is called an *Integrating Factor (I.F.)* of the DE (1).

Remark: Integrating factors can often be obtained by *inspection*. A DE can have more than one I.F. There are certain rules for finding I.F.s for certain types of Des.

4.2 Linear Differential Equations of 2nd and Higher Orders

The general linear DE of nth order is of the form

$$\frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + a_2(x) \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n(x) y = f(x), \quad \dots(1)$$

where $a_1(x), a_2(x), \dots, a_n(x)$ and $f(x)$ are functions of x only. If $a_1(x), a_2(x), \dots, a_n(x)$ are constants, then equation (1) will be called linear DE of constant coefficients. Let us now confine our discussion for 2nd order linear DE of constant coefficients only, i.e., for $n = 2$, the equation (1) can be written as

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = f(x) \quad \dots(2)$$

where a_1 and a_2 are constants. If y_1 and y_2 are two solutions of the equation,

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \dots(3)$$

then $u = C_1 y_1 + C_2 y_2$ is also a solution of (3). Since y_1 and y_2 are solutions of (2), then

$$\left. \begin{aligned} \frac{d^2 y_1}{dx^2} + a_1 \frac{dy_1}{dx} + a_2 y_1 &= 0 \\ \frac{d^2 y_2}{dx^2} + a_1 \frac{dy_2}{dx} + a_2 y_2 &= 0 \end{aligned} \right\}$$

If $C_1 y_1 + C_2 y_2$ is a solution of (2), then

$$\begin{aligned} \frac{d^2}{dx^2} (C_1 y_1 + C_2 y_2) + a_1 \frac{d}{dx} (C_1 y_1 + C_2 y_2) + a_2 (C_1 y_1 + C_2 y_2) \\ = C_1 \left[\frac{d^2 y_1}{dx^2} + a_1 \frac{dy_1}{dx} + a_2 y_1 \right] + C_2 \left[\frac{d^2 y_2}{dx^2} + a_1 \frac{dy_2}{dx} + a_2 y_2 \right] \\ = C_1 \times 0 + C_2 \times 0 = 0, \text{ by (3).} \\ \Rightarrow \frac{d^2 u}{dx^2} + a_1 \frac{du}{dx} + a_2 u = 0 \quad \dots(4) \end{aligned}$$

If $y = v$ be any particular solution of

$$\begin{aligned} \frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y &= f(x), \\ \text{then } \frac{d^2 v}{dx^2} + a_1 \frac{dv}{dx} + a_2 v &= f(x) \quad \dots(5) \end{aligned}$$

Adding (4) and (5), we get

$$\frac{d^2 (u + v)}{dx^2} + a_1 \frac{d(u + v)}{dx} + a_2 (u + v) = f(x) \quad \dots(6)$$

This shows that $y = u + v$ is the couple solution of (2).

The part u is called **Complementary Function (C.F.)** and the part v is called the **Particular Integral (P.I.)** of (2). Then we can write the complete solution of (2) as

$$y = C.F. + P.I. \quad \dots(7)$$

Observation: In order to solve the equation (2), we have to find first the C.F., i.e., the solution of (3) and then the P.I., i.e., the particular solution of (2).

4.3 Method of finding P.I. of a 2nd order ODE with constant coefficients

Let the given equation is $f(D)y = Q \quad \dots(1)$

Case I: When $Q = e^{ax}$

$$P.I. = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \text{ provided } f(a) \neq 0$$

if $f(a) = 0$

$$P.I. = \frac{1}{f(D)} e^{ax} = e^{ax} \frac{1}{f(D+a)} \text{ then go to Case II.}$$

Case II: When $Q = P_n(x)$ (Polynomial of degree n)

$$\therefore P.I. = \frac{1}{f(D)} P_n(x) = \frac{1}{(1 \pm \phi(D))} P_n(x) = (1 \pm \phi(D))^{-1} P_n(x)$$

then expand $(1 \pm \phi(D))^{-1}$ in series form and operate on $P_n(x)$.

Case III: When $Q = \cos ax / \sin ax$ $P.I. = \frac{1}{f(D)} \cos ax / \sin ax$

(i) If $f(D)$ is an even degree function of D then $f(D)$ can be written in the form

$$f(D) = \phi(D^2) \text{ and } P.I. = \frac{1}{\phi(D^2)} \cos ax / \sin ax = \frac{1}{\phi(-a^2)} \cos ax / \sin ax$$

provided $\phi(-a^2) \neq 0$

if $\phi(-a^2) = 0$ then expand $\cos ax / \sin ax$ in terms of exponential functions and follow Case I.

(ii) If $f(D)$ is not an even degree function of D then multiply the expression $\frac{1}{f(D)}$ in

numerator and denominator by some factor of D to make it an even degree function and follow the subcase (i).

Case IV: When $Q = Ve^{ax}$, where V is any function of x .

$$P.I. = \frac{1}{f(D)} Ve^{ax} = e^{ax} \frac{1}{f(D+a)} V \text{ then apply Case II or Case III according to the form of } V.$$

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The differential equation whose general solution is $y = A \sin x + B \cos x$ is -
[WBSCTE 2004, 2007, 2008, 2013, 2017]

- (a) $\frac{d^2 y}{dx^2} = y$ (b) $\frac{d^2 y}{dx^2} = -y$ (c) $\frac{d^2 y}{dx^2} = 0$ (d) none of these

Answer: (b)

2. The order and degree of the differential equation $\left(\frac{d^2 y}{dx^2}\right)^2 = x^2 \left(\frac{dy}{dx}\right)^3$ is -
[WBSCTE 2006, 2008]

- (a) 2 and 2 (b) 2 and 1 (c) 2 and 3 (d) none of these

Answer: (a)

3. The integrating factor of $\frac{dy}{dx} + 3x^2 y = 6x$ is - [WBSCTE 2008, 2009, 2010]

- (a) e^{3x} (b) e^{x^3} (c) x^3 (d) none

Answer: (b)

4. Integrating factor of the equation $\frac{dy}{dx} + \frac{1}{x} y = e^x$ is - [WBSCTE 2009]

- (a) x (b) $1/x$ (c) $\log x$ (d) e^x

Answer: (a)

5. C.F. of the equation $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 4y = e^x$ is - [WBSCTE 2009]

- (a) $C_1 e^{-2x} + C_2 e^{-2x}$ (b) $(c_1 + xc_2) e^{-2x}$ (c) None of these

Answer: (b)

6. P.I. of the equation $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$ is - [WBSCTE 2009, 2016]

- (a) $\frac{1}{4} e^{4x}$ (b) $\frac{1}{2} e^{4x}$ (c) none of these

Answer: (b)

7. Solution of the differential equation $\frac{dy}{dx} = \frac{y}{a}$ is - [WBSCTE 2010, 2013]

(a) $\frac{1}{2}y^2 = x + c$ (b) $x = a \log y + c$
(c) $x^2 = a \log y$ (d) none of these

Answer: (b)

8. C.F. of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x$ is - [WBSCTE 2010]

(a) $(c_1 + c_2)e^{-x}$ (b) $(c_1 + xc_2)e^{-x}$ (c) $(c_1 + xc_2)e^x$ (d) none is true

Answer: (c)

9. P.I. of $(D^2 - 3D + 2)y = e^{2x}$ is - [WBSCTE 2010]

(a) $\frac{1}{2}e^{2x}$ (b) $\frac{1}{2}e^{3x}$ (c) $\frac{xe^{2x}}{2}$ (d) none of these

Answer: (b)

10. General solution of the differential equation $\frac{dy}{dx} = \frac{1}{x}$ is [WBSCTE 2011, 2013]

(a) $xy = c$ (b) $y = x + c$ (c) $y = \log x + c$ (d) none of these

Answer: (c)

11. I.F. of the linear differential equation $\frac{dy}{dx} + \frac{1}{x}y = x^2$ is [WBSCTE 2011]

(a) $\log x$ (b) x (c) e^x (d) $\frac{1}{x}$

Answer: (b)

12. C.F. to the differential equation $(D^2 - 2D + 1)y = e^x$ is [WBSCTE 2011]

(a) $(c_1 + c_2)e^x$ (b) $(c_1 + xc_2)e^x$ (c) $(c_1 + xc_2)e^{-x}$ (d) none of these

Answer: (b)

13. The order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - 3\left(\frac{dy}{dx}\right)^3 + 7\frac{dy}{dx} + 4y = 0$

respectively are

(a) (2 and 3) (b) (3 and 2) (c) (2 and 1) (d) (2 and 2)

[WBSCTE 2011]

Answer: (d)

14. C.F. of $\frac{d^2y}{dx^2} - 4 = 0$ is [WBSCTE 2011]

(a) $c_1x^{2x} + xc_2e^{-2x}$ (b) $c_1x^{-2x} + c_2e^{2x}$ (c) $A \cos 2x + B \sin 2x$ (d) none of these

Answer: (b)

15. Solution of the equation $xy dx = dy$ is [WBSCTE 2012]

(a) $xy = c$ (b) $x^2 + \log y = c$ (c) $\frac{1}{2}x^2 = \log y + c$ (d) none of these.

Answer: (c)

16. I.F. of the linear equation $\frac{dy}{dx} - \frac{1}{x}y = x^2$ is [WBSCTE 2012]

(a) x (b) $\frac{1}{x}$ (c) $-x$ (d) none of these

Answer: (c)

17. C.F. of the equation $\frac{d^2x}{dx^2} - 3\frac{dy}{dx} + 2y = x$ is [WBSCTE 2012]

(a) $c_1e^{2x} + c_2e^x$ (b) $(c_1 + xc_2)e^{2x}$ (c) $c_1e^{-2x} + c_2e^{-x}$ (d) none is correct

Answer: (a)

18. P.I. of $(D^2 - 4)y = \sin 3x$ is [WBSCTE 2012]

(a) $\frac{1}{13} \sin 3x$ (b) $-\frac{\sin 3x}{13}$ (c) $-\frac{3 \cos 3x}{13}$ (d) none of these

Answer: (b)

19. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - 3y\left(\frac{dy}{dx}\right)^3 + 2y = 0$ respectively are

(a) (2, 3) (b) (3, 2) (c) (2, 2) (d) none

[WBSCTE 2012]

Answer: (c)

20. C.F. of $\frac{d^2y}{dx^2} - y = e^x$ is [WBSCTE 2012]

(a) $(c_1 + xc_2)e^x$ (b) $c_1e^x + c_2e^{-x}$ (c) $(c_1 + xc_2)e^{-x}$ (d) none of these

Answer: (b)

21. P.I. of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{2x}$ is

[WBSCTE 2012]

- (a) $-e^{2x}$ (b) e^{2x} (c) $-e^{-2x}$ (d) none of these

Answer: (c)

22. The order and degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = c\frac{d^2y}{dx^2}$ are

respectively -

- (a) 2 & 1 (b) 2 & 2 (c) 1 & 2

[WBSCTE 2014]

(d) 2 & 3

Answer: (b)

23. Solve: $\frac{d^2y}{dx^2} = 0$

[WBSCTE 2014]

- (a) $y = x + c$ (b) $y = ax + b$ (c) $x = ab$ (d) none of these

Answer: (b)

24. The particular integral of $\frac{d^2y}{dx^2} - 4y = \sin 2x$ is -

[WBSCTE 2014]

- (a) $2\cos 2x$ (b) $-\frac{\sin 2x}{8}$ (c) $\sin x$ (d) none

Answer: (b)

25. The degree of the homogenous function $u(x, y) = \frac{xy}{x-y}$ is -

[WBSCTE 2014]

- (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

Answer: (a)

26. The differential equation $(xy^2 + x^2)dx + (yx^2 + y)dy = 0$ is of type -

[WBSCTE 2015]

- (a) Linear (b) exact (c) reducible to homogenous (d) non-linear

Answer: (b)

27. The integrating factor of the equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is -

[WBSCTE 2015]

- (a) $\cos x$ (b) $-\sec^2 x$ (c) $\sec x$ (d) $\log \sec x$

Answer: (c)

28. The differential equation of all straight lines having gradient 2 is -

[WBSCTE 2015]

- (a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} + 2 = 0$ (c) $\frac{dy}{dx} = 2$ (d) none of these

Answer: (c)

29. The value of $\frac{1}{1+2D}(3x)$ is -

[WBSCTE 2015]

- (a) $3x - 6$ (b) $3x$ (c) $3x - 2$ (d) none of these

Answer: (a)

30. The differential equation $\frac{dy}{dx} + Py = Qy^n$ where P and Q are either constants of x

only is of type -

[WBSCTE 2015]

- (a) Linear (b) Bernoulli's differential equation
(c) exact (d) none of these

Answer: (b)

31. The value of $\frac{1}{3D^2 + D + 12}(\cos 2x)$ is -

[WBSCTE 2015]

- (a) $-\sin 2x$ (b) $-\frac{\sin 2x}{2}$ (c) $\frac{\sin 2x}{2}$ (d) none of these

Answer: (c)

32. Complementary Function of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$ is -

[WBSCTE 2016]

- (a) $(c_1 + c_2)e^x$ (b) $(c_1 + c_2x)e^x$ (c) $(c_1 + c_2)e^{-x}$ (d) $(c_1 + c_2x)e^{-x}$

Answer: (b)

33. The order and degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{2}{3}} = \frac{d^3y}{dx^3}$ are -

[WBSCTE 2016]

- (a) 3 and 2 (b) 3 and 3 (c) 2 and 3 (d) 3 and 1

Answer: (b)

34. The differential equation of the family of circles with centre at the origin is [WBSCTE 2017]

- (a) $x dx + y dy = 0$ (b) $y dx + x dy = 0$
(c) $x^2 dx + y^2 dx = 0$ (d) $x dy - y dx = 0$

Answer: (a)

35. The number of arbitrary constant in the general solution of $3 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + x = 0$ is - [WBSCTE 2017]

- (a) 1 (b) 2 (c) 3 (d) 4

Answer: (b)

36. The order and degree of the differential equation $\frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{2}{3}}$ are - [WBSCTE 2017]

- (a) (2, 3) (b) (3, 2) (c) (1, 3) (d) none

Answer: (a)

37. I.F of the differential equation $\frac{dy}{dx} + 2xy = 6$ is [WBSCTE 2017]

- (a) e^{3x} (b) e^{x^2} (c) e^{x^3} (d) none

Answer: (b)

38. Complementary Function of the differential equation $\frac{d^2 y}{dx^2} + 8 \frac{dy}{dx} + 16y = e^x$ is [WBSCTE 2018]

- (a) $(c_1 + c_2)e^{4x}$ (b) $(c_1 + c_2x)e^{-4x}$ (c) $(c_1 + c_2)e^{-4x}$ (d) $(c_1 + c_2x)e^{-x}$

Answer: (b)

39. The integrating factor of the differential equation $\frac{dy}{dx} = \frac{y}{x} + x^2$ is - [WBSCTE 2018, 2019]

- (a) x (b) $\frac{1}{x}$ (c) $\log_e x$ (d) None of these

Answer: (b)

40. If the differential equation $(2x^2 + 4y)dx + k(4x + y - 1)dy = 0$ is exact, then the value of k is - [WBSCTE 2018]

- (a) 1 (b) 2 (c) -1 (d) -2

Answer: (a)

B. Answer the following questions:

41. Find the differential equation of all straight lines passing through origin. [WBSCTE 2005, 2013]

Answer:

Let $y = mx$ be the equation of all straight lines passing through origin.

$$\therefore m = y/x,$$

$$\therefore \frac{dy}{dx} = m = \frac{y}{x}$$

$$\text{or, } x \frac{dy}{dx} = y \text{ or } x \frac{dy}{dx} - y = 0 \text{ is the required differential equation.}$$

42. Solve the equation $y dx + x dy = 0$. [WBSCTE 2005, 2006, 2007, 2008, 2009]

Answer:

$$y dx + x dy = 0 \text{ or, } \frac{y dx + x dy}{x^2} = 0 \text{ or, } d(y/x) = 0$$

$$\therefore y/x = C \text{ or, } y = Cx. [\text{Here, } C \text{ is the integrating constant.}] \text{ Ans.}$$

43. Solve : $\frac{dy}{dx} + y = x$ [WBSCTE 2005, 2008]

Answer:

$$\frac{dy}{dx} + y = x \text{ or, } \frac{dy}{dx} = x - y$$

$$\text{Let } x - y = z, \therefore 1 - \frac{dy}{dx} = \frac{dz}{dx} \text{ or, } \frac{dy}{dx} = 1 - \frac{dz}{dx}$$

$$\therefore \text{The equation becomes } 1 - \frac{dz}{dx} = z$$

$$\text{or, } \frac{dz}{dx} = 1 - z \text{ or, } \frac{dz}{1 - z} = dx$$

$$\text{or, } \int \frac{dz}{1 - z} = \int dx \text{ or } -\log(1 - z) = x + c$$

$$\text{or, } \log(z - 1) = x + c \therefore z - 1 = e^{x+c} = e^x \cdot e^c$$

$$\text{or, } x - y - 1 = A e^x \text{ where } A = e^c. \text{ Ans.}$$

44. Solve: $\frac{d^2 y}{dx^2} - 4 = 0$ [WBSCTE 2005, 2007]

Answer:

$$\frac{d^2 y}{dx^2} - 4 = 0 \text{ or, } \frac{d^2 y}{dx^2} = 4$$

$$\text{or, } \frac{d}{dx} \left(\frac{dy}{dx} \right) = 4. \text{ or, } \int d \left(\frac{dy}{dx} \right) = 4 \int dx$$

$$\text{or, } \frac{dy}{dx} = 4x + c_1$$

$$\text{or, } dy = 4x dx + c_1 dx$$

$$\text{or, } \int d(y) = 4 \int x dx + c_1 \int dx$$

$$\text{or } y = 2x^2 + \frac{c_1 x}{2} + c_2$$

$$\text{or } y = 2x^2 + c_1 x + c_2 \text{ Ans.}$$

45. Examine whether $(2x^2 + 4y)dx + (4x + y - 1)dy = 0$ is exact.

[WBSCTE 2005, 2007, 2008, 2016, 2017, 2019]

Answer:

Comparing the given equation with $Mdx + Ndy = 0$, We obtain,

$$\text{Let } M = 2x^2 + 4y, N = 4x + y - 1$$

$$\therefore \frac{\partial M}{\partial y} = 4, \frac{\partial N}{\partial x} = 4$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 4$$

So, the given equation is exact.

46. Write the order and degree of the differential equation

$$x \frac{d^2 y}{dx^2} - 2 \left(\frac{dy}{dx} \right)^2 + y = xe^x$$

[WBSCTE 2009]

Answer:

$$x \frac{d^2 y}{dx^2} - 2 \left(\frac{dy}{dx} \right)^2 + y = xe^x$$

Here highest derivative is $\frac{d^2 y}{dx^2}$ $\therefore$ order = 2

degree of the derivative of the highest order = 1

order = 2, degree = 1 Ans.

47. Write C.F. of $\frac{d^2 x}{dt^2} + \mu^2 x = 0$.

[WBSCTE 2009]

Answer:

$$\frac{d^2 x}{dt^2} + \mu^2 x = 0, \text{ roots of A.E are imaginary,}$$

$$\text{A.E. is } m^2 = -\mu^2 \text{ or, } m = \pm i\mu,$$

$$\therefore \text{C.F.} = A \cos \mu t + B \sin \mu t$$

48. The equation $(x^2 + y^2)dx - 2xy dy = 0$ is a homogenous equation True or False justify.

[WBSCTE 2009]

Answer:

$$(x^2 + y^2)dx - 2xy dy = 0$$

$$\text{or, } (x^2 + y^2)dx = 2xy dy$$

$$\text{or, } \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \text{ or, } \frac{dy}{dx} = \frac{1}{2} \left[\frac{x}{y} + \frac{y}{x} \right] \text{ or, } \frac{dy}{dx} = \frac{1}{2} \left[\left(\frac{y}{x} \right)^{-1} + \frac{y}{x} \right]$$

$$\text{or, } \frac{dy}{dx} = y_2 f \left(\frac{y}{x} \right) \left[\text{Let } f \left(\frac{y}{x} \right) = \left(\frac{y}{x} \right)^{-1} + \frac{y}{x} \right]$$

$\therefore$ If we compare the given differential equation with $\frac{dy}{dx} = f(x, y)$ we get

$f(x, y) = \frac{x^2 + y^2}{2xy}$. Now, $f(x, y)$ is a homogenous function of degree 2 in x, y . So, the

given equation is said to be homogenous differential equation. Ans.

49. Show that the equation $\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$ is a homogenous equation. [WBSCTE 2009]

Answer:

$$\frac{3xy + y^2}{3x^2} = \frac{x^2 \left(3 \frac{y}{x} + \frac{y^2}{x^2} \right)}{3x^2} = \frac{y}{x} + \frac{1}{3} \frac{y^2}{x^2} = f \left(\frac{y}{x} \right)$$

So the equation $\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$ is a homogeneous equation.

50. Solve: $\frac{dy}{dx} = 3$

[WBSCTE 2009]

Answer:

$$\frac{dy}{dx} = 3$$

$$\Rightarrow dy = 3dx$$

$$\Rightarrow \int dy = 3 \int dx \Rightarrow y = 3x + c$$

51. Test the equation $(2ax + by)y dx + (ax + 2by)x dy = 0$ for exact type.

[WBSCTE 2009]

Answer:

$$\text{We have, } (2ax + by)y dx + (ax + 2by)x dy = 0 \dots (i)$$

Comparing the eq. (i) with the form $M dx + N dy = 0$ we get,

$$M = (2ax + by)y, N = (ax + 2by)x$$

Now, $\frac{\partial M}{\partial y} = 2ax + 2by$, $\frac{\partial N}{\partial x} = 2ax + 2by$

So, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Hence, the given differential eq. (i) is an exact diff. eq.

52. Solve the equation $\frac{dx}{x} = 3dy$.

[WBSCTE 2010]

Answer: $\int \frac{dx}{x} = 3 \int dy \log|x| = 3y + c$ [c = constant]

53. Find the C.F. of the equation: $\frac{d^2x}{dt^2} + w^2x = 0$

[WBSCTE 2010]

Answer:

$$\frac{d^2x}{dt^2} + w^2x = 0 \quad \dots (1)$$

Let $y = Ae^{mx}$ be a trial solution of (1). Therefore Auxiliary equation is $m^2 + w^2 = 0$.

$$m^2 = -w^2$$

$$m = \pm wi$$

$$C.F. = c_1 \cos wt + c_2 \sin wt.$$

54. The equation $(x^2 + 6xy)dx + (y^2 + 3x^2)dy = 0$ is an exact equation. (True/False - Justify)

[WBSCTE 2011]

Answer: False

$$\frac{\partial(x^2 + 6xy)}{\partial x} = 2x + 6y$$

$$\frac{\partial(y^2 + 3x^2)}{\partial y} = 2y$$

55. Solve: $\frac{dy}{dx} = 2x$.

[WBSCTE 2012]

Answer:

$$\frac{dy}{dx} = 2x \quad \text{or, } dy = 2x dx$$

on integration we get, $y = x^2 + c$, where c is an integrating const.

56. Examine $(2x^2 + 4y)dx + (4x + y - 1)dy = 0$ is exact?

[WBSCTE 2013]

Answer:

$$\text{Let } M = 2x^2 + 4y, \quad N = 4x + y - 1.$$

$$\text{Now, } \frac{\partial M}{\partial y} = 4, \quad \frac{\partial N}{\partial x} = 4.$$

So the given equation is an exact differential equation.

57. Find P.I of $(D^2 - 3D + 2)y = e^{3x}$.

[WBSCTE 2013]

Answer:

$$P.I. = \frac{1}{D^2 - 3D + 2} e^{3x} = \frac{1}{(3)^2 - 3.3 + 2} e^{3x} = \frac{1}{2} e^{3x}$$

58. Find C.F of the differential $(D^2 - 2D + 1)y = x$.

[WBSCTE 2013]

Answer:

Let $y = Ae^{mx}$ be a trial solution reduced equation of the given equation.

$$\therefore \text{Auxiliary equation is } m^2 - 2m + 1 = 0 \quad \text{or, } (m - 1)^2 = 0 \quad \text{or, } m = 1, 1$$

So the C.F. of the given equation is $(c_1 + c_2 x)e^x$

59. Solve the differential equation $dx = 3xdy$.

[WBSCTE 2013]

Answer:

$$\text{We have, } dx = 3xdy \quad \text{or, } \frac{dx}{x} = 3dy$$

on integration; $x = c e^{3y}$, where c is any arbitrary constant.

60. Find the I.F. of the linear equation $dy/dx + 3x^2y = 6x$.

[WBSCTE 2013]

Answer:

$$\text{We have } \frac{dy}{dx} + 3x^2y = 6x$$

So I.F. of the given equation is $e^{\int 3x^2 dx} = e^{x^3}$

61. The equation $(x^2 + y^2)dx - 2xydy = 0$ is a homogeneous equation. True or False justify.

[WBSCTE 2013]

Answer:

$$\text{We have, } (x^2 + y^2)dx - 2xydy = 0 \quad \text{or, } \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} = f(x, y)$$

$$\text{Now, } f(tx, ty) = \frac{(tx)^2 + (ty)^2}{2(tx)(ty)} = \frac{x^2 + y^2}{2xy} = f(x, y)$$

So $f(x, y)$ is a homogeneous function in x, y of degree zero. Thus the differential equation is also a homogeneous differential equation.

62. Is the equation $(x^2 - xy^2)dx - (x^2y - y^3)dy = 0$ an exact equation?

[WBSCTE 2014]

Answer:

The equation $(x^2 - xy^2)dx - (x^2y - y^3)dy = 0$

Let, $M = x^2 - xy^2$ and $N = -x^2y + y^3$

$$\therefore \frac{\partial M}{\partial y} = -2xy \quad \therefore \frac{\partial N}{\partial x} = -2xy$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ So the given differential equation is an exact differential equation.}$$

63. Find the differential equation whose general solutions are $y = Ae^x + Be^{-x}$

[WBSCTE 2014]

Answer:

$$y = Ae^x + Be^{-x}$$

$$\therefore \frac{dy}{dx} = Ae^x - Be^{-x}$$

$$\therefore \frac{d^2y}{dx^2} = Ae^x + Be^{-x} = y$$

$$\therefore \frac{d^2y}{dx^2} - y = 0$$

64. If $u(x, y) = \frac{x}{y} + \frac{y}{x}$, then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ?$

[WBSCTE 2019]

Answer:

$$u(x, y) = \frac{x}{y} + \frac{y}{x} = \frac{x^2 + y^2}{xy} = x^0 \left(\frac{1 + \left(\frac{y}{x}\right)^2}{\frac{y}{x}} \right)$$

So, u is a homogeneous function of degree 0. Therefore $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

65. Particular Integral of the equation $\frac{d^2y}{dx^2} + y = e^{4x}$ is

[WBSCTE 2019]

(a) $\frac{1}{17}e^{4x}$

(b) $\frac{1}{2}e^{4x}$

(c) e^{4x}

(d) None of these

Answer: (d)

[WBSCTE 2022]

66. Solve: $\frac{ydy + xdx}{x^2 + y^2} = 0$

Answer:

$$\frac{ydy + xdx}{x^2 + y^2}$$

$$\Rightarrow \frac{xdy - ydx}{x^2} \times \frac{x^2}{x^2 + y^2} = 0$$

$$\Rightarrow d\left(\frac{y}{x}\right) \times \frac{1}{1 + \left(\frac{y}{x}\right)^2} = 0$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{1 + \left(\frac{y}{x}\right)^2} = 0$$

$$\Rightarrow d\left(\tan^{-1} \frac{y}{x}\right) = 0$$

$$\Rightarrow \int d\left(\tan^{-1} \frac{y}{x}\right) = \int 0$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x}\right) = c$$

Ans.

67. Find particular Integral (P.I) of the differential equation $\frac{d^2y}{dx^2} - y = 1$.

[WBSCTE 2022]

Answer:

$$\frac{d^2y}{dx^2} - y = 1$$

The differential equations can be written as $(D^2 - 1)y = 1$

So, the character equation is $p^2 - 1 = 0$

$$\Rightarrow p = \pm 1$$

$\therefore$ The complementary function is $Ae^x + Be^{-x}$

$$\therefore \text{Particular Integral (PI)} = \frac{1}{D^2 - 1}$$

So, the general solution is, $y = Ae^x + Be^{-x} - 1$

When $x = 0, y = 0$ then $A + B = (-1)$

$$y = Ae^x + Be^{-x} - 1$$

$$\Rightarrow ye^x = Ae^{2x} + B^{-x}$$

When $x \rightarrow \infty$ then $y \rightarrow$ to a finite limit

$$\Rightarrow B = 0 \text{ and } A = (-1) \quad \therefore y = (-e^{x-1})$$

Long Answer Type Questions

➤ 1. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

[WBSCTE 2004, 2015]

Answer:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$$

Let, $y = vx$, $\therefore x \frac{dv}{dx} + 2v = v^2$

$$\text{or, } x \frac{dv}{dx} = v^2 - 2v$$

$$\text{or, } \frac{dv}{v^2 - 2v} = \frac{dx}{x}$$

$$\therefore \int \frac{dv}{v(v-2)} = \int \frac{dx}{x}$$

$$\text{or, } \frac{1}{2} \int \frac{v - (v-2)}{v(v-2)} dv = \int \frac{dx}{x} \quad \text{or, } \left[\int \frac{dv}{v-2} - \int \frac{dv}{v} \right] = 2 \int \frac{dx}{x}$$

$$\text{or, } \log \frac{(v-2)}{v} = 2 \log x + \log c = \log cx^2$$

$$\therefore \frac{v-2}{v} = ex^2$$

$$\text{or, } 1 - \frac{2}{v} = ex^2$$

$$\text{or, } \frac{2x}{y} - (1 - ex^2) \Rightarrow y = \frac{2x}{1 - ex^2}$$

➤ 2. (a) Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

[WBSCTE 2005, 2009, 2010, 2013]

Answer:

$$\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$$

Let $\frac{y}{x} = v$ or, $y = vx$ $\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\therefore$ The above equation reduces to $v + x \frac{dv}{dx} = v + \tan v$

Ordinary Differential Equation

$$\text{or, } \frac{dv}{\tan v} = \frac{dx}{x} \text{ or, } \cot v \, dv = \frac{dx}{x}$$

Integrating both sides we get, $\int \cot v \, dv = \int \frac{dx}{x}$ or, $\log |\sin v| = \log x + A$

$A =$ integrating const.

$$\text{or, } \log |\sin v| = \log Cx \text{ where } A = \log C$$

$$\therefore \sin v = Cx \text{ or } \sin \left(\frac{y}{x} \right) = Cx$$

$$\text{or, } \frac{y}{x} = \sin^{-1} Cx \text{ or, } y = x \sin^{-1} Cx \text{ Ans.}$$

➤ 2. (b) Solve: $\frac{dy}{dx} = \frac{y}{x} + x^2$. Given $y = 1$, when $x = 1$.

[WBSCTE 2005, 2010, 2013, 2016, 2019]

Answer:

$$\frac{dy}{dx} - \frac{y}{x} = x^2 \text{ Given } y = 1 \text{ when } x = 1$$

$$\text{or, } \frac{dy}{dx} - \frac{y}{x} = x^2 \text{ or, } \frac{dy}{dx} + (-x^{-1})y = x^2$$

This is a linear equation in y .

$$\therefore I.F. = e^{\int -x^{-1} dx} = e^{-\log x} = x^{-1}$$

Multiplying both sides by x^{-1} , we get $x^{-1} \frac{dy}{dx} - x^{-2} y = x$.

$$\text{or, } \frac{d}{dx}(yx^{-1}) = x \text{ or, } d(yx^{-1}) = x dx$$

$$\therefore \int d(yx^{-1}) = \int x dx \quad \text{or, } yx^{-1} = \frac{x^2}{2} + c$$

When $x = 1, y = 1$

$$\therefore 1 = \frac{1}{2} + C \text{ or } C = \frac{1}{2}$$

$$\therefore \frac{y}{x} = \frac{1}{2}(1 + x^2) \text{ or, } y = \frac{1}{2}x(1 + x^2) \text{ Ans.}$$

➤ 3. (a) Solve $\frac{dy}{dx} = 1 + y + x^2 + yx^2$

[WBSCTE 2006, 2008, 2017]

Answer:

$$\frac{dy}{dx} = (1 + y) + x^2(1 + y) = (1 + y)(1 + x^2)$$

$$\text{or, } \int \frac{dy}{1+y} = \int \frac{dx}{1+x^2}$$

$$\text{or, } \log(1+y) + \log C = \tan^{-1} x$$

$$\text{or, } \log C(1+y) = \tan^{-1} x \quad [\log C = \text{integrating constn.}]$$

$$\text{or, } C(1+y) = e^{\tan^{-1} x}$$

$$\text{or, } (1+y) = C^1 e^{\tan^{-1} x} \text{ where } C^1 = \frac{1}{C} \text{ ans.}$$

3. (b) Find the particular solution of $\cos y dx + (1+2e^{-x}) \sin y dy = 0$ when $x=0, y=\pi/4$ [WBSCTE 2006, 2007, 2008, 2010, 2013]

Answer:

$$\cos y dx + (1+2e^{-x}) \sin y dy = 0$$

$$\text{or, } \cos y dx = -(1+2e^{-x}) \sin y dy$$

$$\text{or, } \frac{dx}{1+2e^{-x}} = -\tan y dy \quad \text{or, } \int \frac{e^x dx}{e^x + 2} = -\int \tan y dy$$

$$\text{Let } e^x + 2 = z \quad \therefore e^x dx = dz$$

$$\therefore \int \frac{e^x dx}{e^x + 2} = \int \frac{dz}{z} = \log z = \log(e^x + 2)$$

$$\therefore \log(e^x + 2) = \log \cos y + C$$

$$\text{or, } \log \frac{e^x + 2}{\cos y} = C$$

$$\text{When } x=0, y=\pi/4, C = \log \left(\frac{1+2}{\cos \frac{\pi}{4}} \right) = \log \left(\frac{3}{\frac{1}{\sqrt{2}}} \right) = \log 3\sqrt{2}$$

$$\therefore \log 3\sqrt{2} = \log \frac{e^x + 2}{\cos y} \text{ or, } \cos y (3\sqrt{2}) = e^x + 2$$

$$\text{or, } \cos y = \frac{e^x + 2}{3\sqrt{2}} \text{ or, } y = \cos^{-1} \left(\frac{e^x + 2}{3\sqrt{2}} \right) \text{ Ans.}$$

4. Solve: $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x$

[WBSCTE 2007, 2014]

Answer:

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x$$

Let $y = e^{mx}$ be a trial solution

So the auxiliary equation is $m^2 - 2m + 5 = 0$

$$\text{or, } m^2 - 2m + 1 = -4 \quad \text{or, } (m-1)^2 = 4i^2$$

$$\text{or, } m-1 = \pm 2i \quad \text{or, } m = 1 \pm 2i$$

$\therefore C.F.(y) = e^x (A \cos 2x + B \sin 2x)$ where A and B are arbitrary constant.

$$\text{Now P.I.}(y) = \frac{1}{D^2 - 2D + 5} 10 \sin x = \frac{10}{-1 - 2D + 5} \sin x \text{ [putting } D^2 = -1]$$

$$= \frac{10}{4 - 2D} \sin x = \frac{5}{2 - D} \sin x$$

$$= \frac{5(2+D)}{4 - D^2} \sin x = \frac{5(2+D)}{4 - (-1)} \sin x \text{ [Again putting } D^2 = -1]$$

$$= (D+2) \sin x = D(\sin x) + 2 \sin x = \cos x + 2 \sin x$$

$\therefore$ Complete solution will be

$$y = C.F.(y) + P.I.(y) = e^x (A \cos 2x + B \sin 2x) + (\cos x + 2 \sin x) \text{ Ans.}$$

5. (a) Solve: $(1+x^2)dy - x^2 y dx = 0$, if $y=2$, when $x=1$.

[WBSCTE 2009]

Answer:

$$(1+x^2)dy = x^2 y dx$$

$$\text{or, } \int \frac{x^2}{1+x^2} dx = \int \frac{dy}{y}$$

$$\text{or, } \frac{1}{3} \int \frac{3x^2}{1+x^2} dx = \int \frac{dy}{y} \text{ [Let } 1+x^2=z \text{ or } x^2 dx = \frac{1}{3} dz]$$

$$\text{or, } \frac{1}{3} \log |z| = \log y + c$$

$$\text{or, } \frac{1}{3} \log(1+x^2) = \log y + c$$

$$\text{or, } \log(1+x^2) = 3 \log y + 3c'$$

$$\text{or, } \log(1+x^2) = \log y^3 + \log c \text{ where } 3c' = \log c$$

$$\text{or, } (1+x^2) = cy^3 \text{ Ans.}$$

5. (b) Solve: $x \log x \frac{dy}{dx} + y = 2 \log x$

[WBSCTE 2009, 2012]

Answer:

$$x \log x \frac{dy}{dx} + y = 2 \log x \quad \text{or, } \frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$$

This is a first order linear differential equation.

$$\text{Here I.F.} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{d(\log x)}{\log x}} = e^{\log(\log x)} = \log x$$

$\therefore$ Multiplying both sides by $\log x$, we get

$$\log x \frac{dy}{dx} + \frac{y}{x} = \frac{2 \log x}{x} \text{ or, } \int d(y \log x) = 2 \int \frac{\log x}{x} dx$$

$$\text{or, } y \log x = 2 \int \frac{\log x}{x} dx \quad \text{Let } \log x = z \therefore \frac{dx}{x} = dz$$

$$= 2 \int z dz = 2 \cdot \frac{z^2}{2} + c = \frac{z(\log x)^2}{2} + c = (\log x)^2 + c$$

$\therefore$ Required solution is $y \log x = (\log x)^2 + c$ Ans.

6. Solve: $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = 0$, where L, R, C are all constant and $R^2 C = 4L$.

[WBSCTE 2009, 2016]

Answer:

Let $I = Ae^{mt}$

So the auxiliary equation will be

$$Lm^2 e^{mt} + Rme^{mt} + \frac{R^2}{4L} e^{mt} = 0 \therefore C = \frac{4L}{R^2}$$

$$\text{or, } (4L^2 m^2 + 4LmR + R^2) e^{mt} = 0 \therefore \frac{1}{C} = \frac{R^2}{4L}$$

Since $e^{mt} \neq 0$, $\therefore (2Lm + R)^2 = 0$

or, $(2Lm + R)(2Lm + R) = 0$ Hence the roots of the

$$\text{or, } m = -\frac{R}{2L}, -\frac{R}{2L}$$

$$R^2 C = 4L$$

A.E. are real and equal. So the required solution

$I = e^{mt} (A + Bt) = e^{-Rt/2L} (A + Bt)$ where A and B are arbitrary constant.

$I = (A + Bt)e^{-Rt/2L}$ Ans.

7. Test the equation $(x^2 + 2ye^{2x})dy + (2xy + 2y^2e^{2x})dx = 0$ for exact and solve it.

[WBSCTE 2009]

Answer:

We have, $(2xy + 2xy^2e^{2x})dx + (x^2 + 2ye^{2x})dy = 0$... (i)

Comparing the eq. (i) with the form $M dx + N dy = 0$ we get,

$$M = 2xy + 2xy^2e^{2x}, N = x^2 + 2ye^{2x}$$

$$\text{Now, } \frac{\partial M}{\partial y} = 2x + 4xye^{2x}, \frac{\partial N}{\partial x} = 2x + 4xye^{2x}$$

$$\text{Therefore, } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

So, the given differential eq. (i) is an exact diff. eq.

Hence, the required solution is:

$\int (x^2 + 2ye^{2x})dy$ (Keeping y as constant) $= c$, where c is an arbitrary constant.

$$\text{Or, } x^2 y + y^2 e^{2x} = c$$

8. Solve $(2x^2 + 4y)dx + (4x + y - 1)dy = 0$

[WBSCTE 2010]

Answer:

$$(2x^2 + 4y)dx + (4x + y - 1)dy = 0$$

$$\text{or, } 2x^2 dx + 4y dx + 4x dy + y dy - dy = 0$$

$$\Rightarrow 2 \int x^2 dx + 4 \int d(xy) + \int y dy - \int dy = 0$$

$$\Rightarrow 2 \cdot \frac{x^3}{3} + 4xy + \frac{y^2}{2} - y = 0$$

$$\Rightarrow 4x^3 + 4xy + 3y^2 - 6y = k$$

$$\Rightarrow 4x^3 + 3y^2 - 6y + 4xy = k$$

9. Solve: $(D^2 - 1)y = xe^{2x}$

[WBSCTE 2010]

Answer:

$$(D^2 - 1)y = xe^{2x}$$

The auxiliary equation is $m^2 - 1 = 0$ or, $m^2 = 1$ or, $m = \pm 1$, which are real and unequal.

$$\therefore C.F. = c_1 e^x + c_2 e^{-x}$$

$$\text{Particular Integral} = \frac{1}{D^2 - 1} (e^{2x}) = e^{2x} \cdot \frac{1}{(D+2)^2 - 1} x$$

$$= e^{2x} \cdot \frac{1}{D^2 + 4D - 1} (x) = e^{2x} \cdot \frac{1}{D^2 + 4D + 3} (x)$$

$$= e^{2x} \cdot \frac{1}{3 \left(1 + \frac{D^2 + 4D}{3} \right)} (x) = \frac{e^{2x}}{3} \left(1 + \frac{D^2 + 4D}{3} \right)^{-1} (x)$$

$$= \frac{e^{2x}}{3} \left(1 - \frac{D^2 + 4D}{3} + \dots \right) (x) = \frac{e^{2x}}{3} \left(x - \frac{4}{3} \right)$$

$$\text{Therefore, the required general solution is } y = c_1 e^x + c_2 e^{-x} + \frac{1}{3} e^{2x} \left(x - \frac{4}{3} \right)$$

where c_1 and c_2 are arbitrary constant independent to each other.

10. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 10 \sin x$.

[WBSCTE 2010]

Answer:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 10 \sin x$$

 It can be written as, $(D^2 - 4D + 3)y = 10 \sin x$

 The auxiliary equation is $m^2 - 4m + 3 = 0$

$$\text{or, } m^2 - m = 3m + 3 = 0 \quad \text{or, } (m-1)(m-3) = 0$$

$$\text{or, } m = 1, 3$$

 $\therefore$ Complimentary function is $c_1 e^x + c_2 e^{3x}$.

$$\text{Particular Integral} = \frac{1}{D^2 - 4D + 3} (10 \sin x) = 10 \cdot \frac{1}{D^2 - 4D + 3} (\sin x)$$

$$= 10 \times \frac{1}{-1^2 - 4D + 3} (\sin x) = 10 \times \frac{1}{2 - 4D} (\sin x)$$

$$= 5x \frac{1}{2 - 2D} (\sin x) = 5 \times \frac{1 + 2D}{1 - 4D^2} (\sin x) = 5 \times \frac{\sin x + 2 \cos x}{1 - 4(-1^2)}$$

$$\therefore y = C.F. + P.I.$$

 $= c_1 e^x + c_2 e^{3x} + \sin x + 2 \cos x$ is the required general solution. Where c_1 and c_2 are two arbitrary constants.

11. If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$ show that $x \frac{du}{dx} + y \frac{du}{dy} = \sin 2u$.

[WBSCTE 2010]

Answer:

$$u = \tan^{-1} \frac{x^3 + y^3}{x - y} \quad \text{or, } \tan u = \frac{x^3 + y^3}{x - y} = \frac{x^3 \left(1 + \frac{y^3}{x^3}\right)}{x \left(1 - \frac{y}{x}\right)} = x^2 \phi\left(\frac{y}{x}\right)$$

 Which shows that, $\tan u$ is a homogenous function in two variables x and y of degree 2. Therefore, by Euler's theorem we get,

$$x \frac{d}{dx} (\tan u) + y \frac{d}{dy} (\tan u) = 2 \tan u$$

$$\text{or, } x \sec^2 u \frac{du}{dx} + y \sec^2 u \frac{du}{dy} = 2 \tan u$$

$$\text{or, } x \frac{du}{dx} + y \frac{du}{dy} = 2 \frac{\sin u}{\cos u} \cos^2 u = 2 \sin u \cos u = \sin 2u$$

$$\text{or, } x \frac{du}{dx} + y \frac{du}{dy} = \sin 2u \quad (\text{Proved})$$

12. Solve the equation $y \sin 2x dx = (1 + y^2 + \cos^2 x) dy$.

[WBSCTE 2011]

Answer:

$$\text{We have, } y \sin 2x dx - (1 + y^2 + \cos^2 x) dy = 0 \quad \dots (1)$$

 Comparing the given equation with $Mdx + Ndy = 0$ we get,

$$M = y \sin 2x, \quad N = -(1 + y^2 + \cos^2 x)$$

$$\text{Now, } \frac{\partial M}{\partial y} = \sin 2x, \quad \frac{\partial N}{\partial x} = \sin 2x$$

So, equation (1) is an exact differential equation

Thus, solution of equation (1) is

$$\int y \sin 2x dx \text{ (keeping } y \text{ as constant)} + \int (-1 - y^2) dy = c$$

 where, c is any arbitrary constant.

$$\text{or, } -y \frac{\cos 2x}{2} - y - \frac{y^3}{3} = c$$

$$\text{or, } 3y \cos 2x + 6y + 2y^3 + 6c = 0.$$

13. Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x}$

[WBSCTE 2011]

Answer:

$$\text{We have } \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x} \quad \dots (1)$$

 Let $y = Ae^{mx}$ be a trial solution of the reduces equation of (1)

 So, the auxiliary equation is $m^2 + 4m + 3 = 0$ or, $m = -3, -1$

$$\text{Thus } C.F. = c_1 e^{-3x} + c_2 e^{-x}$$

 where c_1, c_2 are any two arbitrary constants.

$$P.I. = \frac{1}{(D+1)(D+3)} e^{-3x}, \quad D = \frac{1}{dx}$$

$$= \frac{1}{-2} \frac{1}{D+3} e^{-3x} = -\frac{1}{2} e^{-3x} \frac{1}{D} 1 = -\frac{x}{2} e^{-3x}$$

 Hence the general solution of equation (1) is $y = c_1 e^{-3x} + c_2 e^{-x} - \frac{x}{2} e^{-3x}$.

14. Solve: $(D^2 + 2D + 10)y = -37 \sin 3x$

[WBSCTE 2011]

Answer:

$$\text{We have } (D^2 + 2D + 10)y = -37 \sin 3x \quad \dots (1)$$

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1)

So, the auxiliary equation is $m^2 + 2m + 10 = 0$

$$\text{or, } m = \frac{-2 \pm \sqrt{4 - 40}}{2} = -1 \pm 3i$$

Thus CF = $e^{-x} (c_1 \cos 3x + c_2 \sin 3x)$ where c_1, c_2 are any two arbitrary constants.

$$\begin{aligned} PI &= -37 \frac{1}{D^2 + 2D + 10} \sin 3x = -37 \frac{1}{-9 + 2D + 10} \sin 3x \\ &= -37 \frac{1}{2D + 1} \sin 3x = -37 \frac{2D - 1}{4D^2 - 1} \sin 3x \\ &= -37 \frac{2D - 1}{-36 - 1} \sin 3x = 6 \cos 3x - \sin 3x. \end{aligned}$$

Hence the required solution is $y = e^{-x} (c_1 \cos 3x + c_2 \sin 3x) + 6 \cos 3x - \sin 3x$.

➤ 15. Solve: $\frac{dy}{dx} = \frac{y}{x} \left(\log \frac{y}{x} + 1 \right)$

[WBSCTE 2012, 2016]

Answer:

We have, $\frac{dy}{dx} = \frac{y}{x} \left(\log \frac{y}{x} + 1 \right)$ (1)

Let, $\frac{y}{x} = v$ or, $y = vx$ or, $\frac{dy}{dx} = v + x \frac{dv}{dx}$

Therefore, from equation (1) we get,

$$v + x \frac{dv}{dx} = v (\log v + 1)$$

$$\text{or, } x \frac{dv}{dx} = v \log v \quad \text{or, } \frac{dv}{v \log v} = \frac{dx}{x}$$

$$\text{or, } \log(\log v) = \log x + \log c, \quad (c \text{ being any arbitrary constant})$$

$$\text{or, } \log(v)^c = 0$$

$$\text{or, } \left(\frac{y}{x} \right)^c = 1.$$

➤ 16. Solve: $\frac{d^2x}{dt^2} + w^2x = 0$, when $t = 0, \frac{dx}{dt} = 0$ and $x = a$. [WBSCTE 2012, 2017]

Answer:

$$\text{We have, } \frac{d^2x}{dt^2} + w^2x = 0 \quad \dots (1)$$

Let, $x = Ae^{mt}$ be a trial solution of equation (1).

So, the auxiliary equation is $m^2 + w^2 = 0$

$$\text{or, } m = iw$$

$$\text{or, } x = c_1 \cos(wt) + c_2 \sin(wt) \quad \dots (2)$$

[c_1, c_2 are two arbitrary constant.]

Again, it is given that at $t = 0, x = a$

$$\therefore a = c_1 \cos(w \times 0) \quad \text{or, } c_1 = a$$

$$\text{and at } t = 0, \frac{dx}{dt} = 0$$

$$\therefore 0 = c_2 w$$

$$\text{or, } c_2 = 0 \quad [\because w \neq 0]$$

Thus, $x = a \cos(wt)$, which is the required solution.

➤ 17. Solve: $(D^2 - 4D + 4)y = xe^{2x}$.

[WBSCTE 2012]

Answer:

$$\text{We have, } (D^2 - 4D + 4)y = xe^{2x} \quad \dots (1)$$

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1).

So the auxiliary equation is $m^2 - 4m + 4 = 0$ or, $m = 2, 2$

$$\therefore CF = (c_1 + c_2 x) e^{2x}$$

$$\text{Now, } PI = \frac{1}{(D-2)^2} x e^{2x} = e^{2x} \cdot \frac{1}{D^2} x = \frac{x^2}{6} e^{2x}.$$

$$\text{So, the general solution of equation (1) is } y = (c_1 + c_2 x) e^{2x} + \frac{x^2}{6} e^{2x}.$$

➤ 18. Solve $(x dx + y dy) + (x dy - y dx) / (x^2 + y^2) = 0$.

[WBSCTE 2013, 2017]

Answer:

$$\text{We have } xdx + ydy + (xdy - ydx) / (x^2 + y^2) = 0$$

$$\text{or, } \frac{1}{2} (2xdx + 2ydy) + \frac{\frac{xdy - ydx}{x^2 + y^2}}{\frac{x^2 + y^2}{x^2}} = 0 \quad \text{or, } \frac{1}{2} (2xdx + 2ydy) + \frac{d\left(\frac{y}{x}\right)}{1 + \left(\frac{y}{x}\right)^2} = 0$$

$$\text{On integration; } \frac{1}{2} (x^2 + y^2) + \tan^{-1} \left(\frac{y}{x} \right) = c, \text{ where } c \text{ is the arbitrary constant.}$$

19. Solve $(D^2 + 1)y = xe^{2x}$.

[WBSCTE 2013]

Answer:

We have, $(D^2 + 1)y = xe^{2x}$... (1)

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1).

So the auxiliary equation is $m^2 + 1 = 0$ or, $m = \pm i$

$\therefore$ C.F. of (1) is $c_1 \cos x + c_2 \sin x$, where c_1 and c_2 are two arbitrary constant.

Now, P.I. = $\frac{1}{D^2 + 1} xe^{2x} = x \frac{1}{(D+1)^2 + 1} e^{2x} = x \frac{1}{D^2 + 2D + 2} e^{2x} = \frac{1}{10} xe^{2x}$.

Hence the general solution of equation (1) is $y = \text{C.I.} + \text{P.I.}$

$= c_1 \cos x + c_2 \sin x + \frac{1}{10} xe^{2x}$.

20. Solve $(D^2 + 4)y = x^2$.

[WBSCTE 2013]

Answer:

We have $(D^2 + 4)y = x^2$... (1)

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1).

So the auxiliary equation is $m^2 + 4 = 0 \Rightarrow m = \pm 2i$

$\therefore$ C.F. of equation (1) is $c_1 \cos 2x + c_2 \sin 2x$; where c_1, c_2 are two arbitrary constants.

Now, P.I. = $\frac{1}{D^2 + 4} x^2 = \frac{1}{4} \frac{1}{1 + D^2} x^2$
 $= \frac{1}{4} \left(1 + \frac{D^2}{4} \right)^{-1} x^2 = \frac{1}{4} \left(1 - \frac{D^2}{4} + \dots \right) x^2 = \frac{1}{4} \left(x^2 - \frac{2}{4} \right) = \frac{1}{4} \left(x^2 - \frac{1}{2} \right)$

Hence the general solution of the equation (1) is

$y = \text{C.F.} + \text{P.I.} = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4} \left(x^2 - \frac{1}{2} \right)$

21. Solve: $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = x^2 e^{2x}$

[WBSCTE 2014]

Answer:

Here the auxiliary equation $(m^2 - 4m + 4) = 0$

or, $(m - 2)^2 = 0$ has two real and equal roots $m = 2, 2$

$\therefore$ The complementary function = $(A + Bx)e^{2x}$

$\therefore$ Particular integral (P.I.) = $\frac{1}{(D-2)^2} x^2 \cdot e^x$

$= e^x \cdot \frac{1}{(D+2-2)^2} x^2 = e^x \cdot \frac{1}{D^2} x^2 = e^x \cdot \frac{1}{D} \cdot \frac{x^2}{3} = e^x \cdot \frac{x^3}{12}$

$\therefore$ The complete solution is

$y = (A + Bx)e^{2x} + e^x \cdot \frac{x^3}{12}$ (Ans.)

22. (a) Solve: $\frac{dy}{dx} = (x+y)^2$, when $x=0, y=1$

[WBSCTE 2014]

Answer:

$\frac{dy}{dx} = (x+y)^2$

$\therefore x + y = z$

$\therefore 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{dz}{dx} - 1 = z^2$

$\therefore \frac{dz}{dx} = z^2 + 1$

$\therefore \int \frac{dz}{z^2 + 1} = \int dx + C$

$\therefore \tan^{-1} |z| = x + C$

$\therefore \tan^{-1} (x+y) = x + C$ [$\because x=0, y=1$]

$\therefore C = \frac{\pi}{4}$

$\therefore$ The solution is $\tan^{-1} (x+y) - x = \frac{\pi}{4}$.

22. (b) Solve: $\frac{dy}{dx} + y \cot x = 2 \cos x$

[WBSCTE 2014]

Answer:

$\frac{dy}{dx} + y \cot x = 2 \cos x$, which is a Leibnitz's equation

Here, $P = \cot x$

and $\int P dx = \int \cot x dx = \log |\sin x|$

$$\therefore \text{I.F.} = e^{\int \rho dx} = e^{\log |\sin x|} = \sin x$$

$\therefore$ The solution is

$$y \sin x = \int 2 \cos x \cdot \sin x dx + C$$

$$\therefore y \sin x = \int \sin 2x dx + C$$

$$\therefore y \sin x = -\frac{1}{2} \cos 2x + C$$

$$\therefore 2y \sin x + \cos 2x = C \quad (\text{Ans.})$$

23. Solve $\frac{dy}{dx} = -\frac{x+y \cos x}{1+\sin x}$

Answer:

$$\frac{dy}{dx} = -\frac{x+y \cos x}{1+\sin x}$$

$$\text{or, } (1+\sin x)dy + (x+y \cos x)dx = 0 \quad \dots (1)$$

$$\text{Now, } \frac{\partial}{\partial x}(1+\sin x) = \cos x; \quad \frac{\partial}{\partial y}(x+y \cos x) = \cos x$$

So equation (1) is an exact differential equation
So the solution of equation (1) is

$$\int (x+y \cos x)dx \quad (\text{keeping } y \text{ as constant})$$

$$+ \int dy = c$$

$$\text{or, } xy + y \sin x + y = c$$

24. (a) Solve $\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = 2 \sin 2x$

(b) Solve $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

Answer:

a) $\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = 2 \sin 2x \quad \dots (1)$

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1)

$$\therefore \text{Auxiliary equation is } m^2 - 6m + 9 = 0$$

$$\text{or, } (m-3)^2 = 0$$

$$\text{or, } m = 3, 3$$

$$\text{So C.F.} = (C_1 + C_2 x)e^{3x}$$

[WBSCTE 2015]

[WBSCTE 2015]

$$\begin{aligned} \text{Now, P.I.} &= \frac{1}{D^2 - 6D + 9} 2 \sin 2x = 2 \frac{1}{-2^2 - 6D + 9} \sin 2x = 2 \frac{1}{5 - 6D} \sin 2x \\ &= 2 \frac{5 + 6D}{25 - 36D^2} \sin 2x = 2 \frac{5 + 6D}{25 - 36(-2^2)} \sin 2x = 2 \frac{5 + 6D}{169} \sin 2x \\ &= \frac{2}{169} (\sin 2x + 12 \cos 2x) \end{aligned}$$

$$\text{So the general solution of (1) is } y = (C_1 + C_2 x)e^{3x} + \frac{2}{169} (\sin 2x + 12 \cos 2x)$$

b) We have $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x} \quad \dots (1)$

$$\text{Auxiliary equation is } m^2 - 5m + 6 = 0$$

$$\text{or, } (m-2)(m-3) = 0 \quad \text{or, } m = 2, 3$$

$$\text{So the C.F.} = C_1 e^{2x} + C_2 e^{3x}$$

$$\text{Now, P.I.} = -\frac{1}{(D-2)(D-3)} e^{4x} = \frac{1}{(4-2)(4-3)} e^{4x} = \frac{e^{4x}}{2}$$

$$\text{So the general solution is } y = C_1 e^{2x} + C_2 e^{3x} + \frac{e^{4x}}{2}$$

25. Solve the differential equation $(D^2 - 3D + 2)y = e^{3x}$

[WBSCTE 2016]

Answer:

$$\text{We have } (D^2 - 3D + 2)y = e^{3x} \quad \dots (1)$$

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of (1).

So the auxiliary equation is

$$m^2 - 3m + 2 = 0$$

$$\text{or, } (m-2)(m-1) = 0$$

$$\text{or, } m = 2, 1$$

So C.F. of (1) is $c_1 e^{2x} + c_2 e^x$, where c_1 and c_2 are two arbitrary constants.

$$\text{P.I.} = \frac{1}{(D-1)(D-2)} e^{3x} = \frac{1}{(3-1)(3-2)} e^{3x} = \frac{e^{3x}}{2}$$

$$\text{So the general solution of (1) is } y = c_1 e^{2x} + c_2 e^x + \frac{e^{3x}}{2}$$

26. Solve: $(D^2 - 2D + 1)y = x^2 e^x$

[WBSCTE 2017]

Answer:

$$\text{We have } (D^2 - 2D + 1)y = x^2 e^x$$

.... (1)

Let $y = Ae^{mx}$ be a trial solution of the reduced equation of Eqn. (1).

∴ A.E. is $m^2 - 2m + 1 = 0$

$$\text{or, } (m-1)^2 = 0$$

$$\text{or, } m = 1, 1$$

So, C.F. = $(C_1 + C_2 x)e^x$

$$P.I. = \frac{1}{(D-1)^2} x^2 e^x = e^x \frac{1}{(D+1-1)^2} x^2 = e^x \frac{1}{D^2} x^2 = e^x \frac{1}{D} \left(\frac{x^3}{3} \right) = e^x \frac{x^4}{12}$$

So the G.S. of Eqn. (1) is $y = (c_1 + c_2 x)e^x + \frac{x^4 e^x}{12}$

27. (a) Solve the differential equation: $x \frac{dy}{dx} + y = x e^x$, given that $y = 1$ when $x = 0$.

(b) Solve the differential equation: $(D^2 - 4D + 4)y = e^{-2x}$. [WBSCTE 2018]

Answer:

a) We have $x \frac{dy}{dx} + y = x e^x$

$$\text{or, } \frac{dy}{dx} + \frac{y}{x} = e^x \quad \dots (1)$$

This is a linear in y .

$$\therefore I \cdot F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Now, multiplying equation (1) by x we get,

$$\frac{d}{dx}(xy) = x e^x$$

On integration we get,

$$xy = x e^x - e^x + c \quad \dots (2)$$

Given, $y = 1$ when $x = 0 \Rightarrow 0 = 0 - 1 + c$

$$\text{or } c = 1$$

∴ $xy = (x-1)e^x + 1$ which is the required solution.

b) We have $(D^2 - 4D + 4)y = e^{-2x}$... (1)

Let $y = Ae^{mx}$ be a trial solution of equation (1).

∴ Auxiliary equation is $m^2 - 4m + 4 = 0$

$$\text{or, } (m-2)^2 = 0$$

$$\text{or, } m = 2, 2$$

$$\therefore C.F. = (c_1 + c_2 x)e^{2x}$$

$$P.I. = \frac{1}{(D-2)^2} e^{-2x} \\ = \frac{1}{(-4)^2} e^{-2x} = \frac{e^{-2x}}{16}$$

So the general solution is $y = (c_1 + c_2 x)e^{2x} + \frac{e^{-2x}}{16}$

28. (a) Find by integration the area of the region bounded by the curve $y = x^2$ and the line $y = 4$. [WBSCTE 2019]

Answer:

Given that, $y = 4$

Let line represent

$$y = 4$$

Also, $y = x^2$

$$\Rightarrow x^2 = y$$

Here, AOB represents

$$x^2 = y$$

We have to find area of AOBA

∴ Area of AOBA = 2 × Area of BONB

$$= 2 \int_0^4 x dy$$

$$\therefore x^2 = y$$

$$\Rightarrow x = \pm \sqrt{y}$$

Since, $x = \pm y$

$$\text{Area of AOBA} = 2 \int_0^4 x dy = 2 \int_0^4 \sqrt{y} dy = 2 \int_0^4 (y)^{\frac{1}{2}} dy$$

$$= 2 \left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^4 = 2 \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 = 2 \left[\frac{2}{3} \left[y^{\frac{3}{2}} \right]_0^4 \right] \\ = \frac{4}{3} \left[(2)^{\frac{3}{2}} \right] = \frac{4}{3} \times 2^{\frac{3}{2}} = \frac{32}{3}$$

(b) If $u(x, y) = \tan^{-1} \frac{y}{x} - \frac{x^2 y}{x^3 + y^3}$, then by Euler's Homogeneous Theorem show

$$\text{that } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0.$$

[WBSCTE 2019]

Answer:

$$\text{Let } u(x, y) = \tan^{-1}\left(\frac{y}{x}\right) - \frac{x^2 y}{x^3 + y^3} = \tan^{-1}\left(\frac{y}{x}\right) + \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^3} = x^0 \phi\left(\frac{y}{x}\right)$$

So, $u(x, y)$ is a homogeneous function in x, y of degree zero.

Therefore, from Euler's theorem we get

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0 \cdot u(x, y) = 0$$

29. Solve the differential equation $2 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} - 3y = 0$, Given $y = 9, \frac{dy}{dx} = 1$ when $x = 0$. [WBSCTE 2019]

Answer:

We have

$$2 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} - 3y = 0 \quad \dots (1)$$

Let $y = Ae^{mx}$ be a trial solution of Eqn. (1)

So the auxiliary equation is

$$2m^2 + 5m - 3 = 0$$

$$\text{or, } 2m^2 + 6m - m - 3 = 0$$

$$\text{or, } 2m(m+3) - 1(m+3) = 0$$

$$\text{or, } (2m-1)(m+3) = 0$$

$$\text{or, } m = \frac{1}{2}, m = -3$$

So the general solution of (1) is

$$y = C_1 e^{\frac{x}{2}} + C_2 e^{-3x}; \quad C_1, C_2 \text{ are arbitrary constant.}$$

Now, given $x = 0, y = 9$

Therefore,

$$9 = C_1 + C_2 \quad \dots (2)$$

$$\text{Again, } x = 0, \frac{dy}{dx} = 1$$

Therefore,

$$1 = \frac{C_1}{2} - 3C_2 \quad \dots (3)$$

Solving (2) and (3) we get,

$$\frac{7C_1}{2} = 28 \Rightarrow C_1 = 8 \quad \text{and} \quad C_2 = 1$$

So the required solution of the differential equation (1) is

$$y = 8e^{\frac{x}{2}} + e^{-3x}$$

30. Solve: $2xy + 3x^2 = \frac{dy}{dx}(2y - x^2 + 1)$

[WBSCTE 2022]

Answer:

$$2xy + 3x^2 = \frac{dy}{dx}(2y - x^2 + 1)$$

$$\Rightarrow (3x^2 + 2xy + x^2)dx = (2y + 1)dy$$

$$\Rightarrow (2xy + 3x^2)dx = (x^2 - 2y + 1)dy$$

$$M = 2xy + 3x^2; N = (x^2 - 2y + 1)$$

$$\therefore \frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$$

General solution of exact differential equation is-

$$\int Mdx + \int Ndy = C$$

$$\Rightarrow \int (2xy + 3x^2)dx + \int (-2y)dy = C$$

$$\Rightarrow x^2 y + x^3 - y^2 = C$$

31. Solve: $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = x^3$.

[WBSCTE 2022]

Answer:

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = x^3$$

Let, $y = Ae^{mx}$ be a trial solution of the reduces equation of (i)

The auxiliary equation is $m^2 + 2m + 1 = 0$

$$\text{or, } m = -1, -1$$

Thus, $CF = c_1 e^{-x} + c_2 x e^{-x}$

Where c_1, c_2 are any arbitrary constants.

$$PI = \frac{1}{(D+1)(D+1)} e^{-x}, D = \frac{1}{dx}$$

$$= -\frac{1}{2} \frac{1}{D+1} e^{-x}$$

$$= -\frac{1}{2} e^x \cdot \frac{1}{D} \cdot 1$$

$$= -\frac{x}{2} e^{-x}$$

∴ Hence the general solution of equation (i) is, $y = c_1 e^{-x} + c_2 e^{-x} - \frac{x}{2} e^{-x}$

⇒ 32. Solve: $y dy = (y^2 - 1)(x + 1) dx$

Answer:

$$y dy = (y^2 - 1)(x + 1) dx$$

$$\therefore y \frac{dy}{dx} = (y^2 - 1)(x + 1)$$

$$\Rightarrow \frac{y}{(y^2 - 1)} dy = (x + 1) dx$$

$$\Rightarrow \frac{2y}{2(y^2 - 1)} dy = (x + 1) dx$$

Taking integration both sides,

$$\int \frac{2y}{2(y^2 - 1)} dy = \int (x + 1) dx$$

$$\Rightarrow \frac{1}{2} \int \frac{2y}{y^2 - 1} dy = \frac{x^2}{2} + x + c$$

$$\Rightarrow \frac{1}{2} \log(y^2 - 1) = \frac{x^2}{2} + x + c$$

$$\Rightarrow \sqrt{(y^2 - 1)} = \frac{x^2}{2} + x + c$$

⇒ 33. Solve $\frac{dy}{dx} + xy = xy^3$.

Answer:

$$\frac{dy}{dx} + xy = xy^3$$

$$\Rightarrow \frac{1}{y^3} \cdot \frac{dy}{dx} + \frac{1}{y^2} x = x$$

$$\text{Let, } \frac{1}{y^2} = z$$

[WBSCTE 2022]

[WBSCTE 2022]

$$\therefore \frac{dz}{dx} - (2z)x = -2x$$

$$P(x) = -2x$$

$$\frac{d}{dx} \{ z e^{\int P(x) dx} \} = e^{\int P(x) dx} \cdot (-2x)$$

$$\Rightarrow \frac{d}{dx} (z e^{-x^2}) = e^{-x^2} \times (-2x) \left[\therefore \int P(x) dx = \int -2x dx = -x^2 \right]$$

$$\Rightarrow z e^{-x^2} = \int e^{-x^2} \times (-2x) dx$$

$$\Rightarrow z e^{-x^2} = -e^{-x^2} + c$$

$$\Rightarrow z = 1 + c e^{x^2}$$

$$\Rightarrow \frac{1}{y^2} = 1 + c e^{x^2}$$

$$\Rightarrow y^2 = \frac{1}{1 + c e^{x^2}}$$

$$\Rightarrow \frac{dz}{dx} = -2 \cdot \frac{1}{y^3} \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y^3} \frac{dy}{dx} = -\frac{1}{z} \frac{dz}{dx}$$

Chapter 5: Partial Differentiation

Chapter at a glance:

5.1 Partial Derivatives

A partial derivatives of a function of several variables is the ordinary derivative with respect to one of the variables when all the remaining variables are held constant.

Consider a function u of three independent variables x, y, z

$$u = f(x, y, z)$$

Keeping y, z constant and varying only x , the partial derivatives of u with respect to x is denoted by $\frac{\partial u}{\partial x}$ and is defined as the limit $\frac{\partial u}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y, z) - f(x, y, z)}{\Delta x}$, when this limit exists.

Partial derivatives of u w.r.to. y and z can be defined similarly and are denoted by

$$\frac{\partial u}{\partial y} \text{ and } \frac{\partial u}{\partial z}$$

$$\therefore \frac{\partial u}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y, z) - f(x, y, z)}{\Delta y}$$

$$\frac{\partial u}{\partial z} = \lim_{\Delta z \rightarrow 0} \frac{f(x, y, z + \Delta z) - f(x, y, z)}{\Delta z}$$

Notation: The partial derivative $\frac{\partial u}{\partial x}$ is also denoted by $\frac{\partial f}{\partial x}$ or $f_x(x, y, z)$ or f_x or $D_x f$ or $f_1(x, y, z)$ where the subscripts x and 1 denote the variable w.r.t. which the partial derivative is carried out. Thus we can have $\frac{\partial u}{\partial y} = \frac{\partial f}{\partial y} = f_y(x, y, z) = f_y =$

$D_y f = f_2(x, y, z)$. The value of the partial derivative at a point (a, b, c) is denoted

$$\text{by } \left. \frac{\partial u}{\partial x} \right|_{(x=a, y=b, z=c)} = \left[\frac{\partial u}{\partial x} \right]_{(a, b, c)} = f_x(a, b, c)$$

Example: If $f(x, y) = 2x^2 - xy + 2y^3$ then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1, 2)$

Answer:

$$\text{Now } \frac{\partial f}{\partial x} = 4x - y \Rightarrow \left[\frac{\partial f}{\partial x} \right]_{(1, 2)} = 4 - 2 = 2$$

$$\frac{\partial f}{\partial y} = -x + 6y \Rightarrow \left[\frac{\partial f}{\partial y} \right]_{(1, 2)} = -1 + 12 = 11$$

5.2 Higher Order Partial Derivatives

Partial derivatives of higher order of a function $f(x, y, z)$ are calculated by successive differentiation. Thus if

$u = f(x, y, z)$ then

$$f_{xx} = f_{11} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \lim_{h \rightarrow 0} \frac{f_x(x+h, y) - f_x(x, y)}{h}$$

$$f_{xy} = f_{12} = \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \lim_{h \rightarrow 0} \frac{f_y(x+h, y) - f_y(x, y)}{h}$$

$$f_{yx} = f_{21} = \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \lim_{k \rightarrow 0} \frac{f_x(x, y+k) - f_x(x, y)}{k}$$

$$f_{yy} = f_{22} = \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) = \lim_{k \rightarrow 0} \frac{f_y(x, y+k) - f_y(x, y)}{k}$$

- In general the crossed or mixed partial derivatives $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ are equal,

$$\text{i.e., } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

i.e., the order of differentiation is immaterial if the derivatives involved are continuous.

- In the subscript notation, the subscripts are written in the same order in which differentiation is carried out, while in the ' ∂ ' notation the order is opposite, for example $2''$
- A function of 2 variables has two first order derivatives, four second order derivatives and 2^n of n th order derivatives. A function of m independent variable will have m^n derivative of order n .

5.3 Homogeneous function

A function $f(x, y)$ is said to be homogeneous of degree (or order) n in the variables x and

y if it can be expressed in the form $f(x, y) = x^n \phi\left(\frac{y}{x}\right)$ or $f(x, y) = y^n \psi\left(\frac{x}{y}\right)$

or, $f(tx, ty) = t^n f(x, y)$

Similarly, a function $f(x, y, z)$ is said to be homogeneous of degree n in the variables x, y, z if it can be expressed in the form.

$$f(x, y, z) = x^n \phi\left(\frac{y}{x}, \frac{z}{x}\right) \text{ or } f(x, y, z) = y^n \psi\left(\frac{x}{y}, \frac{z}{y}\right)$$

$$\text{or } f(x, y, z) = z^n g\left(\frac{x}{z}, \frac{y}{z}\right) \text{ or } f(tx, ty, tz) = t^n f(x, y, z).$$

Example: Show that $f(x, y) = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of x, y and find its

degree. But $\sin^{-1} \frac{\sqrt{x^3+y^3}}{x+y}$ is not a homogeneous function.

Answer:

$$f(x, y) = \frac{x+y}{\sqrt{x}+\sqrt{y}} = \frac{x\left(1+\frac{y}{x}\right)}{\sqrt{x}\left(1+\sqrt{\frac{y}{x}}\right)} = x^{1/2} \phi\left(\frac{y}{x}\right)$$

$\therefore f$ is homogeneous function of x, y of degree $\frac{1}{2}$.

Another method:

$$f(tx, ty) = \frac{tx+ty}{\sqrt{tx}+\sqrt{ty}} = \frac{t(x+y)}{\sqrt{t}(\sqrt{x}+\sqrt{y})} = t^{1/2} \frac{x+y}{\sqrt{x}+\sqrt{y}} = t^{1/2} f(x, y)$$

$\therefore f$ is homogeneous function of x, y of degree $\frac{1}{2}$.

$$\text{Let } g(x, y) = \sin^{-1} \frac{\sqrt{x^3+y^3}}{x+y}$$

$$g(tx, ty) = \sin^{-1} \frac{\sqrt{t^3 x^3 + t^3 y^3}}{tx+ty} = \sin^{-1} \frac{t^{3/2} \sqrt{x^3+y^3}}{t(x+y)} = \sin^{-1} \frac{t^{1/2} \sqrt{x^3+y^3}}{x+y} \neq t^{1/2} \sin^{-1} \frac{\sqrt{x^3+y^3}}{x+y}$$

$\therefore \sin^{-1} \frac{\sqrt{x^3+y^3}}{x+y}$ is not homogeneous function in x, y .

5.4 Euler's Theorem

a) If $f(x, y)$ is a homogeneous function of two variables x, y of degree n

$$\text{then i) } x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf$$

$$\text{ii) } \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right)^2 f = x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1)f$$

b) If $f(x, y, z)$ is a homogeneous function of three variables x, y, z of degree n then

$$\text{i) } x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = nf \quad \text{ii) } \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}\right)^2 f = n(n-1)f$$

Short Answer Type Questions

1. Verify Euler's theorem, when $f(x, y) = x^2 + y^2$

[WBSCTE 2005, 2007]

Answer:

$$f(x, y) = x^2 + y^2 = x^2 \left[1 + \left(\frac{y}{x}\right)^2\right] = x^2 \phi(y/x), \text{ Say}$$

The function is a homogeneous function of degree 2

Let $u = x^2 + y^2$

$$\frac{\partial u}{\partial x} = 2x, \frac{\partial u}{\partial y} = 2y$$

$$\therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = x \cdot 2x + y \cdot 2y = 2(x^2 + y^2) = 2u$$

$\therefore f(x, y)$ satisfy Euler's theorem.

2. Evaluate $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ when $Z = \log(x^2 + y^2)$

[WBSCTE 2005, 2007]

Answer:

$$z = \log(x^2 + y^2).$$

$$\text{Let } u = x^2 + y^2 \therefore z = \log u, \frac{\partial z}{\partial u} = \frac{1}{u}$$

$$\therefore \frac{\partial u}{\partial x} = 2x, \frac{\partial u}{\partial y} = 2y \quad \therefore \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} = \frac{1}{u} \cdot 2x = \frac{2x}{x^2 + y^2}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} = \frac{1}{u} \cdot 2y = \frac{2y}{x^2 + y^2} \text{ Ans.}$$

3. If U is a homogeneous function of degree n in x, y then $\frac{\partial u}{\partial x} \frac{\partial u}{\partial y}$ are each of

homogenous function of x and y of degree -

- (a) $n+1$ (b) n (c) $n-1$

[WBSCTE 2006, 2008]

(d) none of these

Answer: (b)

4. If $u = \log(x^2 + y^2)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2$.

[WBSCTE 2009, 2013]

Answer:

$$u = \log(x^2 + y^2), \frac{\partial u}{\partial x} = \frac{1 \times 2x}{x^2 + y^2} = \frac{2x}{x^2 + y^2}$$

$$\frac{\partial u}{\partial y} = \frac{2y}{x^2 + y^2}, \therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \left(\frac{x^2}{x^2 + y^2} + \frac{y^2}{x^2 + y^2} \right) = 2 \text{ [Proved]}$$

5. If $u = x^3 + y^3$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to -

[WBSCTE 2009]

- (a) $2u$ (b) $3u$ (c) $\frac{1}{3}u$ (d) none of these

Answer: (b)

6. If $u = y/x$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

[WBSCTE 2009, 2016]

Answer:

$$u = \frac{y}{x} = yx^{-1}, \frac{\partial u}{\partial x} = -1 \cdot y \cdot x^{-2} = -\frac{y}{x^2}$$

$$\frac{\partial u}{\partial y} = x^{-1} = \frac{1}{x}, x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{y}{x} + \frac{y}{x} = 0 \text{ [Proved]}$$

Partial Differentiation

[WBSCTE 2009]

7. If $z = \tan^{-1} \frac{4}{x}$, find the value of $\frac{\partial z}{\partial x}$

Answer:

$$Z = \tan^{-1} \frac{4}{x}, \frac{\partial z}{\partial x} = \frac{1}{1 + \frac{16}{x^2}} \cdot 4(-x^{-1}) = \frac{x^2}{x^2 + 16} \times \left(\frac{-4}{x^2} \right) = -\frac{4}{x^2 + 16} \text{ Ans.}$$

8. If $u = \tan^{-1} \frac{y}{x}$, find the value of $x \frac{du}{dx} + y \frac{du}{dy}$

[WBSCTE 2010]

Answer:

$$u = \tan^{-1} \frac{y}{x}$$

$$\therefore \frac{du}{dx} = \frac{1}{1 + \frac{y^2}{x^2}} \cdot \left(-\frac{y}{x^2} \right) = \frac{-y}{x^2 + y^2}; \quad \therefore \frac{du}{dy} = \frac{1}{1 + \frac{y^2}{x^2}} \cdot \left(\frac{1}{x} \right) = \frac{x}{x^2 + y^2}$$

$$\therefore x \frac{du}{dx} + y \frac{du}{dy} = -\frac{xy}{x^2 + y^2} + \frac{xy}{x^2 + y^2} = 0 \text{ Ans.}$$

9. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ prove that $x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = 0$

[WBSCTE 2010]

Answer:

$$\text{Given } u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$$

Therefore, $\frac{du}{dx} = -\frac{z}{x^2} + \frac{1}{y}$ [differentiating partially with respect to x]

$$\text{or, } x \frac{du}{dx} = -\frac{z}{x} + \frac{x}{y} \text{ [multiplying both sides by } x] \quad \dots (1)$$

$$\therefore \frac{du}{dy} = \frac{1}{z} - \frac{x}{y^2} \text{ [differentiating partially with respect to } y]$$

$$\text{or, } y \frac{du}{dy} = \frac{y}{z} - \frac{x}{y} \quad \dots (2)$$

$$\therefore \frac{du}{dz} = -\frac{y}{z^2} + \frac{1}{x} \text{ [differentiating partially with respect to } z]$$

$$\text{or, } z \frac{du}{dz} = -\frac{y}{z} + \frac{z}{x} \quad \dots (3)$$

Adding (1), (2) & (3) we get,

$$x \frac{dy}{dx} + y \frac{du}{dy} + z \frac{dz}{dz} = \frac{z}{x} + \frac{x}{y} + \frac{y}{x} - \frac{x}{y} - \frac{y}{z} + \frac{z}{x} = 0 \text{ (Proved).}$$

10. If $Z = \sin^{-1} \frac{y}{x}$ then $\frac{du}{dx}$ is -

[WBSCTE 2010]

(a) $\frac{1}{\sqrt{x^2 + y^2}}$

(b) $\frac{xy}{\sqrt{x^2 - y^2}}$

(c) $\frac{-y}{\sqrt{x^2 - y^2}}$

(d) none of these

Answer: (c)

11. Evaluate $\frac{\partial z}{\partial x}$, where $z = \log_e(x^2 + y^2)$

[WBSCTE 2010]

Answer:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \log_e(x^2 + y^2) = \frac{1}{x^2 + y^2} \cdot \frac{\partial}{\partial x} (x^2 + y^2) = \frac{1}{x^2 + y^2} [2x + 2y \frac{dy}{dx}]$$

12. If $u = \sin^{-1} \frac{y}{x}$, find the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

[WBSCTE 2011]

Answer:

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1 - \frac{y^2}{x^2}}} \left(\frac{-y}{x^2} \right) = \frac{-y}{x\sqrt{x^2 - y^2}} = \frac{1}{\sqrt{x^2 - y^2}}$$

13. If $z = \frac{x}{y}$, the value of $\frac{x\partial z}{\partial x} + \frac{y\partial z}{\partial y}$ is -

[WBSCTE 2011]

(a) 0

(b) 1

(c) -1

(d) none of these

Answer: (a)

14. If $u = \tan^{-1} \frac{y}{x}$ find $\frac{\partial u}{\partial x}$.

[WBSCTE 2011, 2016, 2019]

Answer: $\frac{\partial u}{\partial x} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(\frac{-y}{x^2} \right) = \frac{-y}{x^2 + y^2}$

15. If $u = e^{\frac{y}{x}}$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

[WBSCTE 2012]

Answer:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = x e^{\frac{y}{x}} \cdot \frac{1}{y^2} \cdot \frac{y}{x} - 0 + y e^{\frac{y}{x}} \cdot \frac{0 - 1}{x^2} = 0$$

16. If $u = \frac{y}{x}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

[WBSCTE 2012]

Answer:

$u = \frac{y}{x}$ is a homogeneous function of order 0.

So, by Evles's formula of homogeneous function, we get $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

17. If $z = \tan^{-1} \left(\frac{y}{x} \right)$, find the value of $\frac{\partial z}{\partial x}$.

[WBSCTE 2013]

Answer: Let $z = \tan^{-1} \frac{y}{x}$

18. If $u = \sin^{-1} \left(\frac{y}{x} \right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

[WBSCTE 2014]

(a) 1

(b) 0

(c) 2

(d) None of these

Answer: (b)

19. Euler's theorem can be applied to a function which is -

[WBSCTE 2015]

(a) Linear

(b) homogenous

(c) differentiable

(d) none of these

Answer: (b)

20. If $u = xy + yz + zx$ then the value of $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right)$ at the point (1,1,1) is -

[WBSCTE 2015]

(a) 2

(b) 6

(c) 3

(d) 0

Answer: (b)

21. If $z = 3x^2y$ then $\frac{\partial^2 z}{\partial x \partial y} =$

[WBSCTE 2017]

(a) 3y

(b) 6x

(c) 6y

(d) none of these

Answer: (b)

22. If $u = x^2 + xy + y^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ equals to

[WBSCTE 2017]

- (a) u (b) $2u$ (c) $3u$ (d) none

Answer: (b)

23. If $u(x, y) = \sqrt{\frac{x^2}{x+y}}$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ?$

[WBSCTE 2022]

Answer:

$$u(x, y) = \sqrt{\frac{x^2}{x+y}}$$

$$u(x, y) = \sqrt{\frac{x^2 t^2}{xt + yt}} = \sqrt{\frac{x^2 t^2}{t(x+y)}} = \frac{\sqrt{x^2 t^2}}{\sqrt{t(x+y)}} = \frac{t^2}{t^{1/2}} = t^{3/2} = t^{\frac{3}{2}}$$

$$u(x, y) = t^{\frac{3}{2}} u(x, y) \quad [u \text{ is a homogeneous function, } n = \frac{3}{2}]$$

According to Euler's theorem,

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n u$$

$$= \frac{3}{2} u \quad \text{Ans.}$$

24. Is the differential equation homogenous: $\frac{dy}{dx} = \frac{3xy^2 + 2x^2y}{5y^3 + x^2}$? Yes/No

[WBSCTE 2022]

Answer: Yes

25. Write the degree of the differential equation $\left(1 - 9 \frac{d^2 y}{dx^2}\right)^{\frac{1}{2}} + 7 \frac{dy}{dx} - y^2 = 0$

[WBSCTE 2022]

Answer: Degree = 2

Long Answer Type Questions

1. If $U = \tan^{-1} \frac{x^3 + y^3}{x - y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2U$.

[WBSCTE 2004, 2010, 2012, 2018]

Answer:

$$U = \tan^{-1} \frac{x^3 + y^3}{x - y}$$

$$\text{or, } \tan U = \frac{x^3 + y^3}{x - y} = \frac{x^3 \left(1 + \frac{y^3}{x^3}\right)}{x \left(1 - \frac{y}{x}\right)} = x^2 \frac{\left(1 + \frac{y^3}{x^3}\right)}{\left(1 - \frac{y}{x}\right)}$$

$\therefore \tan U$ is a homogenous function of degree 2. So by applying Euler's theorem, we get

$$x \frac{\partial u}{\partial x} (\tan U) + y \frac{\partial}{\partial y} (\tan U) = 2 \tan U$$

$$\text{or, } x \frac{\partial}{\partial x} (\tan U) \frac{\partial u}{\partial x} + y \frac{\partial}{\partial y} (\tan U) \frac{\partial u}{\partial y} = 2 \tan U$$

$$\text{or, } x \sec^2 U \frac{\partial u}{\partial x} + y \sec^2 U \frac{\partial u}{\partial y} = 2 \tan U$$

$$\text{or, } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2U. \quad (\text{Proved})$$

2. (a) If $z(x, y) = x^2 + y^2$ then show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4 \left(1 - \frac{\partial z}{\partial x} \frac{\partial z}{\partial y}\right)$

[WBSCTE 2004, 2014]

Answer:

$$z(x, y) = x^2 + y^2 \quad \text{or, } z = \frac{x^2 + y^2}{x + y}$$

$$\therefore \frac{\partial z}{\partial x} = \frac{(x+y)2x - (x^2 + y^2)}{(x+y)^2} = \frac{x^2 + 2xy - y^2}{(x+y)^2}$$

$$\therefore \frac{\partial z}{\partial y} = \frac{(x+y)2y - (x^2 + y^2)}{(x+y)^2} = \frac{y^2 + 2xy - x^2}{(x+y)^2}$$

$$\begin{aligned} \therefore \left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 &= \left[\frac{x^2 + 2xy - y^2 - y^2 - 2xy + x^2}{(x+y)^2} \right]^2 = \left[\frac{2(x^2 - y^2)}{(x+y)^2} \right]^2 \\ &= \left[2 \frac{(x+y)(x-y)}{(x+y)^2} \right]^2 = \frac{4(x-y)^2}{(x+y)^2} \\ \therefore 4 \left\{ 1 - \left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right) \right\} &= 4 \left\{ 1 - \frac{x^2 + 2xy - y^2 + y^2 + 2xy - x^2}{(x+y)^2} \right\} \\ &= 4 \left\{ 1 - \frac{4xy}{(x+y)^2} \right\} = 4 \left\{ \frac{(x+y)^2 - 4xy}{(x+y)^2} \right\} = 4 \frac{(x-y)^2}{(x+y)^2} \\ \therefore \left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 &= 4 \left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 \quad \text{(Proved).} \end{aligned}$$

2. (b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

[WBSCTE 2004, 2005, 2007, 2009, 2011, 2015]

Answer:

$$\begin{aligned} u &= \log(x^3 + y^3 + z^3 - 3xyz) = \log\{(x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)\} \\ &= \log(x+y+z) + \log(x^2 + y^2 + z^2 - xy - yz - zx) \end{aligned}$$

$$\begin{aligned} \therefore \frac{\partial u}{\partial x} &= \frac{1}{x+y+z} + \frac{2x-y-z}{x^2 + y^2 + z^2 - xy - yz - zx} \\ \frac{\partial u}{\partial y} &= \frac{1}{x+y+z} + \frac{2y-x-z}{x^2 + y^2 + z^2 - xy - yz - zx} \\ \frac{\partial u}{\partial z} &= \frac{1}{x+y+z} + \frac{2z-y-x}{x^2 + y^2 + z^2 - xy - yz - zx} \\ \therefore \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} &= \frac{3}{x+y+z} + \frac{2x-y-z+2y-x-z+2z-y-x}{x^2 + y^2 + z^2 - xy - yz - zx} \\ &= \frac{3}{x+y+z} + 0 = \frac{3}{x+y+z} \quad \text{Proved.} \end{aligned}$$

3. If $u = \cos^{-1} \left\{ \frac{x+y}{\sqrt{x}+\sqrt{y}} \right\}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$.

[WBSCTE 2005, 2006, 2015]

Answer:

$$\begin{aligned} u &= \cos^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}} \quad \text{or, } \cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}} \\ \text{or, } \cos u &= \frac{x \left(1 + \frac{y}{x} \right)}{x^{\frac{1}{2}} \left[1 + \left(\frac{y}{x} \right)^{\frac{1}{2}} \right]} = x^{\frac{1}{2}} f \left(\frac{y}{x} \right) \end{aligned}$$

So $\cos u$ is homogenous function of degree $\frac{1}{2}$

$$\therefore x \frac{\partial}{\partial x} (\cos u) + y \frac{\partial}{\partial y} (\cos u) = \frac{1}{2} \cos u \quad \text{or, } -x \sin u \frac{\partial u}{\partial x} - y \sin u \frac{\partial u}{\partial y} = \frac{1}{2} \cos u$$

$$\text{or, } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u \quad \text{or, } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0. \quad \text{Proved.}$$

4. (a) If $u = \sin^{-1} \left(\frac{x}{y} \right) + \tan^{-1} \left(\frac{y}{x} \right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

[WBSCTE 2005, 2006, 2008, 2013, 2017]

Answer:

$$\begin{aligned} u &= \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x} \\ \frac{\partial u}{\partial x} &= \frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \cdot \frac{1}{y} + \frac{1}{\left(1 + \frac{y^2}{x^2} \right)} \cdot y(-1)x^{-2} = \frac{1}{\sqrt{y^2-x^2}} - \frac{x^2}{x^2+y^2} \cdot \frac{y}{x^2} \\ \therefore x \frac{\partial u}{\partial x} &= \frac{x}{\sqrt{y^2-x^2}} - \frac{xy}{x^2+y^2} \\ \therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} &= 0 \quad \text{Proved.} \end{aligned}$$

4. (b) If $u = \sin^{-1} \frac{x^2+y^2}{x+y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.

[WBSCTE 2005, 2007, 2009, 2011]

Answer:

$$u = \sin^{-1} \frac{x^2 + y^2}{x + y} \quad \text{or, } \sin u = \frac{x^2 + y^2}{x + y} = \frac{x^2 \left(1 + \frac{y^2}{x^2}\right)}{x \left(1 + \frac{y}{x}\right)} = x^2 \left[\frac{1 + \left(\frac{y}{x}\right)}{1 + \left(\frac{y}{x}\right)} \right] = x^2 \phi(y/x)$$

$\therefore \sin u$ is a homogeneous function of degree 1. So by Euler's theorem we get

$$\therefore x \frac{\partial}{\partial x} (\sin u) + y \frac{\partial}{\partial y} (\sin u) = \sin u$$

$$\text{or, } x \cos u \frac{\partial u}{\partial x} + y \cos u \frac{\partial u}{\partial y} = \sin u \quad \text{or, } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u. \text{ Proved.}$$

5. Verify Euler's theorem for $U = x^3 + 3x^2y + 3xy^2 + y^3$ [WBSCTE 2006, 2013]

Answer:

$$U = x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3 = x^3 \left(1 + \frac{y}{x}\right)^3$$

$\therefore U$ is homogeneous function of degree 3.

$$\frac{\partial U}{\partial x} = 3x^2 + 6xy + 3y^2 \cdot 1 = 3x^2 + 6xy + 3y^2$$

$$\frac{\partial U}{\partial y} = 3x^2 \cdot 1 + 3x \cdot 2y + 3y^2 = 3x^2 + 6xy + 3y^2$$

$$\therefore x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} = 3x^3 + 6x^2y + 3xy^2 + 3x^2y + 6xy^2 + 3y^3$$

$$= 3x^3 + 9x^2y + 9xy^2 + 3y^3 = 3(x^3 + 3x^2y + 3xy^2 + y^3) = 3U$$

Euler's theorem is valid for $U = x^3 + 3x^2y + 3xy^2 + y^3$

6. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ prove that $x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = 0$ [WBSCTE 2010, 2012]

Answer:

$$\text{Given } u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$$

$$\text{Therefore, } \frac{du}{dx} = -\frac{z}{x^2} + \frac{1}{y} \quad [\text{differentiating partially with respect to } x]$$

$$\text{or, } x \frac{du}{dx} = -\frac{z}{x} + \frac{x}{y} \quad [\text{multiplying both sides by } x] \quad \dots (1)$$

$$\therefore \frac{du}{dy} = \frac{1}{z} - \frac{x}{y^2} \quad [\text{differentiating partially with respect to } y]$$

$$\text{or, } y \frac{du}{dy} = \frac{y}{z} - \frac{x}{y} \quad \dots (2)$$

$$\therefore \frac{du}{dz} = -\frac{y}{z^2} + \frac{1}{x} \quad [\text{differentiating partially with respect to } z]$$

$$\text{or, } z \frac{du}{dz} = -\frac{y}{z} + \frac{z}{x} \quad \dots (3)$$

Adding (1), (2) and (3) we get,

$$x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = \frac{z}{x} + \frac{y}{y} - \frac{x}{y} - \frac{y}{z} + \frac{z}{x} = 0 \quad (\text{Proved}).$$

7. If $u = \log_e \left(\frac{x^4 + y^4}{x + y} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ [WBSCTE 2014]

Answer:

$$u = \log_e \frac{x^4 + y^4}{x + y} = \log_e (x^4 + y^4) - \log_e (x + y)$$

$$\therefore \frac{du}{dx} = \frac{4x^3}{x^4 + y^4} - \frac{1}{x + y} \quad ; \quad \therefore \frac{\partial u}{\partial y} = \frac{4y^3}{x^4 + y^4} - \frac{1}{x + y}$$

$$\therefore x \frac{\partial u}{\partial y} = \frac{4x^4}{x^4 + y^4} - \frac{x}{x + y} \quad ; \quad \therefore y \frac{\partial u}{\partial y} = \frac{4y^4}{x^4 + y^4} - \frac{y}{x + y}$$

$$\therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{4x^4}{x^4 + y^4} - \frac{x}{x + y} + \frac{4y^4}{x^4 + y^4} - \frac{y}{x + y} \\ = 4 \left(\frac{x^4 + y^4}{x^4 + y^4} \right) - \frac{(x + y)}{(x + y)} = 4 - 1 = 3 \quad (\text{Proved})$$

8. If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x} - \frac{xy}{x^2 + y^2}$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

[WBSCTE 2016]

Answer:

$$u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x} - \frac{xy}{x^2 + y^2}$$

$$\begin{aligned}\frac{\partial u}{\partial x} &= \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \cdot \frac{1}{y} + \frac{1}{1+\left(\frac{y}{x}\right)^2} \left(\frac{-y}{x^2}\right) - \frac{y}{x^2+y^2} + \frac{2x^2y}{(x^2+y^2)^2} \\ &= \frac{1}{\sqrt{y^2-x^2}} + \frac{(-y)}{x^2+y^2} - \frac{y}{x^2+y^2} + \frac{2x^2y}{(x^2+y^2)^2} = \frac{1}{\sqrt{y^2-x^2}} - \frac{2y}{x^2+y^2} + \frac{2x^2y}{(x^2+y^2)^2} \\ &= \frac{1}{\sqrt{y^2-x^2}} - \frac{2y^3}{(x^2+y^2)^2}\end{aligned}$$

$$\begin{aligned}\text{and } \frac{\partial u}{\partial y} &= \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \left(\frac{-x}{y^2}\right) + \frac{1}{1+\left(\frac{y}{x}\right)^2} \left(\frac{+4}{x}\right) - \frac{x}{x^2+y^2} + \frac{2xy^2}{(x^2+y^2)^2} \\ &= \frac{-x}{y\sqrt{y^2-x^2}} + \frac{x}{x^2+y^2} - \frac{x}{x^2+y^2} + \frac{2xy^2}{(x^2+y^2)^2} = \frac{-x}{y\sqrt{y^2-x^2}} + \frac{2xy^2}{(x^2+y^2)^2}\end{aligned}$$

$$\therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{x}{\sqrt{y^2-x^2}} - \frac{2xy^3}{(x^2+y^2)^2} - \frac{x}{\sqrt{y^2-x^2}} + \frac{2xy^3}{(x^2+y^2)^2} = 0$$

9. If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{2}{x+y}$ [WBSCTE 2016]

Answer:

$$\begin{aligned}\text{We have, } u &= \log(x^3 + y^3 - x^2y - xy^2) = \log(x^3 + y^3 - xy(x+y)) \\ &= \log((x+y)^3 - 3xy(x+y) - xy(x+y)) = \log((x+y)^3 - 4xy(x+y)) \\ &= \log((x+y)((x+y)^2 - 4xy)) = \log((x+y)(x-y)^2) = \log(x+y) + 2\log(x-y)\end{aligned}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{1}{x+y} + \frac{2}{x-y}; \quad \frac{\partial u}{\partial y} = \frac{1}{x+y} - \frac{2}{x-y}$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{2}{x+y} \quad (\text{approved})$$

10. If $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$ [WBSCTE 2017]

Answer:

$$u = e^{xyz}$$

$$\therefore \frac{\partial u}{\partial x} = yze^{xyz}$$

$$\frac{\partial^2 u}{\partial x \partial y} = ze^{xyz} + xyz^2e^{xyz} = (z + xyz^2)e^{xyz}$$

$$\therefore \frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 2xyz)e^{xyz} + xy(z + xyz^2)e^{xyz} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

11. If $u = \frac{x^2+y^2}{x+y}$, then show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right) = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$. [WBSCTE 2018]

Answer:

$$\text{Given } u = \frac{x^2+y^2}{x+y}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{2x}{x+y} - \frac{x^2+y^2}{(x+y)^2}$$

$$\text{and } \frac{\partial u}{\partial y} = \frac{2y}{x+y} - \frac{x^2+y^2}{(x+y)^2}$$

$$\therefore \text{L.H.S} = \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = \frac{2(x-y)}{x+y}$$

$$\text{R.H.S} = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= 4\left(1 - \frac{2(x+y)}{x+y} + \frac{2(x^2+y^2)}{(x+y)^2}\right) = 4\left(1 - 2 + \frac{(x^2+y^2)}{(x+y)^2}\right)$$

$$= 4\left(\frac{2(x^2+y^2)}{(x+y)^2} - 1\right) = 4\left(\frac{2(x^2+y^2) - x^2 - y^2 - 2xy}{(x+y)^2}\right)$$

$$= 4\left(\frac{x^2+y^2-2xy}{(x+y)^2}\right) = 4\left(\frac{(x-y)^2}{(x+y)^2}\right)$$

$$\therefore \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$

12. If $z = \tan^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{1}{4} \sin 2z$ [WBSCTE 2022]

Answer:

$$z = \tan^{-1} \left(\frac{x-y}{\sqrt{x}+\sqrt{y}} \right)$$

$$\tan z = \frac{x-y}{\sqrt{x}+\sqrt{y}} f(x, y) \text{ (say)}$$

$$\text{Now, } f(x, y) = \frac{x-y}{\sqrt{x}+\sqrt{y}} = \frac{x \left(1 + \frac{y}{x} \right)}{\sqrt{x} \left(1 + \sqrt{\frac{y}{x}} \right)} = x^{\frac{1}{2}} \phi \left(\frac{y}{x} \right)$$

Therefore, $f(x, y)$ is a homogeneous function of order $\frac{1}{2}$.

Now, by Euler's theorem,

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{2} f(x, y)$$

$$\Rightarrow x \frac{\partial}{\partial x} \tan z + y \frac{\partial}{\partial y} \tan z = \frac{1}{2} \tan z$$

$$\Rightarrow x \left(\sec^2 z \right) \frac{\partial z}{\partial x} + y \sec^2 z \left(\frac{\partial z}{\partial y} \right) = \frac{1}{2} \tan z$$

$$\Rightarrow x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \left(\frac{1}{2} \sin 2z \right) \frac{1}{2}$$

$$\Rightarrow x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{4} \sin 2z$$

Chapter 6: Statistics & Probability

Chapter at a glance:

6.1 Introduction

The most important types of the measure of central tendency are *Mean*, *Median*, *Quartiles* and *Mode*.

6.1.1 Mean

There are three types of means, namely, *Arithmetic Mean (A.M.)*, *Geometric Mean (G.M.)* and *Harmonic Mean (H.M.)*.

Let a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$ with frequencies $f_1, f_2, f_3, \dots, f_n$ respectively.

▪ **Arithmetic Mean (A.M.):** Arithmetic Mean may be of two types – *Simple A.M.* and *Weighted A.M.*

○ **Simple A.M.** $(\bar{x}) = \frac{1}{n} \sum_{i=1}^n x_i$

○ **Weighted A.M.** $(\bar{x}) = \frac{1}{N} \sum_{i=1}^n f_i x_i$ where $N = \sum_{i=1}^n f_i$

▪ **Geometric Mean (G.M.):** Geometric Mean may be of two types – *Simple G.M.* and *Weighted G.M.*

○ **Simple G.M.** $= (x_1 \times x_2 \times x_3 \times \dots \times x_n)^{\frac{1}{n}}$

○ **Weighted G.M.**

$$= (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \dots \times x_n^{f_n})^{\frac{1}{N}} \text{ where } N = \sum_{i=1}^n f_i$$

▪ **Harmonic Mean (H.M.):** Geometric Mean may be of two types – *Simple H.M.* and *Weighted H.M.*

○ **Simple H.M.** $= \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}}$

o **Weighted H.M.** = $\frac{1}{\frac{1}{N} \sum_{i=1}^n \frac{f_i}{x_i}}$ where $N = \sum_{i=1}^n f_i$

Determination of A.M. of Grouped Frequency Distribution

For a grouped frequency distribution, the mid-value (i.e., average of two class limits) of a class is taken as representative value of the class. So, $x_1, x_2, x_3, \dots, x_n$ will represent the mid values of the classes with frequencies $f_1, f_2, f_3, \dots, f_n$ respectively. If the class intervals have a common width then width = i , and if not, i = H.C.F. of all the class widths. Then A.M. is given by

$$\bar{x} = A + \frac{\sum f_i d}{N} \times i \text{ where } N = \sum_{i=1}^n f_i \text{ and } d = \frac{x - A}{i}$$

Relations among A.M., G.M. and H.M.

The relations among A.M., G.M. and H.M. is as follows:

(i) $(A.M.) \times (H.M.) = (G.M.)^2$ (ii) $A.M. \geq G.M. \geq H.M.$

6.1.2 Median

The median of a variable is defined as the middlemost value when its values are arranged in ascending or descending order of magnitude.

Steps for finding median

• For simple data

- Arrange the data in ascending or descending order of magnitude.
- If the total number of values of the variable, say n , is odd then the median is the

$\frac{n+1}{2}$ th value in the arrangement. If the total number of values of the variable,

say n , is even then the median is the average of the $\frac{n}{2}$ th and $\frac{n+1}{2}$ th value in the arrangement.

• For simple frequency data

- Arrange the data in ascending or descending order of magnitude

- Compute the cumulative frequency ('less than' type) of each item. If the total frequency is $N (= \sum_{i=1}^n f_i)$ then the value of the variable corresponding to the cumulative frequency $\frac{N+1}{2}$ gives the median of the distribution.

• For grouped frequency data

- Compute the cumulative frequency ('less than' type) of each class. If the total frequency is $N (= \sum_{i=1}^n f_i)$ then the class corresponding to the cumulative frequency $\frac{N}{2}$ is called the *median class*.

- Compute the median using the formula

$$\text{Median } (M) = l + \frac{\frac{N}{2} - C}{f} \times i$$

where l = lower boundary of the median class

N = total frequency

C = cumulative frequency of the class preceding the median class

f = frequency of the median class

i = class width of the median class

= upper class boundary - lower class boundary)

6.1.3 Mode

Mode of a distribution is the value of the variable that occurs with highest frequency.

Steps for finding Mode

• For simple data

Sort out that value of the variable occurring most frequently in the given data. It is the mode.

• For simple frequency distribution

Sort out that value of the variable having highest frequency in the distribution. It is the mode.

• For grouped frequency distribution

- Locate the *modal class* (i.e., the class having highest frequency)
- Compute mode using the following formula:

$$\text{Mode}(m) = l + \frac{f - f_1}{2f - f_1 - f_2} \times i$$

where l = lower boundary of the modal class

f = frequency of the modal class

f_1 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

i = width of the modal class.

Relation among Mean, Median and Mode

The following is the empirical relation among mean, median and mode:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

6.2 Standard Deviation (S.D.)

(i) **For simple data:** Let $x_1, x_2, x_3, \dots, x_n$ be a set of n observations and $\bar{x}$ be their A.M.. Then their Standard Deviation is denoted by σ and defined as

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

(ii) **For frequency data:** Let $x_1, x_2, x_3, \dots, x_n$ be a set of n observations with frequencies $f_1, f_2, f_3, \dots, f_n$ respectively and $\bar{x}$ be their A.M.. Then their Standard

Deviation is denoted by σ and defined as $\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2}$ where $N = \sum_{i=1}^n f_i$.

6.3 Random experiment

Experiment in which we know all possible outcomes but we do not know the outcome in a particular performance.

Sample space: Set of all possible outcomes is called the sample space corresponding to the random experiment E .

Event: Any subset of the sample space is called an event of the random experiment E .

Event is classified into three types:

- i) simple event
- ii) compound event
- iii) impossible event.

Simple event: Event which denotes only one outcome is called the simple event.

Compound event: Event which can be decomposed into two or more simple events.

Impossible event: Event which can't be occurred in the random experiment.

Mutually exclusive events: Any two events A and B are said to mutually exclusive if $A \cap B = \phi$.

Exhaustive events: A set of events $A_1, A_2, \dots, A_n$ are said to be exhaustive if

$$A_1 \cup A_2 \cup \dots \cup A_n = S \text{ (Sample space)}$$

Equally likely events: If the possibility of occurrence of any two events A and B are equal then they are called equally likely events.

6.4 Classical Definition of Probability

Let A be any event related to the random experiment E in which the sample space is S . If S containing $n(S)$ points and A contains $n(A)$ number of points then the probability of

$$\text{the event } A \text{ is } P(A) = \frac{n(A)}{n(S)}$$

Axiomatic definition of probability: If probability of an event A is denoted by $P(A)$ then $P(A)$ satisfy the following conditions:

- i) $P(A) \geq 0$
- ii) $P(S) = 1$
- iii) For any set of pairwise mutually exclusive events $A_1, A_2, A_3, \dots, A_n, \dots$
 $P(A_1 \cup A_2 \cup \dots \cup A_n \cup \dots) = P(A_1) + P(A_2) + \dots + P(A_n) + \dots$

Short Answer Type Questions

1. If $\text{var}(x) = 5$ and $y = 5x + 6$ then $\text{var}(y)$ is equal to – [WBSCTE 2004, 2013]
 (a) 125 (b) 150 (c) 6 (d) none

Answer: (a)

2. Find the standard deviation of the following numbers : 1, 2, 3, 4, 5, 6, 7, 8, 9. [WBSCTE 2004, 2013, 2016]

Answer:

$$\text{Mean } \bar{x} = \frac{1}{9}(1 + 2 + 3 + \dots + 9) = 5$$

X_i	$di = X_i - \bar{X}$	di^2
1	-4	16
2	-3	9

3	-2	4
4	-1	1
5	0	0
6	1	1
7	2	4
8	3	9
9	4	16
$\sum di = 0$	$\sum di^2 = 60$	$N = 9$

$$\therefore Sd = \sqrt{\frac{1}{N} \sum di^2 - \left(\frac{\sum di}{N} \right)^2} = \sqrt{\frac{1}{9} \times 60 - 0} = \sqrt{\frac{20}{3}} = 2.58 \text{ Ans.}$$

3. Three coins are tossed at random, then the probability that there will be at least one head is –
 (a) $3/8$ (b) $7/8$ (c) $2/9$ (d) None [WBSCTE 2004, 2013]

Answer: (b)

4. Find the chance of throwing an even number with an ordinary six faced die.
 [WBSCTE 2005, 2007]

Answer: $\frac{3}{4}$

5. Two unbiased coins are tossed. Then the probability of obtaining at least one tail is –
 (a) $4/3$ (b) $1/4$ (c) $1/3$ (d) one of these [WBSCTE 2006, 2008]

Answer: (b)

6. One card is drawn from a standard pack of 52. The probability which is either king or queen is –
 (a) $1/13$ (b) $2/13$ (c) $3/13$ (d) none of these [WBSCTE 2006, 2008, 2010, 2017]

Answer: (b)

7. A die is thrown once, then the probability of obtaining a six is
 (a) $1/3$ (b) $1/2$ (c) $1/6$ (d) none of these [WBSCTE 2006, 2013]

Answer: (c)

8. The mean, Median, Mode of the data: 0,5,-1,5,2,1,-1,4,3,0,0,3,3, are –
 (a) 2,3,3 (b) 2,2,3 (c) 3,3,3 (d) none of these [WBSCTE 2006, 2008]

Answer: (b)

9. An unbiased dice is rolled, the probability that an odd point or a six will appear on the top of the dice is –
 (a) $1/2$ (b) $1/3$ (c) $2/3$ (d) none of these [WBSCTE 2009, 2010, 2013]

Answer: (c)

10. A coin is tossed three times in succession, write the sample space.
 [WBSCTE 2009, 2013, 2016, 2018]

Answer:

All possible outcomes of 3 throws are (H B H), (H T T), (H H T), (H T H), (T H H), (T H T), (T T H) and (T T T) i.e.8.

11. Find the means, median and mode of the following
 33, 32, 33.5, 33, 31.5, 32, 31, 32, 33, 32. [WBSCTE 2009]

Answer:

Frequency distribution table for mean, median and mode:

Serial No.	Variable (x_i)	Frequency (f_i)	Deviation (d_i)	$f_i d_i$	Cumulative Frequency F
1	31.0	1	-1	-1	10
2	31.5	1	-0.5	-0.5	9
3	32.0	4	0	0	5
4	32.5	0	+0.5	0	5
5	33.0	3	+1	+3	2
6	33.5	1		+1.5	
		$\sum f_i = N$		$\sum f_i d_i = +3$	

(i) Calculation of Mean ($\bar{x}$):

Assumed mean (a) = 32, class width (h) = 0.5

Total frequency (N) = 10, $\sum f_i d_i = +3$

$$\therefore \text{Mean } (\bar{x}) = a + \frac{h}{N} \sum f_i d_i = 32 + \frac{0.5}{10} \times (+3) = 32 + 0.15 = 32.15 \text{ Ans.}$$

(ii) **Calculation of Median:** So median is the value of the variate for which cumulative frequency $F \geq \frac{N}{2}$, which is 32 here.

$\therefore$ Median = 32 Ans.

(iii) **Calculation of Mode:** According to the definition of the mode, it is the value of the variable corresponding to the highest frequency. So here mode = 32. Ans.

12. The probability of getting at least any one of the following events, point 'six' or 'one' on the top in rolling of an unbiased die once is [WBSCTE 2010]

- (a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) none of these

Answer: (c)

13. The means of the following data:

Age 'x' - year:	12	13	14	15
No. of boys f:	2	3	2	1

is

- (a) 12.5 yrs (b) 13 yrs (c) 13.25 yrs (d) none of these

[WBSCTE 2010]

Answer: (c)

14. The Median of the following data 3.1, 2.6, 5.0, 4.7, 4.2, 3.9, 51.1 and 3.6 is

- (a) 3.75 (b) 4.7 (c) 3.6 (d) none of these

[WBSCTE 2010]

Answer: (d)

15. Median of the data 2.5, 2.0, 2.6, 2.2, 2.9 and 2.1 is -

- (a) 2.2 (b) 2.383 (c) 2.35 (d) none of these

[WBSCTE 2011, 2013, 2016, 2019]

Answer: (c)

16. Two coins are tossed, the probability of getting at least one head is - [WBSCTE 2011]

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) none of these

Answer: (c)

17. One unbiased die is rolled, the probability of an even point or one point will appeared top of the die is - [WBSCTE 2011, 2012]

- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) none of these

Answer: (c)

18. The mean and median of the data 0, 2, 1, 5, 7, 2, 3, 4, respectively are -

- (a) (2, 3) (b) (2.4, 2.5) (c) (3, 3) (d) none

[WBSCTE 2012]

Answer: (c)

19. The probability of any event A satisfies -

- (a) $P(A) \geq 1$, (b) $P(A) < 0$, (c) $0 \leq P(A) \leq 1$, (d) none

[WBSCTE 2013]

Answer: (c)

20. An urn contains 6 white balls and 12 black balls. A ball is drawn at random. What is the probability of being it to be a white? [WBSCTE 2014, 2016]

- (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

Answer: (d)

21. A die is rolled, the probability of getting an odd point or six is -

- (a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) None

[WBSCTE 2014]

Answer: (c)

22. If $\text{var}(x) = 5$ and $y = 5x + 6$ then $\text{var}(y)$ is equal to -

- (a) 125 (b) 150 (c) 25 (d) None

[WBSCTE 2014]

Answer: (a)

23. Median of the data 2.5, 2.0, 2.6, 2.2, 2.9 and 2.1 is -

- (a) 2.2 (b) 2.383 (c) 2.35 (d) 2

[WBSCTE 2014]

Answer: (c)

24. The probability of obtaining an even number in the throw of an unbiased die is -

- (a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) 1 (d) $\frac{1}{2}$

[WBSCTE 2015]

Answer: (d)

25. A coin is tossed. The events $\{H\}$, $\{T\}$ are –

- (a) mutually exclusive (b) independent
(c) not exhaustive (d) none of these

Answer: (a)

26. The relation among mean, median and mode is –

- (a) Mode = 3(Median) – 2(Mean) (b) Mode = 3(Median) + 2(Mean)
(c) Mode = 2(Median) – 3(Mean) (d) Mode = 2(Median) + 3(Mean)

Answer: (a)

27. The median of the distribution 7, 9, 5, 3, 10, 15, 21, 19, 17 is –

- (a) 12 (b) 10 (c) 13 (d) 11

Answer: (b)

28. Let A and B be two events and $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$ then the value of $P(A \cup B)$ is –

- (a) 0.4 (b) 0.6 (c) 0.06 (d) None of these

Answer: (a)

29. If $Var(X) = 3$ and $Y = 2X + 3$ then $Var(Y)$ is equal to –

- (a) 6 (b) 12 (c) 15 (d) 9

Answer: (b)

30. Let ' A ' be any event then $P(A)$ is always –

- (a) $0 \leq P(A) \leq 1$ (b) $1 \leq P(A) \leq 0$ (c) $-1 \leq P(A) \leq 1$ (d) $1 \leq P(A) \leq 2$

Answer: (a)

31. The probability of any event A satisfies

- (a) $P(A) \geq 1$ (b) $P(A) < 0$ (c) $0 \leq P(A) \leq 1$ (d) none

Answer: (c)

32. Two events A and B are mutually exclusive if

- (a) $P(A \cap B) = P(A)P(B)$ (b) $P(A \cap B) = 1$
(c) $P(A \cap B) = 0$ (d) none of these

Answer: (c)

[WBSCTE 2015]

[WBSCTE 2015]

[WBSCTE 2015]

[WBSCTE 2016]

[WBSCTE 2016]

[WBSCTE 2016]

[WBSCTE 2017]

[WBSCTE 2017]

33. The median of the data 6, 4, -1, 0, 4, 8, 9 is

- (a) 0 (b) 4 (c) 6 (d) none

Answer: (b)

34. If $P(A) = p$, $P(B) = q$ and $A \subseteq B$ then the correct relation is –

- (a) $p + q = 1$ (b) $p + q = 0$
(c) $p \leq q$ (d) $p \geq q$

Answer: (c)

35. The median value of the set of numbers {4, 12, 15, 3, 7, 10} is –

- (a) 7 (b) 10 (c) 9 (d) 8.5

Answer: (c)

36. If $Var(X) = 3$, $Var(Y) = 27$ and $Y = aX + b$ then the value of ' a ' is equal to –

- (a) 3 (b) -3 (c) ± 3 (d) None of these

Answer: (c)

37. An urn contains 12 white balls and 6 black balls. A ball is drawn at random. What is the probability of being it to be a white? –

- (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{4}{9}$ (d) 1

Answer: None of these

38. Let ' A ' be any event and $P(A) = 0.4$ then find $P(A^c) = ?$

Answer:

$$P(A) = 0.4$$

$$P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6$$

39. Let A and B be two events and $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cap B) = 0.4$ then the value of $P(A \cup B)$ is

- (a) 0.4 (b) 0.1 (c) 0.06 (d) None of these

Answer: (b)

40. A coin is tossed two times in succession, write the sample space.

[WBSCTE 2019]

Answer:

A coin is tossed two times in succession.

The sample space = {HH, HT, TH, TT}

41. Find the arithmetic mean of first five prime numbers.

[WBSCTE 2019]

Answer:

The first five prime numbers are 2, 3, 5, 7 and 11.

The mean of these 5 numbers = $\frac{2+3+5+7+11}{5} = 5.6$

42. A die is thrown at random one time. The probability of appearing 'multiple of 2' on top of the face of die is

[WBSCTE 2019]

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{4}{9}$ (d) 1

Answer: (b)

43. If two coins are tossed, the probability of getting at most one head is _____

[WBSCTE 2022]

Answer: $\frac{3}{4}$

44. If $P(A) = 0.4$, $P(B) = 0.35$, Find $P(\overline{A}) = P(\overline{B}) - 1 = ?$

[WBSCTE 2022]

Answer:

$$P(A) = 0.4$$

$$P(B) = 0.35$$

$$P(\overline{A}) = 1 - 0.4 = 0.6$$

$$P(\overline{B}) = 1 - 0.35 = 0.65$$

$$P(\overline{A}) + P(\overline{B}) - 1$$

$$= 0.6 + 0.65 - 1$$

$$= 0.25$$

45. Find the probability of obtaining at least one '6' in a game of throwing of two dice.

[WBSCTE 2022]

Answer:

The outcomes in which there is at least one 6 are -

(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5).

So, there are 11 cases where at least one six appears in 2 die rolls.

$\therefore$ The probability of at least one six in two die rolls is - $\frac{11}{36}$

46. The mode of the data: 10,12, 8, 10, 12,11,10,12,7,11,12 is _____. [WBSCTE 2022]

Answer:

12. (Mode is maximum occurring observation).

47. Compute the median of the data : 10,5,9,6,4,8,7,6,5.

[WBSCTE 2022]

Answer:

Arranging data in ascending order, 4, 5, 5, 6, 6, 7, 8, 9, 10

Here, number of observation, $n = 9$ (odd)

Since n is odd,

$$\text{Median} = \left(\frac{9+1}{2} \right)^{\text{th}} = \frac{10}{2} = 5.$$

Long Answer Type Questions

1. Find standard deviation of the length of the belts produced in two different machines and compare s.d.s. [WBSCTE 2004, 2007]

Machine										
A	24	27	26	28	39	31	22	29	27	26
B	28	26	25	26	28	29	27	26	27	28

Answer:

Frequency distribution table for Standard Deviation,

Taking assumed means $a = 27$.

Sl No.	Machine A			Machine B		
	Length in Mm (x)	di	Di ²	Length in Mm (x)	di	di ²
1.	22	-5	25	25	-2	4
2.	24	-3	9	26	-1	1
3.	26	-1	1	26	-1	1
4.	26	-1	1	26	-1	1
5.	27	0	0	27	0	0
6.	27	0	0	27	0	0
7.	28	1	1	28	1	1
8.	29	2	4	28	1	1
9.	30	3	9	28	1	1
10.	31	4	16	29	2	4
		$\sum 0$	$\sum 66$		$\sum 0$	14

$$\sigma = \sqrt{\frac{1}{N} \sum di^2 - \left(\frac{\sum di}{N} \right)^2}$$

$$\therefore \sigma_A (\text{S.d for A}) = \sqrt{\frac{1}{10} \times 66 - 0} = \sqrt{6.6} = 2.57 \text{ mm.}$$

$$\sigma_B (\text{S.d for B}) = \sqrt{\frac{1}{10} \times 14 - 0} = \sqrt{1.4} = 1.18 \text{ mm.}$$

$\therefore$ From the above result we can conclude that performance of machine is better than that of A.

2. (a) Calculate the mean and standard deviation for the following:

Size of Item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

[WBSCTE 2004, 2005, 2007, 2009]

(b) Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second a queen, if the first card is (i) replaced,

(ii) not replaced. [WBSCTE 2004, 2005, 2007, 2009]

Answer:

a) The Table shows the computation of mean and standard deviation

Let, assumed mean $a = 9$.

Sl No.	Size of the item	frequency (f)	Deviation (di)	di ²	f di	f di ²
1.	6	3	-3	9	-9	27
2.	7	6	-2	4	-12	24
3.	8	9	-1	1	0	9
4.	9	13	0	0	0	0
5.	10	8	1	1	8	8
6.	11	5	2	4	10	20
7.	12	4	3	9	12	36
		$N = 48$			$\sum 0$	124

The mean $\bar{x} = a + \frac{h}{N} \sum f di = 9 + 0 = 9$ units.

$$\sigma = h \times \sqrt{\frac{1}{N} \sum f di^2 - \left(\frac{f di}{N} \right)^2} = 1 \times \sqrt{\frac{124}{48}} = \sqrt{\frac{31}{12}} \text{ units.} = \sqrt{2.583} = 1.607 \text{ units}$$

b) (i) In the pack of cards, there are 4 kings and 4 Queens. When the 1st card is drawn, the chance of getting a king is $\frac{4c_1}{52c_1} = \frac{4}{52} = \frac{1}{13}$

When the 2nd card is drawn, there are also

52 cards since the 1st one is replaced. So in this case also chance of getting a Queen is

$$\frac{4c_1}{52c_1} = \frac{1}{13}$$

So, the required chance of getting a king and a queen is $\frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

(ii) Here in the 1st draw.

The chance is equal as (i) i.e. $\frac{1}{13}$

But in the 2nd draw, there are 51 cards left. So the chance of getting a queen in 2nd draw =

$$\frac{4c_1}{51c_1} = \frac{4}{51}$$

$\therefore$ The required chance = $\frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$ Ans.

3. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that one is red, one is white and one is blue.

[WBSCTE 2006, 2013]

Answer:

From the bag, among 6 red balls, one red ball can be chosen in 6C_1 ways. Similarly, one white and one blue ball can be chosen in 4C_1 and 8C_1 respectively.

But $(1+1+1)$ or 3 balls can be chosen from $(6+4+8)$ or, 18 balls in ${}^{18}C_3$ ways.

$$\text{So, the required no. of probability} = \frac{{}^6C_1 \times {}^4C_1 \times {}^8C_1}{{}^{18}C_3} = \frac{6 \times 4 \times 8 \times 2 \times 3}{18 \times 17 \times 16} = \frac{4}{17} \text{ Ans.}$$

4. Two dice are rolled. What is the probability that the sum total of the points on the dice be 9?

[WBSCTE 2007, 2016, 2019]

Answer:

As per rule we know that, in the rolling of 2 dice, total outcome will be: 6×6

Here, the events in which 9 will appear those are,

First die	second die
6	3
3	6
4	5
5	4

$\therefore$ Total number of possible ways.

$N(A) = 4$ and Actual no. of ways $N = 36$.

$$\therefore \text{Required probability } P(A) = \frac{4}{36} = \frac{1}{9} \text{ Ans.}$$

5. A problem in Mechanics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the

problem will be solved?

[WBSCTE 2009, 2017]

Answer:

The probability that A fails to solve the problems $= 1 - \frac{1}{2} = \frac{1}{2}$ Similarly probability of

failure of B and C are $1 - \frac{1}{3} = \frac{2}{3}$ and $1 - \frac{1}{4} = \frac{3}{4}$ respectively.

The events are independent. So the probability that all fail to solve $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$.

Hence the probability that the problem will be solved $= 1 - \frac{1}{4} = \frac{3}{4}$ Ans.

6. There are three gentlemen aged 60, 65 and 70 years. The probability that they will live next 5 years are 0.8, 0.6 and 0.3 respectively. Find the probability that exactly two of them will remain alive 5 years hence.

OR,

A batch of 50 fuses is known to have 10 defective fuses. If 10 fuses are selected at random, find the probability that all will be good.

[WBSCTE 2010]

Answer:

$$\text{The probability of aged 60 yrs will live next 5 yrs} = \frac{8}{10} = \frac{4}{5}$$

$$\text{" " " " " 60 " will die " " " " " } = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\text{The probability of aged 65 years will live next 5 years} = \frac{6}{10} = \frac{3}{5}$$

$$\text{" " " " " 65 " will die " " " " " } = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\text{The probability of aged 70 year will live next 5 yrs} = \frac{3}{10}$$

$$\text{" " " " " 70 " will die " " " " " } = 1 - \frac{3}{10} = \frac{7}{10}$$

$\therefore$ The probability that exact by two remain alive 5 years hence

$$= \frac{4}{5} \times \frac{3}{5} \times \frac{7}{10} + \frac{4}{5} \times \frac{2}{5} \times \frac{3}{10} + \frac{1}{5} \times \frac{3}{5} \times \frac{3}{10} = \frac{84 + 24 + 9}{250} = \frac{117}{250}$$

Or,

Answer:

Given total number of fuses = 50;

Number of defective fuse = 10;

Therefore number of non-defective fuse = 40

10 fuses are selected at random.

Therefore, total number of outcomes $= {}^{50}C_{10}$ which are mutually exclusive and equally likely.

Now, number of good fuse = 40

Hence, the number of cases favorable to the event 'all the selected 10 fuses will be good' $(m) = {}^{40}C_{10}$

Therefore, using the classical definition of probability we have probability that all the selected 10 fuses are good. $= \frac{{}^{40}C_{10}}{{}^{50}C_{10}} = 0.0825$

7. From the following distribution table, compute mean and S.D. of weights of 100 students. [WBSCTE 2010]

Wt. in kg(x)	60-62	63-65	66-68	69-71	72-74
No. students(f)	5	18	42	27	8

OR,

The heights of 100 students of a polytechnic college are given in inches in the following table:

Height in inches (x)	60-62	63-65	66-68	69-71	72-74	
No. of students (f)	5	18	42	27	8	N = 100

Find the mean and S.D. of the above distribution. [WBSCTE 2012]

Answer:

Let the assumed mean: $A = 67$ and class width : $h = 2$.

Sl.No	Class	Interval	Freq.	Mid value(x)	$d = \frac{x-A}{h}$	fd	fd ²
1	60-62	h = 2	5	61	-3	-15	45
2	63-65		18	64	-1.5	-27	40.5
3	66-68		42	67	0	0	0
4	69-71		27	70	1.5	40.5	60.75
5	72-74		8	73	3	24	72
total			N = 100			$\sum fd = 22.5$	$\sum fd^2 = 218.25$

$$\text{Mean of } d : \bar{d} = \frac{\sum fd}{\sum f} = \frac{22.5}{100} = 0.225$$

Therefore Mean of weights $\bar{x} = A + h \times \bar{d} = 67 + 2 \times 0.225 = 67.45 \text{ kg}$

$$\text{Standard deviation of 'd'} = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} = \sqrt{\frac{218.25}{100} - \left(\frac{22.5}{100}\right)^2}$$

$$= \sqrt{2.1825 - 0.050625} = 1.46$$

Therefore standard deviation of the weights $= 2 \times 1.46 = 2.92$

8. (a) A bag contains 4 white balls and 2 black balls and a second bag contains 3 of each colours. A bag is drawn at random and a ball is then selected at random from the bag chosen. What is the probability that the ball selected is a white?

[WBSCTE 2011, 2015]

Answer:

Let 'W' denotes the event that white ball is drawn and A_i denotes the event that box has been chosen.

$$\therefore P(W) = P(A_1)P(W|A_1) + P(A_2)P(W|A_2) = \frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{3}{6} = \frac{1}{2} \times \frac{7}{6} = \frac{7}{12}$$

8. (b) A can hit a target 4 times in 5 shots; B 3 times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that two shots at least hit.

[WBSCTE 2011]

Answer:

Let 'A', 'B', 'C' denotes the event that A, B, C can hit the target respectively.

$$\therefore P(AB) = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}, P(BC) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}, P(CA) = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

$$\text{and } P(ABC) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

Hence, the problem that at least two hit the target is $\frac{3}{5} + \frac{1}{2} + \frac{8}{15} - 2 \times \frac{2}{5} = \frac{5}{6}$

8. (c) The table below gives the weights of 50 school boys nearest to kg.

Wt in kg (x):	40-42	43-45	46-48	49-51	52-54	55-57
No. of boys (f):	3	7	10	14	9	7

Calculate the mean and S.D. of the distribution.

[WBSCTE 2011]

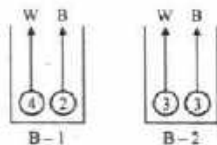
Answer:

x	f	mid of x	$x' = x - 47$	x'^2	fx'	fx'^2
40-42	3	41	-6	36	-16	108
43-45	7	44	-3	9	-21	63
46-48	10	47	0	0	0	0
49-51	14	50	3	9	42	126
52-54	9	53	6	36	54	324
55-57	7	56	9	81	63	567
Total	50	—	9	—	120	1188

$$\therefore SD = \sqrt{\frac{1188}{50} - \left(\frac{120}{50}\right)^2} = \sqrt{23.76 - (2.4)^2} = \sqrt{18} = 4.24$$

9. A bag contains 4 white and 2 black balls and another bag contains 3 of each colour. A ball is drawn from each bag. What is the probability that one is white and one is black. [WBSCTE 2012]

Answer:



Let 'B' denotes the event that drawn ball is black and 'W' denotes the white ball.

$$\therefore P(B) = P(B_1) P(B/B_1) + P(B_2) P(B/B_2) = \frac{1}{2} \times \frac{2}{6} + \frac{1}{2} \times \frac{3}{6} = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$$

$$\therefore P(W) = P(B_1) P(W/B_1) + P(B_2) P(W/B_2) = \frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{3}{6} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$\therefore P(BW) = \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$$

10. Calculate mean and standard deviation from the following data. [WBSCTE 2013, 2015]

Size of the item	6	7	8	9	10	11	12
Frequency	3	5	9	13	8	5	4

Answer:

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{(6 \times 3) + (7 \times 5) + (8 \times 9) + (9 \times 13) + (10 \times 8) + (11 \times 5) + (12 \times 4)}{(3 + 5 + 9 + 13 + 8 + 5 + 4)} = 9.04$$

$$\text{variance} = \frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i} = 2.55$$

$$\therefore \text{S.D.} = \sqrt{\text{variance}} = 1.60$$

11. Five men in a company of twenty are graduates. If three men are picked out of 20 at random, what is the probability that at least one is graduate? [WBSCTE 2014]

[WBSCTE 2014]

Answer:

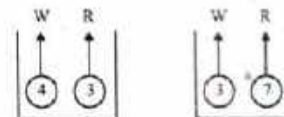
Probability that at least one is graduate

= P(One man is graduate or two man are graduate or three man are graduate)

$$= \frac{{}^5C_1 \times {}^{15}C_2}{{}^{20}C_3} + \frac{{}^5C_2 \times {}^{15}C_1}{{}^{20}C_3} + \frac{{}^5C_3 \times {}^{15}C_0}{{}^{20}C_3} = \frac{525}{1140} + \frac{150}{1140} + \frac{10}{1140} = \frac{685}{1140} = 0.6$$

12. There are two identical urns containing 4 white and 3 red balls; 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. What is the probability that it is from the first urn if the ball drawn is white? [WBSCTE 2016]

Answer:



Let W be the event denotes a white ball is drawn from the urn and A_i denotes that the i^{th} ($i = 1, 2$) urn is chosen.

$$\therefore P(w) = P(A_1)P(W|A_1) + P(A_2)P(W|A_2) = \frac{1}{2} \left[\frac{4}{7} + \frac{3}{10} \right] = \frac{61}{140}$$

$$P(A_1|W) = \frac{P(A_1)P(W|A_1)}{P(W)} = \frac{\frac{1}{2} \times \frac{4}{7}}{\frac{61}{140}} = \frac{2}{7} \times \frac{140}{61} = \frac{40}{61}$$

13. Find Mean, Median and Mode of the following data: 33, 32, 33.5, 33, 31.5, 32, 31, 32, 33, 33, 32. [WBSCTE 2016]

Answer:

$$\text{Mean} = \frac{33 + 32 + 33.5 + 33 + 31.5 + 32 + 31 + 32 + 33 + 33 + 32}{11} = 32.45$$

Now the given data are arranged in an ascending order as follows:

31, 31.5, 32, 32, 32, (32), 33, 33, 33, 33, 33.5

So median is the 6th term i.e. 32.

32 and 33 both the data accrued 4 times so mode is 32 and 33.

14. Calculate mean and standard deviation from the following data:

Size of the item	6	7	8	9	10	11	12
Frequency	3	5	9	13	8	5	4

[WBSCTE 2016]

Answer:

The Table shows the computation of mean and standard deviation.

Let, assumed mean $a = 9$.

Sl. No.	Size of the item	Frequency (f_i)	Deviation (d_i)	d_i^2	$f_i d_i$	$f_i d_i^2$
1	6	3	-3	9	-9	27
2	7	5	-2	4	-10	20
3	8	9	-1	1	-9	9
4	9	13	0	0	0	0
5	10	8	1	1	8	8
6	11	5	2	4	10	20
7	12	4	3	9	12	36
	Total	$N=47$	$\sum d_i = 0$	$\sum d_i^2 = 28$	$\sum f_i d_i = 2$	$\sum f_i d_i^2 = 120$

$$\text{The mean } \bar{x} = a + \frac{h}{N} \sum f_i d_i = 9 + \frac{2}{47} = 9.04$$

$$\text{S.D.} = h \times \sqrt{\frac{1}{N} \sum f_i d_i^2 - \left(\frac{\sum f_i d_i}{N} \right)^2} = \sqrt{\frac{120}{47} - \frac{4}{2209}} = \sqrt{\frac{5638}{2209}} = \sqrt{2.55} = 1.6$$

15. Calculate the Mean and S.D. from the following frequency distribution:

Class:	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency:	5	12	15	20	18	10	6	4

[WBSCTE 2017]

Answer:

Class	Frequency (f_i)	Mid values (x_i)	$X := \frac{(x_i - 55)}{10}$	$X_i f_i$	$X_i^2 f_i$
20-29	5	25	-3	-15	45
30-39	12	35	-2	-24	48
40-49	15	45	-1	-15	15
50-59	20	55	0	0	0
60-69	18	65	1	18	18
70-79	10	75	2	20	40
80-89	6	85	3	18	54
90-99	4	95	4	16	64
$M = \sum f_i =$			$\sum X_i = 4$	$\sum X_i f_i = 18$	$\sum X_i^2 f_i = 284$

POLY-EM

$$\therefore \text{S.D.} = \sqrt{\frac{\sum X_i^2 f_i}{\sum f_i} - \left(\frac{\sum X_i}{\sum f_i} \right)^2} \times 10 = \sqrt{\frac{204}{90} - \left(\frac{4}{90} \right)^2} \times 10 = \sqrt{3.156 - 0.0020} \times 10 = 17.76$$

16. (a) There are two identical urns containing 3 white and 5 red balls; 4 white and 9 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white.

(b) (i) Two dice are rolled. What is the probability that the sum total of the points on the dice is 6?

(ii) If $P(A) = p$, $P(B^c) = 0.7$, $P(A \cup B) = 0.8$ and $P(A \cap B) = 1 - p$, then find the value of p .

[WBSCTE 2018]

Answer:

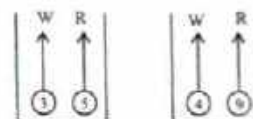
a) Let W be the event that a white ball is drawn and $A_i (i = 1, 2)$ denote the event that i^{th} urn is selected.

$$\therefore P(A_1) = P(A_2) = \frac{1}{2}$$

$$\text{Now, } P(W) = P(W | A_1)P(A_1) + P(W | A_2)P(A_2)$$

$$= \frac{3}{8} \times \frac{1}{2} + \frac{4}{13} \times \frac{1}{2}$$

$$= \frac{1}{2} \left(\frac{3}{8} + \frac{4}{13} \right) = \frac{39 + 32}{2 \times 104} = \frac{71}{208}$$



b) (i) No. of points under this event is 5 i.e.

$$\{ (1, 5), (5, 1), (2, 4), (4, 2), (3, 3) \}$$

So the required probability is $\frac{5}{6^2} = \frac{5}{36}$

(ii) We know $P(A \cap B) = 1 - p$

$$\text{or, } 0.8 = 1 - p$$

$$\text{or, } p = 1 - 0.8$$

$$\text{or } p = 0.2$$

17. Calculate mean and standard deviation from the following data:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

[WBSCTE 2018]

POLY-EM

Answer:

x_i	f_i	$x_i f_i$	$x_i^2 f_i$
4	3	12	48
8	5	40	320
11	9	99	1089
17	5	85	1445
20	4	80	1600
24	3	72	1728
32	1	32	1024
	$\sum f_i = 30$	$\sum x_i f_i = 420$	$\sum x_i^2 f_i = 7254$

$$\therefore \text{Mean } (\bar{x}) = \frac{\sum x_i f_i}{\sum f_i} = \frac{420}{30} = 14$$

$$\therefore SD = \sqrt{\frac{\sum x_i^2 f_i}{\sum f_i} - (\bar{x})^2} = \sqrt{\frac{7254}{30} - (14)^2} = \sqrt{241.8 - 196} = \sqrt{45.8} = 6.77$$

18. (i) Calculate the median for the following frequency distribution

Marks	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	7	15	24	31	42	30	26	15	10

 (ii) If $\text{var}(X) = 5$ and $Y = 5X + 6$ find $\text{var}(Y)$. [WBSCTE 2019]

Answer:

(i)

Class interval	f	C_f
5-10	7	7
10-15	15	22
15-20	24	46
20-25	31	77
25-30	42	119
30-35	30	149
35-40	26	175
40-45	15	190

 $\longrightarrow \text{Medium } \left(\leq \frac{N}{2} \right)$

45-50	10	200
	$N = 200$	

$$\text{Here, } N_i = \frac{N}{2} = \frac{200}{2} = 100, C_f = 77, f = 42, L = 25$$

$$\therefore M = L + \frac{N_i - C_f}{f} \times i = 25 + \frac{100 - 77}{42} \times 5 = 25 + \frac{23}{42} \times 5 = 27.74$$

(ii) Refer to Question No. 1 of Short Answer Type Questions.

19. What is the probability that the sum of outcomes is 13, when three dice are rolled? [WBSCTE 2022]

Answer:

When 3 dices are rolled,

 Total no. of outcomes = $6^3 = 216$

Sum of 13 can be achieved in the following ways:

From the digits 6, 4, 3

 So, there are 3! ways = $3 \times 2 = 6$

From the digits 6, 2, 5

 So, there are 3! ways = $3 \times 2 = 6$

From the digits 5, 4, 4

 So, there are $\frac{3!}{2!}$ ways = 3

From the digits 6, 6, 1

 So, there are $\frac{3!}{2!}$ ways = 3

From the digits 3, 5, 5

 So, there are $\frac{3!}{2!}$ ways = 3

 So, Total numbers whose sum is 13 = $(6+6+3+3+3) = 21$

 So, Probability = $\frac{21}{216}$

 Therefore, the probability of getting sum as 21 on rolling three dice = $\frac{21}{216}$

20. Find the median of the distribution given below:

[WBSCTE 2022]

x	01-10	11-20	21-30	31-40	41-50	51-60
f _i	3	10	9	13	7	15

Answer:

x	f _i	Cumulative frequency(CF)
1-10	3	3
11-20	10	13(CF)
(ℓ) 21-30	9(f)	22
31-40	13	35
41-50	7	42
51-60	15	57
Total:	N = 57	

$$\text{Median} = \ell + \frac{\frac{n}{2} - CF}{f} \times h$$

Here, M size of $\left(\frac{N}{2}\right)$ in item

$$= \frac{57}{2} = 28.5$$

$\therefore$ Median lies in the class 21-30

$$M = \ell + \frac{\frac{n}{2} - CF}{f} \times h$$

$$= 21 + \frac{28.5 - 13}{9} \times 10 = 21 + 17.2 = 38.2$$

Therefore, the median of the distribution is 38.2

APPLIED PHYSICS - II

NOTE:

We have designed the complete APPLIED PHYSICS-II course material in accordance with new and changed syllabus under WBSCTE. Previously the subject was in 2nd semester. Hence we are providing relevant university solutions and some model questions & answers along with new university paper, so that students can get an idea about WBSCTE questions pattern.

20. Find the median of the distribution given below:

[WBSCTE 2022]

x	01-10	11-20	21-30	31-40	41-50	51-60
f _i	3	10	9	13	7	15

Answer:

x	f _i	Cumulative frequency(CF)
1-10	3	3
11-20	10	13(CF)
(l) 21-30	9(f)	22
31-40	13	35
41-50	7	42
51-60	15	57
Total:	N = 57	

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Chapter 1: Wave motion and its applications

Chapter at a glance:

1.1 Preliminary concepts of SHM

Simple Harmonic Motion (SHM) is a periodic motion of a body, in which acceleration of the body (or force on the body) is directly proportional to its displacement from a fixed point and is always directed towards the fixed point.

1.2 Energy of a SHM

- P.E. = $\int_0^x F dx = \int_0^x \text{mass} \times \text{acceleration} dx = \int_0^x m\omega^2 x dx = \frac{m\omega^2 x^2}{2} = \frac{m\omega^2 a^2 \sin^2 \omega t}{2}$
- K.E. = $\frac{mv^2}{2} = \frac{ma^2 \omega^2 \cos^2 \omega t}{2}$
- Total energy P.E + K.E = $E = \frac{ma^2 \omega^2}{2}$
- Average kinetic energy, $\langle E_k \rangle = \frac{1}{T} \left[\frac{1}{2} m \int_0^T \left(\frac{dx}{dt} \right)^2 dt \right] = \frac{1}{4} m\omega^2 a^2$
- Average potential energy, $\langle E_p \rangle = \frac{1}{T} \left[\frac{1}{2} m \int_0^T x^2 dt \right] = \frac{1}{4} m\omega^2 a^2$

1.3 Damped harmonic motion

In reality when a pendulum is allowed to swing in air, frictional force due to air will come into play and as a result its amplitudes decreases down continuously with time. This nature of pendulum's motion is deviating from free vibration as discussed in chapter one. So free vibration is a idealistic case but not a realistic one. In presence of some dissipative force if a body executes a vibrational motion and its amplitude decays down continuously with time then the motion of the body is known as damped vibration.

Examples:

- The motion of a coil in a suspended coil galvanometer
- The motion of a mass-spring system etc...

- Differential Equation of a Damped Harmonic Motion** is $m \frac{d^2 x}{dt^2} = -kx - L \frac{dx}{dt}$
where L is resistive constant, k is restoring force constant.
- If we adjust the above differential equation as

Wave motion and its applications

$$\frac{d^2 x}{dt^2} + \frac{L}{m} \frac{dx}{dt} + \frac{k}{m} x = 0 \quad \text{or} \quad \frac{d^2 x}{dt^2} + 2K \frac{dx}{dt} + \omega^2 x = 0 \quad \left[\text{where } K = \frac{L}{2m} \text{ and } \omega^2 = \frac{k}{m} \right]$$

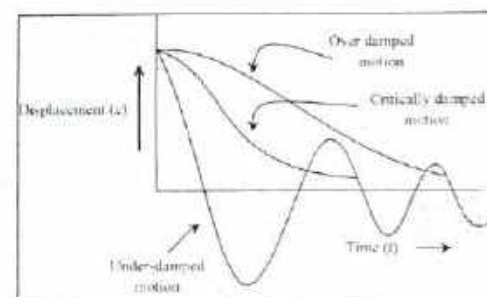
- Solution of the differential equation of a damped vibration as

$$x = A_1 e^{\left(-K \pm \sqrt{K^2 - \omega^2} \right) t} + A_2 e^{\left(-K \mp \sqrt{K^2 - \omega^2} \right) t}$$

Case 1: If $K > \omega$, i.e., $\sqrt{K^2 - \omega^2}$ motion is called as *over damped* or *dead beat*

Case 2: If $K = \omega$, the motion will be critical

Case 3: When $K < \omega$ i.e., $\sqrt{K^2 - \omega^2}$ is imaginary, then motion is called damped oscillatory motion



- Logarithmic decrement can be represented as

$$\lambda = \log_e \frac{C_1}{C_2} = \log_e \frac{C_1}{C_2} = \log_e \frac{C_2}{C_1} = \text{constant.}$$

- Relaxation time means the time taken to fall the amplitude $\frac{1}{e}$ times (37%) of its initial amplitude.
- When damping constant (K) is greater than restoring constant (ω) the motion is damped or dead beat.
- For critically damped motion damping constant is **equal to restoring constant**.
- When damping constant (K) is less than restoring constant (ω) the motion is damped oscillatory.
- Damped oscillatory motion of frequency is

$$f' = \frac{P}{2\pi} = \frac{\sqrt{\omega^2 - K^2}}{2\pi} \quad \text{Time period for under damped motion } T = \frac{2\pi}{\sqrt{\omega^2 - K^2}}$$

- When a body is set into vibrating condition by applying strong periodic force having a frequency different from the natural frequency of the body, such vibration of the body is termed as forced vibration.

Example: When a vibrating tuning fork is pressed by the stem of it on a table a forced vibration is produced.

- Differential Equation of Forced Vibration is $m \frac{d^2x}{dt^2} = -kx - L \frac{dx}{dt} + F_0 e^{i\omega t}$ where ω is the frequency of the applied external force.
- General Solution of the differential Equation of Forced Vibration is sum of the complimentary function and particular integral at the start

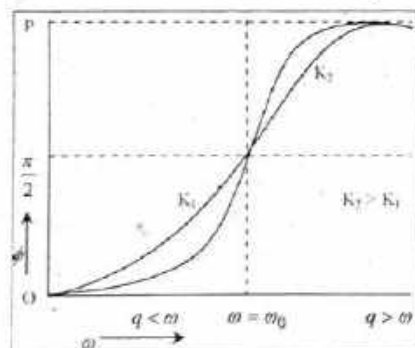
$$\text{i.e., } x = A e^{i(\omega - \omega_0)t} + C e^{-\beta t} \sin(\sqrt{\omega^2 - K^2} t + \delta)$$

- After a lapse of time when the 2nd part becomes negligible, we can write the sustained forced vibration solution as $x = \frac{f_0}{\sqrt{4K^2 q^2 + (\omega^2 - q^2)^2}} e^{i(\omega - \omega_0)t}$, which is represented as **steady state motion**

- Amplitude** is $A = \frac{f_0}{\sqrt{4K^2 q^2 + (\omega^2 - q^2)^2}}$ and that of **phase** is $\phi = \tan^{-1} \left(\frac{2qK}{\omega^2 - q^2} \right)$ of the

steady state displacement of the particle under forced vibration.

- Phase vs. frequency plot**

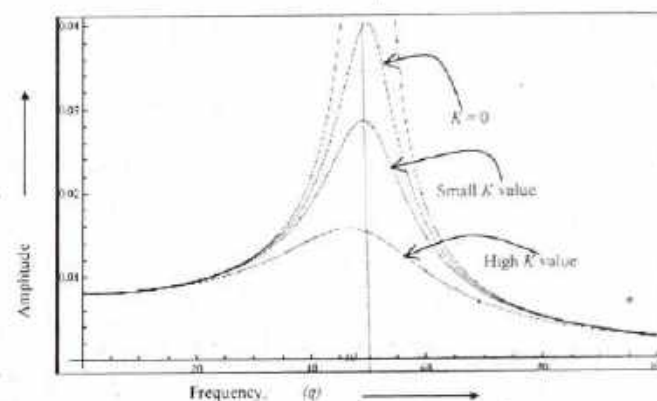


- The phenomenon in which a body vibrates with its natural frequency under the influence of external periodic force is called **resonance vibration**.

- The value of amplitude under **amplitude resonance** condition is $A = \frac{f_0}{2K\sqrt{\omega^2 - K^2}}$

For **weak damping** (i.e., K very small) the amplitude becomes $A_{\text{max}} = \frac{f_0}{2K\omega}$

- The amplitude resonance always occurs at or near $\omega = \omega_0$ provided the damping is not too large.



- Average power supplied by the driving force to the oscillator is equal to average loss of power of the oscillator i.e., $= mKq^2 A^2$
- The sharpness of resonance may be defined (In terms of amplitude resonance) as the rate of fall in a amplitude (or decrement in amplitude) with the change of the frequency of the applied periodic force on each side of resonant frequency.
- We can define also sharpness of resonance in terms of velocity resonance as when the amplitude of velocity of the forced vibration under the action of impressed periodic force is maximum, then the phenomenon is known as velocity resonance with the applied force.

1.4 Superposition of waves:

When two or more waves are passing simultaneously through the same medium, each wave proceeds independently of the others and in the regions in which they meet simultaneously, the resulting displacement of any particle of the medium is the vector sum of the displacements of the particle due to the individual waves. This is known as the principle of superposition.

1.5 Acoustics of building:

In day today life sound engineering plays a vital role in film industries, broadcasting of television signals and even in television signals. So a new field of science is developed which deals with the planning of a building or a hall with a view to provide best audible sound to the audience and is called Acoustics of building.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The phase difference between any two points situated on a wave front is
[WBSCTE 2022]

(a) π (b) 2π (c) 0 (d) $\frac{\pi}{2}$

Answer: (c)

2. For destructive interference the path difference of the waves emitted from the two sources is equal to
[WBSCTE 2022]

(a) odd multiple of $\frac{\lambda}{2}$ (b) even multiple of $\frac{\lambda}{2}$
(c) any multiple of $\frac{\lambda}{2}$ (d) any other number

Answer: (a)

3. Find the velocity when $x=5$ cm
(a) ± 6.28 cm/sec (b) ± 6.00 cm/sec
(c) ± 7.28 cm/sec (d) ± 5.28 cm/sec
[Model Question]

Answer: (a)

[Explanation:

We know that for SHM

$$v = \pm \omega \sqrt{A^2 - x^2}$$

Where A is amplitude and $\omega = \frac{2\pi}{T}$

Solving we get $v = \pm 6.28$ cm/sec.]

4. Find the maximum acceleration and maximum velocity
(a) 3.00 cm²/sec; 6.8 cm/sec (b) 3.56 cm²/sec; 6.0 cm/sec
(c) 3.2 cm²/sec; 6.1 cm/sec (d) 3.56 cm²/sec; 6.8 cm/sec
[Model Question]

Answer: (d)

[Explanation:

$$v_{\max} = \omega A = 6.8 \text{ cm/sec.}$$

$$a_{\max} = \omega^2 A = 3.56 \text{ cm}^2/\text{sec.}]$$

5. The instantaneous displacement of a particle of mass m executing SHM under a force constant k is
[Model Question]

$$x = A \sin(\omega t + \phi), \text{ where } \omega = \sqrt{\frac{k}{m}}$$

The time average of kinetic energy over a Time period T is:

(a) kA^2 (b) $\frac{1}{4}kA^2$ (c) $\frac{1}{3}kA^2$ (d) $\frac{1}{2}kA^2$

Answer: (a)

B. Fill in the blanks with the alternatives given in the bracket:

6. A simple harmonic wave may be of two types (i) longitudinal and (ii) transverse.
[Model Question]

7. Stationary waves are produced by both transverse and longitudinal.
[Model Question]

8. The velocity of the wave does not depend on its frequency or Wavelength or intensity.
[Model Question]

C. Answer the following questions:

9. An oscillator executing SHM has zero displacement at time $t=0$. If the displacement are 1mm and 1.5mm at instants 0.1 and 0.2 seconds, calculate the frequency and amplitude of oscillation.
[Model Question]

Answer:

Using the formula, $x = a \sin \omega t$

we have $a \sin(0.1\omega) = 0.1$ and $a \sin(0.2\omega) = 0.15$

$$\text{or, } 0.15 = 0.1 \frac{\sin(0.2\omega)}{\sin(0.1\omega)} \rightarrow 1.5 = \frac{2\sin(0.1\omega)\cos(0.1\omega)}{\sin(0.1\omega)}$$

$$\text{or, } \cos(0.1\omega) = 0.75 \rightarrow \omega = 0.723 \text{ rad or } 2.3\pi \text{ rad}$$

$$\text{So, Frequency } \nu = \frac{\omega}{2\pi} = \frac{0.723}{2\pi} = 1.15 \text{ Hz}$$

$$\text{and amplitude } a = \frac{0.1}{\sin(0.1\omega)} = \frac{0.1}{\sin 41.4} = 0.151 \text{ cm}$$

10. Can a motion be oscillatory but not simple harmonic? Explain with valid reason.
[Model Question]

Answer:

Yes.

Consider an example of a ball dropping from a height on a perfectly elastic surface. The type of motion involved here is oscillatory but not simple harmonic as restoring force

$F=mg$ is constant and not $F \propto -x$, which is a necessary condition for simple harmonic motion.

11. What is the basic condition for the motion of a particle to be a simple harmonic motion? [Model Question]

Answer:

The restoring force acting on it is proportional to its displacement from the mean position. That is, $F = -kx$.

12. What happens to the oscillation of the body in damped oscillation? [Model Question]

Answer:

The oscillation of the body decreases with time.

13. What is free oscillation? [Model Question]

Answer:

Free oscillation is the oscillation of the particle with a fundamental frequency under the influence of restoring force.

14. In free oscillation, what happens to the amplitude, frequency and energy of the oscillating body? [Model Question]

Answer:

In free oscillation, the amplitude, frequency and energy of the oscillating body remain constant/unchanged.

15. Why is the frequency of free oscillation called natural frequency? [Model Question]

Answer:

The frequency of free oscillation depends on the nature and the structure of the oscillating body.

16. Name a few damping forces. [Model Question]

Answer:

Some damping forces are - Hysteresis force, viscous force and frictional force.

17. In damping oscillation, why does the oscillation decrease exponentially? [Model Question]

Answer:

In damping, oscillation decrease in the energy results in a decrease in amplitude of oscillation.

18. What is forced oscillation? [Model Question]

Answer:

When oscillation of a body is under the influence of external force the oscillation is called forced oscillation.

19. How does the damping effect amplitude in forced oscillation? [Model Question]

Answer:

The forced oscillation does face the damping effect. But the supply of external force results in gaining the energy thereby maintaining constant amplitude.

20. What is resonance? [Model Question]

Answer:

Oscillation of object with maximum amplitude, when the frequency of the applied force equal to the natural frequency of the object is called resonance.

21. When can resonance be observed? [Model Question]

Answer:

When the frequency of the applied force is equal to the natural frequency of the object.

Long Answer Type Questions

1. (a) What do you mean by free and damped oscillations?
 (b) What are beats? Prove that, the number of beats per second produced by two sources of sound is equal to the difference in the frequencies of the two sources.
 (c) What are the requirements of good acoustic in auditorium?
 (d) Write down the medical applications of ultrasonic waves. [WBSCTE 2022]

Answer:

(a) Free Oscillation:

The free oscillation possesses constant amplitude and period without any external force to set the oscillation. Ideally, free oscillation does not undergo damping. But in all-natural systems damping is observed unless and until any constant external force is supplied to overcome damping. In such a system, the amplitude, frequency and energy all remain constant.

Damped Oscillation:

The damping is a resistance offered to the oscillation. The oscillation that fades with time is called damped oscillation. Due to damping, the amplitude of oscillation reduces with time. Reduction in amplitude is a result of energy loss from the system in overcoming external forces like friction or air resistance and other resistive forces. Thus, with the decrease in amplitude, the energy of the system also keeps decreasing.

(b) 1st Part:

Beats is a phenomenon of superposition of two sound waves of slightly different frequencies, travelling in a medium along the same direction. It is found that the intensity of the resultant sound at a particular position rises and falls regularly with time. Thus the phenomenon of regular variation in the intensity of sound with time at a particular position, when two sound waves of nearly equal frequencies and amplitude moving in the same direction superimpose on each other is called **beats**.

2nd Part:

Let us consider two waves of slightly different frequencies n_1 and n_2 ($n_1 \sim n_2 < 10$) having equal amplitude travelling in a medium in the same direction.

At time $t = 0$, both waves travel in same phase. The equations of two waves are

$$y_1 = a \sin \omega_1 t$$

$$y_1 = a \sin(2\pi n_1)t \quad \dots (1)$$

$$y_2 = a \sin \omega_2 t = a \sin(2\pi n_2)t \quad \dots (2)$$

When the two waves superimpose, the resultant displacement is given by

$$y = y_1 + y_2$$

$$y = a \sin(2\pi n_1)t + a \sin(2\pi n_2)t \quad \dots (3)$$

Therefore

$$y = 2a \sin 2\pi(n_1 + n_2/2)t \cos 2\pi(n_1 - n_2/2)t \quad \dots (4)$$

Substitute $A = 2a \cos 2\pi(n_1 - n_2/2)t$ and $n = n_1 + n_2/2$ in equation (4)

$$y = A \sin 2\pi n t$$

The number of beats produced per second is called beat frequency, which is equal to the difference in frequencies of two waves.

(c) The requirements of good acoustic in auditorium are:

- The sound waves from the source should be of an adequate intensity.
- The sound should spread everywhere inside the hall evenly and should be audible enough everywhere.
- The sound notes should be clear and distortions should be minimum.
- an appropriately low background noise.

(d) The medical applications of Ultrasonic waves are:
i) Echocardiography:

In the process of electrocardiography, the ultrasonic waves are used to form an image of the heart using reflection and detection of these waves from various parts.

ii) Ultrasonography:

Medical ultrasound is a diagnostic imaging technique based on it. It is used for the imaging of internal body structures such as muscles, joints and internal organs. Ultrasonic images are known as sonograms. In this process, pulses of ultrasound are sent to the

tissue using a probe. The sound echoes off the tissue, where different tissues reflect sound varying in degrees. These echoes are recorded and displayed an image.

2. What is free, forced and damped oscillations?

[Model Question]

Answer:

There are three main types of Simple Harmonic Motion

- Damped Oscillation
- Forced Oscillation
- Free Oscillation

Free Oscillation:

The free oscillation possesses constant amplitude and period without any external force to set the oscillation. Ideally, free oscillation does not undergo damping. But in all-natural systems damping is observed unless and until any constant external force is supplied to overcome damping. In such a system, the amplitude, frequency and energy all remain constant.

Damped Oscillation:

The damping is a resistance offered to the oscillation. The oscillation that fades with time is called damped oscillation. Due to damping, the amplitude of oscillation reduces with time. Reduction in amplitude is a result of energy loss from the system in overcoming external forces like friction or air resistance and other resistive forces. Thus, with the decrease in amplitude, the energy of the system also keeps decreasing. There are two types of damping

- Natural Damping
- Artificial Damping

Forced Oscillation:

When a body oscillates by being influenced by an external periodic force, it is called forced oscillation. Here, the amplitude of oscillation, experiences damping but remains constant due to the external energy supplied to the system.

For example, when you push someone on a swing, you have to keep periodically pushing them so that the swing doesn't reduce.

3. Light can be reflected by a small mirror but an extended area is required for reflection of sound. Why?

[Model Question]

Answer:

For reflection of an incident wave the reflecting surface of the reflector should be fairly large in comparison to the wavelength of the wave. For regular reflection, the irregularities on the reflecting surface should be of dimensions much smaller than the wavelength of the incident wave.

The wavelength of visible light extends from about 4×10^{-7} cm to about 8×10^{-7} cm. A small mirror is sufficiently large in comparison to the order of the wavelength of visible light and the surface of a mirror is very smooth, so a small mirror can reflect light waves. But wavelength of sound wave is much larger than that of visible light and is of the order from about 1.5 cm to about 16 metre. Hence for reflection of sound waves reflection of

extended area are required. And these reflectors need not be very smooth because the irregularities on the reflecting surface are of lesser dimensions relative to the size of the wavelength of sound wave.

➤ 4. What are the principal differences between light wave and sound wave?

[Model Question]

Answer:

The principal differences between light and sound waves are as follows:

Light	Sound
1. Light waves are electromagnetic waves and produce sensation of vision in our eyes.	1. Sound waves are elastic waves and produce sensation of hearing in our ears.
2. Light waves are transverse. These can be polarised.	2. Sound waves in air are longitudinal. These cannot be polarised.
3. Material medium is not necessary for the propagation of light waves. Light can travel through vacuum.	3. Material medium is necessary for the propagation of sound waves. Sound wave cannot travel through vacuum.
4. The velocity of light wave is very high given as $c = \frac{1}{\sqrt{\mu\epsilon}}$, where μ and ϵ are the permeability and permittivity of the medium respectively. The velocity of light in vacuum is $3 \times 10^8 \text{ m s}^{-1}$.	4. Velocity of sound wave in any medium is given as $V = \sqrt{\frac{E}{\rho}}$, where E is the elasticity appropriate to the medium and ρ is the density of the medium. The velocity of sound waves in air at 0°C is 332 m s^{-1} .
5. The wavelength of light is very small. The wavelength of visible light lies within the range of $4000 \text{ \AA}$ to about $8000 \text{ \AA}$. ($1 \text{ \AA} = 10^{-10} \text{ m}$).	5. The wavelength of audible sound waves is large. The wavelength lies approximately between 1.7 cm to 17 m .

➤ 5. What are the conditions for the clear audibility of beats?

[Model Question]

Answer:

The wave's amplitude and the frequency of beating should be similar; it just depends on column/height that is large for feet than the brain and that is all this cause. It happens when two sound waves of fairly diverse frequencies traveling along a similar direction, path, in a similar medium superpose each other, the intensity of the resultant sound at any point in the medium rises and falls instead with time. Beats are heard when the dissimilarity in frequency is less than 10. (because a human ear can distinguish two sounds distinctly only if they vary in time by 0.1 sec.). The waves traveling with approximately similar frequency and amplitudes in the same direction produce beats.

- Two sound waves constituting beats must originate at the same time.
- Frequencies and amplitudes of the two waves constituting beats should be nearly equal.
- Displacement of the particles of the medium due to two waves should be along the same line.
- The phase difference of the resultant wave at a point must vary with time.
- The amplitude of the resultant wave must vary with time. The amplitude of the two interfering waves should be similar.
- For beats to be audible the difference in the frequency of the two sound waves should be not over 10. The beats can be heard only if the frequency dissimilarity is less than 10.
- The difference between the frequencies of interfering waves should be small.
- But due to the persistence of hearing, our ear is not able to distinguish between two sounds as separate.

➤ 6. Explain Transverse wave and write down the characteristics of propagating waves?

[Model Question]

Answer:

A transverse wave is a wave motion in which individual particles of the medium execute simple harmonic oscillations about their mean positions in a direction perpendicular to the direction of propagation of wave motion.

Some examples of transverse waves are — (i) vibrations of a string of a stringed musical instrument, (ii) vibrations of the membrane of a table etc.

A transverse wave travels through a medium in the form of **crests** and **troughs**. A crest is a position of the medium, which is temporarily raised above the normal positions of the particles of the medium when a transverse wave passes through it. The centre of the crest is the position of maximum displacement in the positive direction. While a trough is a position of the medium, which is depressed temporarily below the normal positions of rest of the particles of the medium, when a transverse wave passes through it. The centre of the trough is the position of maximum displacement in the negative direction.

Characteristics of propagating waves:

Following are the characteristics of wave motion —

- Wave motion is a type disturbance which travels through a medium.
- A material medium is essential for propagation of elastic or mechanical waves. The medium must possess elasticity, inertia and there must be minimum friction among the particles of the medium.
- When wave passes through a medium, particles of the medium only undergo simple harmonic oscillations about their mean positions but do not move with the disturbance.
- There is a continuous phase difference among the successive particles of the medium.
- The velocity of the particles during their vibration is different at different positions.

- vi) The velocity of the wave through a particular medium is constant and it depends only on the nature of the medium.
- vii) The velocity of the wave does not depend on its frequency or wavelength or intensity.
- viii) In the wave propagation, energy is transported along with it and there is no net transport of the medium.

7. Define Longitudinal waves.

[Model Question]

Answer:

A longitudinal wave motion is a wave motion in which individual particles of the medium execute simple harmonic oscillations about their mean positions along the direction of wave propagation.

Some examples of longitudinal waves are — (i) sound waves travelling through gases, (ii) vibrations of air column in an organ pipe etc.

A longitudinal wave travels through a medium through compressions and rarefactions of the medium. A compression is a region of the medium where particles are compressed. There is thus a temporary decrease in volume and corresponding increase in the density of the medium in this region. While a rarefaction is a region of the medium where the particles are rarefied. There is thus a temporary increase in volume and corresponding decrease in the density of the medium in this region.

Longitudinal waves can propagate in any medium as all the substances are compressible to some extent. So longitudinal waves can be transmitted through solid, liquid and gases.

8. Explain the terms:

[Model Question]

- i) Wave velocity
- ii) Frequency
- iii) Wavelength

Answer:

i) In a medium, the distance travelled by the wave in one second is known as the velocity of the wave in the medium. It is usually denoted by v . Its unit is cm s^{-1} or m s^{-1} .

Relation among wavelength, frequency and velocity of a wave:

We know that, during the time a particle in wave motion completes one vibration, the wave advances through a distance equal to one wave length (λ). So if n be the frequency of the wave, then in one second the wave travels a distance $n\lambda$, which is the velocity (v) of the wave in the medium, i.e.,

$$v = n\lambda$$

$$\text{or, } \boxed{\text{velocity of wave} = \text{frequency} \times \text{wavelength}}$$

If T be the time period, then $n = \frac{1}{T}$ and $v = \frac{\lambda}{T}$ or, $\lambda = vT$

$$\boxed{\text{wavelength} = \text{velocity of wave} \times \text{time period}}$$

ii) The number of complete vibrations performed in one second by a particle in the path of the wave is called the frequency of the wave. It may also be defined as the number of complete wavelengths traversed by the wave in one second. It is usually denoted by n . The unit of frequency is cycle per second (cps), which is also called hertz (Hz). We have

$$n = \frac{1}{T} \text{ or, } nT = 1.$$

iii) Wavelength is the distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. It can also be defined as the distance between any two nearest particles of the medium vibrating in the same phase. It is usually denoted by λ and expressed in metre and centimetre in SI and CGS system. Very small wavelengths are measured in angstrom ($\text{\AA}$), where $1\text{\AA} = 10^{-10} \text{ m}$. In transverse wave motion, wavelength is the distance between the centres of two consecutive compressions or the distance between the centres of two consecutive crests or distance between the centres of two consecutive troughs. While in a longitudinal wave motion, wave length is the distance between the centres of two consecutive rarefactions.

9. Write down the equation of plane progressive wave.

[Model Question]

Answer:

Let a plane wave originate at O (Fig. 1) and proceed from left to right in an elastic medium. The particles of the medium would execute SHMs of same amplitude and time period about their mean positions. Let the time be counted from the instant the particle at O crosses its mean position in the positive Y direction. The displacement y of the particle at any instant t is given by

$$y = a \sin \omega t$$

where a is the amplitude of vibration and ω is the angular velocity. As the disturbance is passed from one particle to its next, so there is a gradual fall in phase from left to right. Let the phase of a particle at P lag behind the phase of the particle at O by ϕ , then the displacement of the particle at P at time t is given by,

$$y = a \sin (\omega t - \phi)$$

If OP be equal to x , then phase $\phi = \frac{2\pi}{\lambda} x$. So,

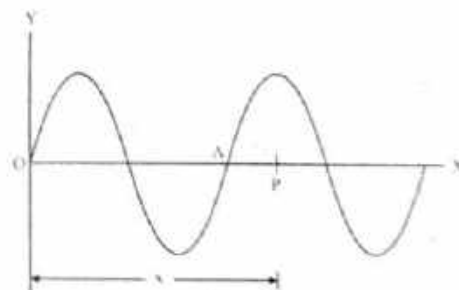


Fig. 1

$$y = a \sin \left(\omega t - \frac{2\pi}{\lambda} x \right)$$

or,

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

or,

$$y = a \sin \frac{2\pi}{\lambda} (vt - x) \quad \left[\because \omega = \frac{2\pi}{T} \right]$$

or,

$$y = a \sin 2\pi n \left(t - \frac{x}{v} \right) \quad \left[\because \text{Wave velocity } v = n\lambda = \frac{\lambda}{T} \right]$$

All these equations give the equations of a progressive wave travelling along the positive x axis in different forms.

For a wave travelling along the negative x -direction, then the equations of the wave take the forms

$$y = a \sin \left(\omega t + \frac{2\pi}{\lambda} x \right) = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) = a \sin \frac{2\pi}{\lambda} (vt + x) = a \sin 2\pi n \left(t + \frac{x}{v} \right)$$

10. State and explain the principle of superposition of waves. [Model Question]

Answer:

When two or more waves are passing simultaneously through the same medium, each wave proceeds independently of the others and in the regions in which they meet simultaneously, the resulting displacement of any particle of the medium is the vector sum of the displacements of the particle due to the individual waves. This is known as the **principle of superposition**.

If $y_1, y_2, y_3, \dots$ be the displacements at a particular instant of a particle due to different individual waves, then the resultant displacement y at the same instant of the same particle would be given by superposition principle as

$$y = y_1 + y_2 + y_3 + \dots$$

The most important property of wave motion is that it preserves its individuality while travelling through a medium and behaving as if nothing to do with other waves. It is to be noted that superposition principle ceases to apply when amplitude of mechanical waves is too large. Further overlapping waves add algebraically to produce a resultant wave. However, the overlapping waves do not affect in any way the motion of one another.

11. What is beats? Graphically explain the formation of beats. [Model Question]

Answer:

1st Part:

Beats is a phenomenon of superposition of two sound waves of slightly different frequencies, travelling in a medium along the same direction. It is found that the intensity of the resultant sound at a particular position rises and falls regularly with time. Thus the phenomenon of regular variation in the intensity of sound with time at a particular

position, when two sound waves of nearly equal frequencies and amplitude moving in the same direction superimpose on each other is called **beats**. A beat consists of one rise and one fall of the amplitude of the wave and hence the intensity. The time interval between two successive beats is called **beat period** and the number of beats produced per second is called **beat frequency**. It can be proved that the number of beats produced per second i.e., beat frequency is equal to the difference in the frequencies of two superposing component waves.

Formation of beats:

(i) **Graphical method:** Let us consider two waves of frequencies n_1 and n_2 travelling in the same direction simultaneously. In Fig. (1) two such waves with $n_1 = 7$ cps and $n_2 = 9$ cps are shown and displacement of the resultant wave obtained by the algebraic sum of the displacements of component waves is shown in Fig. 1. It is seen that at particular instants time A, B and C, the waves are in phase and so the amplitude of resultant wave is maximum and at some other instants D and E, the waves are in opposite phase and amplitude of the resultant wave is minimum. As two successive maxima or minima constitute one beat, from the figure it is evident that two beats are produced in one second i.e., the beat frequency is equal to the difference of the frequencies of the component waves.

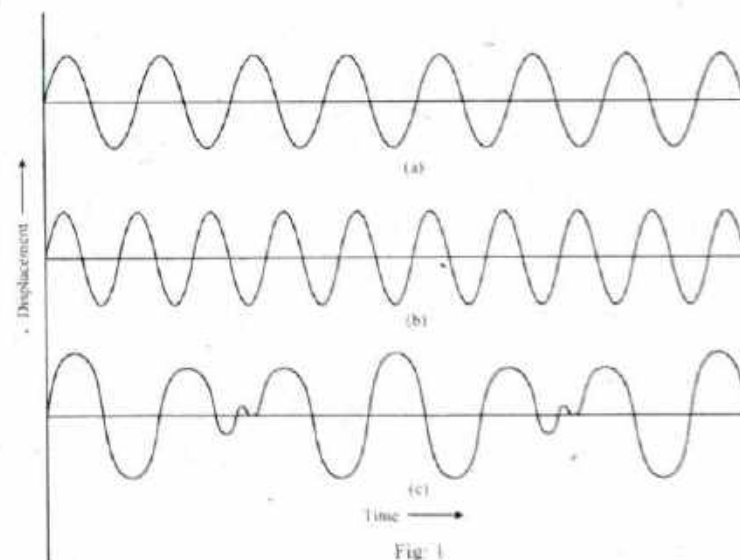


Fig. 1

(ii) **Analytical method:** Consider two waves of equal amplitude a and slightly different frequencies n_1 and n_2 ($n_2 > n_1$) travelling in a medium in the same direction. Let the equations of the waves be given by,

$$y_1 = a \sin 2\pi n_1 t \text{ and } y_2 = a \sin 2\pi n_2 t$$

According to superposition principle, the resultant displacement y is given by,

$$\begin{aligned} y &= y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t \\ &= a (\sin 2\pi n_1 t + \sin 2\pi n_2 t) \\ &= 2a \sin 2\pi \left(\frac{n_1 + n_2}{2} \right) t \cos 2\pi \left(\frac{n_1 - n_2}{2} \right) t \end{aligned}$$

Putting, $A = 2a \cos 2\pi \left(\frac{n_2 - n_1}{2} \right) t$ and $\frac{n_1 + n_2}{2} = n$

We have, $y = A \sin 2\pi n t$

This represents a wave of frequency n and amplitude A . It is seen that the amplitude of the resultant wave varies with time periodically.

$$A = 0, \text{ for } t = \frac{1}{2(n_2 - n_1)}, \frac{3}{2(n_2 - n_1)}, \frac{5}{2(n_2 - n_1)}$$

and $A = 2a$ (maximum), for $t = \frac{1}{n_2 - n_1}, \frac{2}{n_2 - n_1}, \frac{3}{n_2 - n_1}, \dots$

It follows that successive minima or maxima occur after the time of $\frac{1}{n_2 - n_1}$ second.

Hence the number of beats per second i.e., beat frequency is given by $n_2 - n_1$ = difference of frequencies of the component waves.

We know persistence of hearing is $\frac{1}{10}$ th of a second. So in order to hear distinct beats,

time interval between two successive beats must be greater than $\frac{1}{10}$ second i.e., frequency

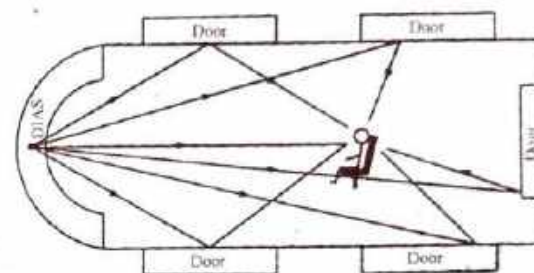
of beats must be less than 10 . Hence the difference in frequencies of two sources must be less than 10 . If the frequency of the beats be too high then due to persistence of hearing, our ears would fail to distinguish fluctuations of loudness and a continuous note is heard, which is called **beat tone**.

12. Explain the term: "Reverberation".

[Model Question]

Answer:

In day today life sound engineering plays a vital role in film industries, broadcasting of television signals and even in television signals. So a new field of science is developed which deals with the planning of a building or a hall with a view to provide best audible sound to the audience and is called Acoustics of building. Therefore to provide a best audible sound in a building or hall a prime factor called Reverberation.



When a sound pulse is generated in a hall, the sound wave travels towards all direction and are reflected back by the walls, floors, doors, windows ceiling etc as shown in the figure.

So a sound wave has two to three hundred repeated reflections, before it becomes inaudible. Therefore, the observer in the hall does not be able to hear a single sharp sound instead a "role of sound" of diminishing intensity (since part of energy is lost at each reflection).

13. Proof that: The reverberation time can be optimized by either varying the surface area of the reflecting surfaces or the absorption coefficient of the materials used inside the hall. [Model Question]

Answer:

According to Sabine, the reverberation time is defined as the time taken by a sound to fall to one millionth of its initial value, when the source of sound is cut off.

Time taken for E to be equal to $E = \frac{E_0}{10^6}$

Therefore condition is at $t = T$: $E = \frac{E_0}{10^6}$

Therefore equation 10 becomes $E = \frac{E_0}{10^6} = E_0 e^{-\alpha t}$

$$10^{-6} = e^{-\alpha t}$$

$$10^{-6} = e^{-\alpha t}$$

$$\alpha t = \log_e 10^6$$

$$\alpha t = 6 \log_e 10$$

$$\alpha t = 6 \times 2.3026 \log_{10} 10$$

$$\alpha t = 6 \times 2.3026 \times 1 \quad \dots (1)$$

We know $\alpha = \frac{vA}{4V} \quad \dots (2)$

Substituting equation 2 in 1, we get,

$$\frac{vA}{4l} T = 6 \times 2.3026$$

$$T = \frac{4l \times 6 \times 2.306}{vA}$$

We know the velocity of sound in air at room temperature $v = 330$ m/s

$$\therefore T = \frac{4 \times 6 \times 2.3026}{330} \times \frac{l}{A}$$

$$T = \frac{0.167l}{A}$$

$$T = \frac{0.167l}{\sum as} \quad \dots (3)$$

Where $\sum as = a_1s_1 + a_2s_2 + \dots$

Equation 3 represents the Reverberation time, which depends on the three factors:

- Volume of the hall (V)
- Surface area (S)
- Absorption coefficient(a) of the materials kept inside the hall.

Among these three actors volume is fixed. Therefore, the reverberation time can be optimized by either varying the surface area of the reflecting surfaces or the absorption coefficient of the materials used inside the hall.

14. Write down the properties and applications of Ultrasonic waves.

[Model Question]

Answer:

An imaging method that uses sound waves to produce images of structures within your body is known as ultrasound.

Properties of ultrasound:

- Ultrasonic waves cannot move in a vacuum.
- Ultrasonic waves constitute high-frequency, short-wavelength sound waves.
- These waves travel at the same velocity as sound in a particular medium.
- Throughout homogenous media, their velocity remains constant.

Applications of ultrasound:

- Bats utilize ultrasound just to detect obstructions in their route.
- Ultrasonography, as well as echocardiography, are methods of visualizing human organs.
- Drilling holes through materials such as glass and cleaning small things
- Defect detection throughout metals.
- Remove cataracts including using SONAR (Sound navigation and ranging) to detect as well as calculate the distance between objects.

15. What is decay constant (or relaxation time)? In damped harmonic motion, calculate the time in which the energy of the system falls to e^{-1} times of its initial value. [Model Question]

Answer:

In damped harmonic motion, how fast amplitude of the vibration of the body decay with time by the factor e^{-kt} depends on the damping coefficient 'K'. The time at which amplitude Ce^{-kt} decreases to e^{-1} i.e., $1/e$ of its original value C is called **relaxation time** or **decay constant**. Let us take that time to be τ . Putting τ in place of t we have $K\tau = 1$ or $\tau = 1/K$. So the time τ when new amplitude becomes $1/e$ i.e., 0.368 of its initial value. This τ is called the **relaxation time** or **decay constant**.

The displacement of damped harmonic motion is $x = Ce^{-kt} \sin(pt + \delta)$ where K is the damping constant and other symbols have their usual meaning.

We can have the average energy $\langle E \rangle = \frac{1}{2} mC^2 \omega^2 e^{-2kt} = E_0 e^{-2kt}$. When $2Kt = 1$, i.e.,

$t = \frac{1}{2K} \langle E \rangle = E_0 e^{-1}$. We define relaxation time τ as τ such that $\tau = \frac{1}{2K}$. So relaxation

time is defined as the time taken to fall the energy of the damped oscillatory motion to $\frac{1}{e}$ (nearly 37%) of its initial energy.

16. At an instant of time displacement of a particle is 12 cm, velocity is 5 cm/sec and when its displacement is 5 cm, the velocity is 12 cm/sec. Calculate the amplitude, frequency and time period. [Model Question]

Answer:

Velocity of the particle, $v = \frac{dy}{dt} = \omega \sqrt{a^2 - y^2}$

In the first case:

$$v_1 = \omega \sqrt{a^2 - y_1^2}$$

where, $v_1 = 5 \text{ cm s}^{-1}$ and $y_1 = 12 \text{ cm}$

$$\text{or, } 5 = \omega \sqrt{a^2 - 144} \quad \dots (1)$$

In the second case:

$$v_2 = \omega \sqrt{a^2 - y_2^2}$$

where, $v_2 = 12 \text{ cm s}^{-1}$ and $y_2 = 5 \text{ cm}$

$$\therefore 12 = \omega \sqrt{a^2 - 25} \quad \dots (2)$$

Dividing (1) by (2) we have, $\frac{5}{12} = \frac{\sqrt{a^2 - 144}}{\sqrt{a^2 - 25}}$

$$\begin{aligned}\frac{25}{144} &= \frac{a^2 - 144}{a^2 - 25} \Rightarrow 25a^2 - 25^2 = 144a^2 - 144^2 \\ &\Rightarrow 119a^2 = 144^2 - 25^2 \\ &\Rightarrow 119a^2 = (144 - 25)(144 + 25) \\ &\Rightarrow a^2 = 169 \Rightarrow a = 13 \text{ cm}\end{aligned}$$

$$5 = \omega\sqrt{13^2 - 144} \Rightarrow 5 = \omega\sqrt{25}$$

$$\omega = 1 \text{ rad s}^{-1}$$

$$\text{So, frequency, } \gamma = \frac{\omega}{2\pi} = \frac{1}{2\pi} \text{ Hz}$$

$$\text{The time period, } T = \frac{1}{\gamma} = 2\pi \text{ s.}$$

Chapter 2: Optics

Chapter at a glance:

2.1 Laws of Optics:

Laws of Reflection:

- The principle when the light rays fall on a smooth surface, the angle of reflection is equal to the angle of incidence.
- The incident ray, the reflected ray, and the normal to the surface all lie in the same plane.

Laws of Refraction:

- The incident ray, refracted ray, and the normal to the interface of two media at the point of incidence all lie on the same plane.
- The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant. This is also known as Snell's law of refraction.

$$\frac{\sin i}{\sin r} = \text{constant.}$$

Brewster's Law:

When an unpolarized light of known wavelength is incident on a transparent substance surface, it experiences maximum plane polarization at the angle of incidence whose tangent is the refractive index of the substance for the wavelength.

$$\mu = \tan i$$

2.2 Lens Definition:

Lenses are basically magnifying glasses with curved sides. A lens is a piece of transparent glass which concentrates or disperses light rays when passes through them by refraction. Due to the magnifying property, lenses are used in telescopes and other magnifying devices. They are employed in cameras for gathering the light rays.

2.2.1 Related Terms Definition:

Pole:

The centre of the spherical refracting surface of the lens is called the pole. The point where the principal axis meets the surface of the lens.

Optical Centre:

The point on the principal axis at the centre of the lens is called Optical centre.

Centre of Curvature:

A lens has two spherical surfaces, these two spherical surfaces form a part of a sphere. The centre of these spheres is known as the centre of curvature.

Principal axis:

The principal axis is an imaginary line passing through the centres of curvature and the pole.

Aperture:

The area of the lens suitable for refraction is called Aperture. The aperture of the lens is the effective diameter of its light-transmitting area.

Focus:

Focus is the point onto which collimated light parallel to the axis is focused.

Focal length:

The focal length is the distance between the optical centre and the focal point or focus of the lens.

Power:

The power of the lens is the reciprocal of its focal length. The S.I unit of power is Dioptre.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The power of two is +5D and -2.5D. The focal length of the two lenses in contact will be [WBSCTE 2022]
(a) +40 cm (b) -40 cm (c) +40 m (d) none of these

Answer: (a)

2. The focal length of a convex lens is maximum for the colour of light. [WBSCTE 2022]
(a) blue (b) yellow (c) green (d) red

Answer: (d)

3. In the interference fringe, if width of dark band is β_1 and that of the bright band is β_2 , then [WBSCTE 2022]
(a) $2\beta_1 = \beta_2$ (b) $2\beta_2 = \beta_1$ (c) $\beta_1 = \beta_2$ (d) $\beta_1 + 3\beta_2 = 1$

Answer: (c)

4. If a lens is surrounded by a medium denser than air, the focal length of the lens [WBSCTE 2022]
(a) decreases (b) increase
(c) remains same (d) cannot be determined

Answer: (b)

5. A lens which converges a beam of parallel rays to a point is called [Model Question]
(a) Diverging / concave lens (b) Converging / convex lens
(c) Plano concave lens (d) Plano convex lens

Answer: (b)

6. The diameter of a lens is called [Model Question]
(a) Focal length (b) Principle axis
(c) Aperture (d) Radius of curvature

Answer: (c)

7. In going from a denser to rarer medium a ray of light is [Model Question]
(a) Undeviated (b) bent away from the normal
(c) bent towards the normal (d) polarised

Answer: (b)

8. The power of a concave lens is [Model Question]
(a) Real (b) Virtual (c) Positive (d) negative

Answer: (d)

9. Magnifying power of a simple microscope [Model Question]
(a) Increases with increase in focal length
(b) Increases with the decrease in focal length
(c) No effect with decrease or increase with the focal length
(d) Least distance of distinct vision

Answer: (b)

10. Magnification of astronomical telescope is [Model Question]
(a) $f_o + f_e$ (b) f_o / f_e (c) f_e / f_o (d) $(1 + f_o / f_e) L / f_o$

Answer: (b)

11. The magnifying power of an astronomical telescope is 10 and the focal length of its eye-piece is 20 cm. The focal length of its objective will be [Model Question]
(a) 200 cm (b) 2 cm (c) 0.5 cm (d) 0.5×10^{-2} cm

Answer: (a)

12. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eye-piece is found to be 20 cm. The focal lengths of the lenses are [Model Question]
(a) 18 cm, 2 cm (b) 11 cm, 9 cm
(c) 10 cm, 10 cm (d) 15 cm, 5 cm

Answer: (a)

13. In a compound microscope, magnifying power is 95 and the distance of the object from the objective lens is $1/38$ cm. The focal length of the objective lens $1/4$ cm. What is the magnification of eyepiece? [Model Question]

(a) 5 (b) 10 (c) 100 (d) 200

Answer: (a)

14. A hypermetropic person having "near point" at a distance of 0.75 m puts on spectacles of power 2.5 D. The "near point" now is at [Model Question]

(a) 0.75 m (b) 0.83 m (c) 0.26 cm (d) 0.26 m

Answer: (d)

15. Which of the following lights deviates the most when it passes through a prism?

(a) Red light (b) Violet light
(c) Neither (a) nor (b) (d) Both (a) nor (b)

[Model Question]

Answer: (b)

16. Which of the following phenomena of light results in a mirage? [Model Question]

(a) refraction of light (b) reflection of light
(c) Total internal reflection (d) diffraction of light

Answer: (c)

17. For which of the following is the field of view maximum? [Model Question]

(a) Concave mirror (b) Convex mirror
(c) Plane mirror (d) Cylindrical mirror

Answer: (b)

18. What happens when the light is refracted into a medium? [Model Question]

(a) Both frequency and wavelength of the light increases
(b) The wavelength increases but the frequency remain unchanged
(c) Both wavelength and frequency decreases
(d) The wavelength decreases but the frequency remain constant

Answer: (b)

19. If a glass prism is dipped in water, what happens to its dispersive power?

(a) Increases (b) Decreases
(c) Does not change (d) No effect

[Model Question]

Answer: (b)

20. What should be increased to increase the angular magnification of a simple microscope? [Model Question]

(a) The power of the lens (b) The focal length of the lens
(c) Lens Aperture (d) Object Size

Answer: (a)

21. Which of the following phenomenon is used in optical fibre?

(a) Refraction (b) Diffraction
(c) Scattering (d) Total Internal Reflection

[Model Question]

Answer: (d)

22. Which of the following statements is true of total internal reflection?

[Model Question]

(a) Light travels from rarer medium to denser medium
(b) Light travels from denser medium to rarer medium
(c) Light travels in water only
(d) Light travels in the air only

Answer: (b)

23. A convex lens is dipped in a liquid whose refractive index is equal to the refractive index of the lens. Then what is its focal length? [Model Question]

(a) Focal Length will become zero
(b) Focal Length will become infinite
(c) Focal length will reduce, but not become zero
(d) Remains unchanged

Answer: (b)

24. For a telescope, the larger the diameter of the objective lens

[Model Question]

(a) Greater the resolving power
(b) Greater the magnifying power
(c) Smaller the resolving power
(d) Smaller the magnifying power

Answer: (b)

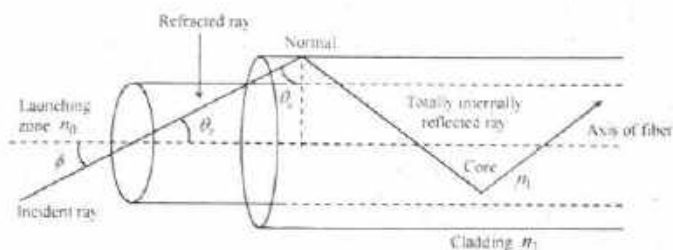
B. Answer the following Questions:

25. Explain the mechanism of light propagation through optical fiber.

[WBSCTE 2022]

Answer:

The propagation of light through the optical fiber is possible due to the **total internal reflection**. When a light ray goes from denser medium to a rarer medium it bends away from the normal. Snell's law tells us that if the angle of incidence increases, the angle of refraction also increases. The angle of incidence for which the angle of refraction is 90° is called **critical angle**. Now If the angle of incidence is slightly increased the critical angle, the light is completely reflected at the surface of rarer mediums, this is known as total internal reflection.



Let us consider three light rays enter the fibre at the launching end making different angles with the axis of the fibre as shown in the Fig. It is clear that the ray 2 strikes the boundary of the cladding at an angle equal to the critical angle and travels through the fibre parallel to the core-cladding interface. The ray 1 which enters the fibre at an angle greater than ray 2, strikes the boundary of the cladding at an angle less than critical angle and get refract into cladding. On the other hand, ray 3, which enters the fibre at an angle less than ray 2, undergoes total internal reflection at the core-cladding interface.

It is clear that the light propagates through the fibre only when its angle of incidence at launching end is less than or equal to a certain value. This maximum possible angle with the axis of the fibre up to which a light ray accepted into the core of the fiber is called **acceptance angle**. The light rays that enter the fibre beyond acceptance angle refracts into cladding.

26. Which type of reflection results in a clear and sharp reflection? [Model Question]

Answer:

Specular or regular reflection produces a clear and sharp reflection.

Long Answer Type Questions

1. (a) State the criteria for the phenomenon of total internal reflection of light to take place.
- (b) How does focal length of a lens change when red light incident on it is replaced by violet light?
- (c) Draw a ray diagram of a compound microscope. Write the expression for its magnifying power.
- (d) How will the interference pattern in Young's double slit experiment get affected, when
 - i) distance between the slits S_1 and S_2 reduced and
 - ii) the entire set up is immersed in water? Justify your answer.

[WBSCTE 2022]

Answer:

(a) The criteria for total internal reflection to take place are:

- (i) Light must travel from a denser medium to a rarer medium and
- (ii) The angle of incidence inside the denser medium must be greater than the critical angle.

(b) The refractive index of the material of a lens increases with the decrease in wavelength of the incident light. So, focal length will decrease with decrease in wavelength according to lens makers formula.

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Thus, when we replace red light with violet light, then due to decrease in wavelength the focal length of the lens will decrease.

(c) The diagram of a compound microscope is shown in Figure. The lens near the object, called the **objective**, forms a real, inverted, magnified image of the object. This serves as the object for the second lens which is the **eyepiece**. Eyepiece serves as a simple microscope that produces finally an enlarged and virtual image. The first inverted image formed by the objective is to be adjusted close to, but within the focal plane of the eyepiece so that the final image is formed nearly at infinity or at the near point. The final image is inverted with respect to the original object and obtain the magnification for a compound microscope.

Magnification of compound microscope

From the ray diagram, the linear magnification due to the objective is,

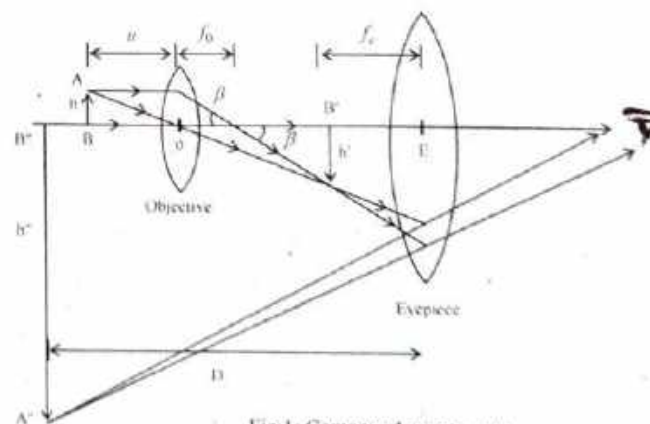


Fig 1: Compound microscope

$$m_o = \frac{h'}{h} \quad \dots (1)$$

From the figure 1, $\tan \beta = \frac{h}{f_e} = \frac{h'}{L}$, then

$$\frac{h'}{h} = \frac{L}{f_o} \quad \dots (2)$$

$$m_o = \frac{L}{f_o} \quad \dots (3)$$

Here, the distance L is between the first focal point of the eyepiece to the second focal point of the objective. This is called the tube length L of the microscope as f_o and f_e are comparatively smaller than L .

If the final image is formed at P (near point focussing), the magnification m_e of the eyepiece is,

$$m_e = 1 + \frac{D}{f_e} \quad \dots (4)$$

The total magnification m in near point focusing is,

$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(1 + \frac{D}{f_e} \right) \quad \dots (5)$$

If the final image is formed at infinity (normal focusing), the magnification m_e of the eyepiece is,

$$m_e = \frac{D}{f_e} \quad \dots (6)$$

The total magnification m in normal focusing is,

$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(\frac{D}{f_e} \right) \quad \dots (7)$$

(d) Fringe width of interference pattern = β

$$(i) \therefore \beta = \frac{\lambda D}{d}$$

fringe width is inversely proportional to the distance between slits s_1 and s_2 .

$$(ii) \therefore \beta = \frac{\lambda D}{d}$$

$$\Rightarrow \beta \propto \lambda$$

In water λ (wavelength) of light decreases so fringe width will decrease.

$$\lambda_m = \frac{\lambda_{\text{vac}}}{\mu}$$

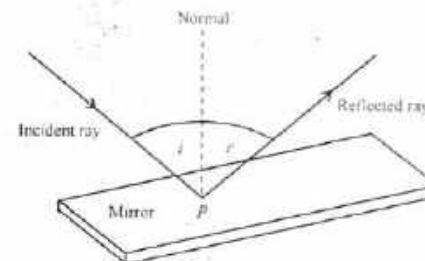
2. What is Reflection of light?

[Model Question]

Answer:

When a ray of light approaches a smooth polished surface and the light ray bounces back, it is called the reflection of light. The incident light ray that land on the surface is reflected off the surface. The ray that bounces back is called the reflected ray. If a perpendicular were drawn on a reflecting surface, it would be called normal. The figure below shows the reflection of an incident beam on a plane mirror.

Here, the angle of incidence and angle of reflection are with respect to normal and the reflective surface.



Laws of Reflection:

The laws of reflection determine the reflection of incident light rays on reflecting surfaces, like mirrors, smooth metal surfaces and clear water. Let's consider a plane mirror as shown in the figure above. The law of reflection states that

- The incident ray, the reflected ray and the normal all lie in the same plane.
- The angle of incidence = Angle of reflection.

3. What is Refraction of light? Explain the causes of refraction.

[Model Question]

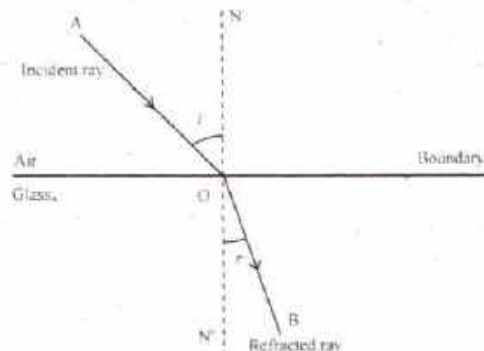
Answer:

Refraction is the bending of a wave when it passes from one medium to another. The bending is caused due to the differences in density between the two substances.

Causes of Refraction:

(i) Change of Speed Results in Change in Direction:

A light ray refracts whenever it travels at an angle into a medium of different refractive index. This change in speed results in a change in direction. As an example, consider air travelling into water. The speed of light decreases as it continues to travel at a different angle.



The refraction of light in glass is shown in the figure above. When light travels from air into glass, the light slows down and changes direction slightly. When light travels from a less dense substance to a denser substance, the refracted light bends more towards the normal line. If the light wave approaches the boundary in a perpendicular direction, the light ray doesn't refract despite the change in speed.

Laws of Refraction of Light

Laws of refraction state that:

- The incident ray, refracted ray, and the normal to the interface of two media at the point of incidence all lie on the same plane.
- The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. This is also known as Snell's law of refraction.

$$\frac{\sin i}{\sin r} = \text{constant.}$$

4. Light travelling in air enters into an optical fibre of refractive index 1.44.

(a) In which direction does the light bend?

(b) If the angle of incidence on one end of the fibre is 22° , then what is the angle of refraction? [Model Question]

Answer:

a) The light travels from a rarer medium (air) to a denser medium (optical fibre). Hence the refracted ray will bend towards the normal.

b) The angle of refraction can be calculated as follows:

let air be medium 1 and optical fibre be medium 2. Therefore, $n_1 = 1.00$, $n_2 = 1.44$ and $\theta_i = 22^\circ$

Now, substituting the values in the equation as follows:

$$(1.00) \sin 22^\circ = 1.44 \sin \theta_r$$

$$\theta_r = \sin^{-1}(0.260) = 15^\circ$$

5. Explain: The formation of an image by convex mirror

[Model Question]

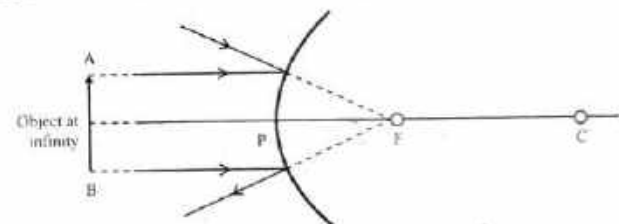
Answer:

Rays emerging from a point actually meet at another point after reflection, and that point is called the image of the first point. The image is real if the rays converge to the point and it is virtual if the rays do not meet it but appear to diverge from a point when the rays are produced backwards.

During image formation we assume that the rays are paraxial, i.e., they are incident at points close to the pole P of the mirror and make small angles with the principal axis. For a convex mirror, we consider two positions of the object before the mirror.

- When an object is placed at infinity.
- When the object is placed between infinity and the pole of the mirror.

For the image formation by a convex mirror in this condition, we consider two rays parallel to the principal axis and one directed towards the pole of the mirror originating from the object. These rays after reflection form an image at F, the principal focus of the mirror, which is behind the convex mirror. The image thus formed is highly diminished, virtual and erect.



6. Write down the lens maker equation for thin lenses. Write down its limitations. [Model Question]

Answer:

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where, f is the focal length (half the radius of curvature)

n is the refractive index of the material used

R_1 is the radius of curvature of sphere 1

R_2 is the radius of curvature of sphere 2

As previously mentioned, this equation can be used only for thin lenses. Lenses where the thickness is lesser, such that they are considered negligible in comparison to the radius of curvature, are referred to as thin lenses.

If the thickness of lens has to be considered in comparison to the radius of curvature, the following lens formula for thick lenses can be used.

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{n \times R_1 \times R_2} \right]$$

where d is the thickness of the lens in consideration.

Another situation that has to be considered is the lens maker's formula accounting for objects that are present in different media. The equation is as follows.

$$\frac{1}{f} = \left[\frac{n_1}{n_2} - 1 \right] \times \left[\frac{R_1 - R_2}{R_1 \times R_2} \right]$$

where,

n_1 is the refractive index of the lens in consideration

n_2 is the refractive index of the medium outside.

Limitations of Lens Maker's Formula:

1. The lens needs to be thin. This is because the separation between the two refracting surfaces will also be small.
 2. The medium on either side of the lens needs to be the same.
- On violation of the limitations then the refraction at the curved surface formula is used for both the surfaces.

7. What is Critical angle? What is refractive index?

[Model Question]

Answer:

In Optics, The angle of incidence to which the angle of refraction is 90° is called the critical angle. The ratio of velocities of a light ray in the air to the given medium is a refractive index.

8. Explain the relation between critical angle and refractive index.

[Model Question]

Answer:

The relationship between critical angle and refractive index can be derived as –

Consider a ray of light,

- Let the angle of incidence i be critical angle C
- Let the angle of refraction $r = 90^\circ$
- Refractive index of the rarer medium be μ_a
- Refractive index of the denser medium be μ_b

Applying Snell's Law

$$\begin{aligned} \frac{\sin i}{\sin r} &= \frac{\mu_a}{\mu_b} \\ \Rightarrow \mu_b \sin C &= \mu_a \sin 90^\circ \\ \Rightarrow \frac{\mu_b}{\mu_a} &= \frac{1}{\sin C} \end{aligned}$$

Thus, we arrive at a formula expressing the critical angle and refractive index relation –

$$\mu_b = \frac{1}{\sin C}$$

9. Explain the terms:

(i) Power of lens

(ii) Magnification

[Model Question]

Answer:

(i) Power of lens:

Power of a lens is its ability to converge or diverge the rays of light falling on it. Power of a lens is equal to reciprocal of the focal length of the lens. SI unit of power is dioptre (D).

(ii) Magnification:

When a linear object is placed perpendicular to the principal axis of lens, a linear image is formed perpendicular to the principal axis due to the refraction of the lens. The position, size and the nature of the image formed depend on the position of the object with respect to the lens.

The ratio of the linear size of the image to the linear size of the object is called the magnification of the lens.

m = linear size of the image / linear size of the object

10. What is Total internal reflection?

[Model Question]

Answer:

Total internal reflection is defined as the phenomenon which occurs when the light rays travel from a more optically denser medium to a less optically denser medium.

A ray of light passes from a medium of water to that of air. Light ray will be refracted at the junction separating the two media. Since it passes from a medium of a higher refractive index to that having a lower refractive index, the refracted light ray bends away from the normal. At a specific angle of incidence, the incident ray of light is refracted in such a way that it passes along the surface of the water. This particular angle of incidence is called the critical angle. Here the angle of refraction is 90 degrees. When the angle of incidence is greater than the critical angle, the incident ray is reflected back to the medium. We call this phenomenon total internal reflection.

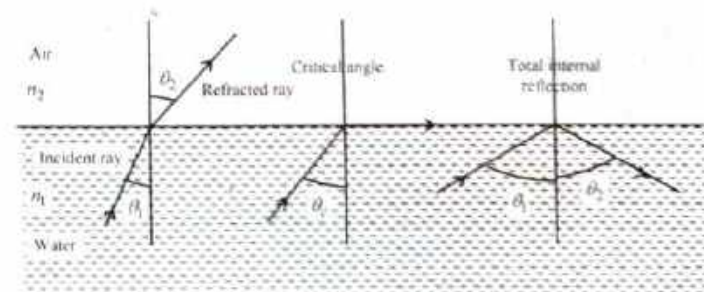


Fig: Total Internal Reflection

Formula of Total Internal Reflection

$$\text{Total Internal reflection, } \frac{n_1}{n_2} = \frac{\sin i}{\sin r}$$

$$\text{Critical angle, } \theta \quad \sin \theta = \frac{n_2}{n_1}$$

Notations used in the total internal reflection formula and critical angle.

- r is the angle of refraction
- i is the angle of incidence
- n_1 is the refractive index in medium 1
- n_2 is the refractive index in medium 2
- θ is the critical angle.

11. The focal length of a lens is 50 cm. Calculate the power of that lens. Write the type of this lens. [Model Question]

Answer:

Given,

Focal length $f = 50$ cm

$$\text{Power, } P = \frac{100}{f} = \frac{100}{50}$$

$$P = 2D$$

Since, focal length in +ve lens is convex.

12. A convex lens is of focal length 25 cm. Calculate the power of the lens. [Model Question]

Answer:

$$\text{Power of lens} = \frac{1}{\text{Focal length (in metre)}} = \frac{100}{\text{Focal length (in centimetre)}} = \frac{100}{25} = 4D$$

13. Write down the applications of Total internal reflection in Optical fiber. [Model Question]

Answer:

When the incident ray falls on the cladding, it suffers total internal reflection as the angle formed by the ray is greater than the critical angle. Optical fibers have revolutionized the speed with which signals are transferred, not only across cities but across countries and continents making telecommunication one of the fastest modes of information transfer. Optical fibers are also used in endoscopy.

14. Find the refractive index of the medium whose critical angle is 40° . [Model Question]

Answer:

Critical angle, $\theta = 40^\circ$

Refractive index of the medium, $\mu = ?$

$$\mu = \frac{1}{\sin \theta}, \mu = \frac{1}{\sin 40^\circ}, \mu = \frac{1}{0.65}, \mu = 1.6$$

15. An optical fibre made up the glass with refractive index $n_1 = 1.5$ which is surrounded by another glass of refractive index n_2 of the cladding such that the critical angle between the two cladding is 80° . [Model Question]

Answer:

Critical angle, $\theta = 80^\circ$

Refractive index, $n_1 = 1.5$

Refractive index $n_2 = ?$

Using the below formula, we can calculate n_2

$$\sin \theta = \frac{n_2}{n_1}$$

$$\sin 80^\circ = \frac{n_2}{1.5}$$

$$n_2 = 1.5 \times \sin 80^\circ$$

$$n_2 = 1.48$$

16. Explain the ray diagram of a compound microscope. Write the expression for its magnifying power. [Model Question]

Answer:

The diagram of a compound microscope is shown in Figure 1. The lens near the object, called the **objective**, forms a real, inverted, magnified image of the object. This serves as the object for the second lens which is the **eyepiece**. Eyepiece serves as a simple microscope that produces finally an enlarged and virtual image. The first inverted image formed by the objective is to be adjusted close to, but within the focal plane of the eyepiece so that the final image is formed nearly at infinity or at the near point. The final image is inverted with respect to the original object and obtain the magnification for a compound microscope.

Magnification of compound microscope

From the ray diagram, the linear magnification due to the objective is,

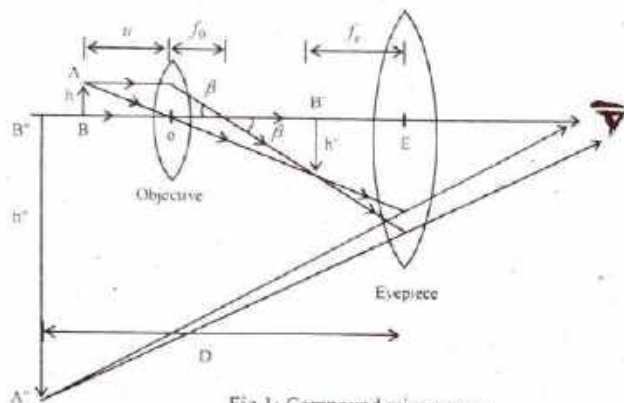


Fig 1: Compound microscope

$$m_o = \frac{h'}{h} \quad \dots (1)$$

From the figure 1, $\tan \beta = \frac{h}{f_o} = \frac{h'}{L}$, then

$$\frac{h'}{h} = \frac{L}{f_o} \quad \dots (2)$$

$$m_o = \frac{L}{f_o} \quad \dots (3)$$

Here, the distance L is between the first focal point of the eyepiece to the second focal point of the objective. This is called the tube length L of the microscope as f_o and f_e are comparatively smaller than L .

If the final image is formed at P (near point focussing), the magnification m_e of the eyepiece is,

$$m_e = 1 + \frac{D}{f_e} \quad \dots (4)$$

The total magnification m in near point focusing is,

$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(1 + \frac{D}{f_e} \right) \quad \dots (5)$$

If the final image is formed at infinity (normal focusing), the magnification m_e of the eyepiece is,

$$m_e = \frac{D}{f_e} \quad \dots (6)$$

The total magnification m in normal focusing is,

$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(\frac{D}{f_e} \right) \quad \dots (7)$$

Chapter 3: Electrostatics

Chapter at a glance:

3.1 Electric field

The electric field is defined mathematically as a vector field that can be associated with each point in space, the force per unit charge exerted on a positive test charge at rest at that point.

The electric field is generated by the electric charge or by time-varying magnetic fields. In the case of atomic scale, the electric field is responsible for the attractive forces between the atomic nucleus and electrons which hold them together.

According to coulomb's law, a particle with electric charge q_1 at position x_1 exerts a force on a particle with charge q_0 at position x_0 of,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{(x_1 - x_0)^2} \hat{r}_{1,0}$$

Where,

$\hat{r}_{1,0}$ is the unit vector in the direction from point x_1 to point x_0

ϵ_0 is the electric constant also known as absolute permittivity of free space $C^2 m^{-2} N^{-1}$

When the charges q_0 and q_1 have the same sign then the force is positive, the direction is away from other charge which means they repel each other. When the charges have unlike signs then the force is negative and the particles attract each other.

Electric field is force per unit charge,

$$E(x_0) = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{(x_1 - x_0)^2} \hat{r}_{1,0}$$

Electric field can be found easily by using Gauss law which states that the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

Or total flux linked with a surface is $1/\epsilon_0$ times the charge enclosed by the closed surface. The electric field can also be calculated by coulomb's law but the Gauss law method is easier. Besides, Gauss law is just a replica of the coulomb's law. If we apply Gauss theorem to a point charge enclosed by a sphere, we will get to coulomb's law.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The electric flux through each of the faces of a cube of side 1 m if a charge q coulomb is placed at its centre is [Model Question]

(a) $\frac{q}{4\epsilon_0}$ (b) $4\epsilon_0 q$ (c) $\frac{q}{6\epsilon_0}$ (d) $\frac{\epsilon_0}{6q}$

Answer: (c)

2. Work done to displace a charge q from a point P_1 having potential V_1 to a point P_2 having potential V_2 is [Model Question]

(a) $q(V_2 - V_1)$ (b) $q(V_1 V_2)$ (c) $q(V_2 + V_1)$ (d) $q(V_2 / V_1)$

Answer: (a)

3. The unit of $\iint \vec{D} \cdot d\vec{s}$ is [Model Question]

(a) Coulomb (b) Farad (c) Watt (d) Volt

Answer: (a)

4. The electric field between two oppositely charged plates with equal charge density σ is [Model Question]

(a) 0 (b) $\frac{2\sigma}{\epsilon_0}$ (c) $\frac{\sigma}{\epsilon_0}$ (d) $\frac{\sigma}{2\epsilon_0}$

Answer: (c)

5. The total polarization of a polyatomic gas is [Model Question]

(a) $\vec{P} = N(\alpha_e + \alpha_i)\vec{E}$ (b) $\vec{P} = N\left(\alpha_e + \alpha_i + \frac{\mu}{KT}\right)\vec{E}$
 (c) $\vec{P} = N\left(\alpha_e + \alpha_i + \frac{\mu^2}{3KT}\right)\vec{E}$ (d) $\vec{P} = \frac{N\mu\vec{E}}{KT}$

Answer: (c)

6. A vector field is said to be solenoidal only when [Model Question]

(a) its divergence is zero
 (b) its curl is zero
 (c) both of its curl and divergence are zero
 (d) both of its curl and divergence are infinite

Answer: (a)

7. Flux of the electric field for a point charge $(+q)$ at the origin through a spherical surface centered at the same origin is [Model Question]

(a) $\frac{q}{\epsilon_0}$ (b) $\frac{q}{4\pi\epsilon_0}$ (c) $\frac{2q}{\epsilon_0}$ (d) $\frac{4q}{\epsilon_0}$

Answer: (a)

8. The change in net charge inside a dielectric before and after polarization remains [Model Question]

(a) negative (b) positive
 (c) unchanged (d) net charge becomes zero

Answer: (d)

9. Total electric flux (ϕ) of electric field ($\vec{E}$) over a surface S is [Model Question]

(a) $\vec{E} \cdot d\vec{S}$ (b) $\oint_S \vec{E} \cdot d\vec{S}$ (c) $\vec{E} \times d\vec{S}$ (d) $\oint_S \vec{E} \times d\vec{S}$

Answer: (b)

10. Electrical susceptibility χ_e is [Model Question]

(a) $\chi_e = \frac{P}{\epsilon_0 E}$ (b) $\chi_e = \frac{P}{3\epsilon_0 E}$ (c) $\chi_e = \epsilon_0 EP$ (d) $\chi_e = \frac{3\epsilon_0 E}{P}$

Answer: (a)

11. The unit of polarizability is [Model Question]

(a) Nm^2/C^2 (b) debye (c) Fm^2 (d) V/m

Answer: (c)

12. The electrical conductivity of an ideal dielectric is [Model Question]

(a) infinity (b) zero (c) very large (d) moderate

Answer: (b)

13. When a dielectric material is inserted between two plates of a capacitor, the capacitance will be [Model Question]

(a) increased (b) decreased (c) same (d) zero

Answer: (a)

14. In a conservative field [Model Question]

(a) work done is zero (b) line integral is independent of path
 (c) curl is not equal to zero (d) divergence is zero

Answer: (b)

15. Line integral of an electrostatic field is the

- (a) work done by the field per unit charge (b) electric field
(c) work done by magnetic field (d) total charge in the field

Answer: (a)

[Model Question]

Long Answer Type Questions

- Q 1. (a) State Gauss' law in electrostatics. Using the law, derive an expression for the electric field due to a uniformly charged sphere.
(b) Why electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface?
(c) Net capacitance of three identical capacitors in series is $1\mu F$. What will be their net capacitance if connected in parallel?
(d) Define electric flux. Write its SI unit.

[WBSCTE 2022]

Answer:

a) 1st Part:

Gauss's law-statement: The net outward electric flux (i.e., integral of the normal component of the electric field) over any closed surface of any shape drawn in an electric field is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

Mathematically, it can be represented as $\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} q$ where

$\oint_S$ indicates the surface integral over whole of the closed surface

and q is net charge enclosed by the surface.

From Gauss's law we have $\oint_S \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} q$

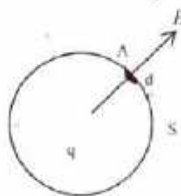
Considering the sphere of radius r , and elementary area dS as shown in the figure. Electric field at A due to charge q at the center is E . The direction of E is along $\hat{n}$ i.e., normal to dS . Hence $\vec{E} \cdot d\vec{S} = E dS$

From above Gauss theorem we can write $E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} q$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

If another charge q' is placed on A then force on the charge q'

$$\vec{F} = \vec{E} q' = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2} \hat{r} \text{ which is coulomb's law.}$$



2nd Part:

Electric field at a point inside a uniformly charged sphere

Let us consider a point P at a distance r from the sphere centre as shown in figure 2. Let the electric field intensity at the point P be $\vec{E}$.

$\therefore$ Applying Gauss's law, we have

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\text{or, } E \times (4\pi r^2) = \frac{q_m}{\epsilon_0}$$

where q_m is the charge enclosed by the sphere of radius r (shown with dotted lines).

$\therefore$ Considering eqn. (2) we can get the value of q_m by substituting $R = r$

$$\therefore q_m = \frac{4}{3} \pi r^3 \rho$$

$$\text{Hence we get } E \times (4\pi r^2) = \frac{\frac{4}{3} \pi r^3 \rho}{\epsilon_0}$$

$$\text{or, } E = \frac{\frac{4}{3} \pi r^3 \rho}{4\pi r^2 \epsilon_0} = \frac{r \rho}{3\epsilon_0}$$

$$\therefore \vec{E} = \frac{r \rho}{3\epsilon_0} \hat{r} \quad \dots (4)$$

Thus electric field varies linearly with the distance from the centre of the sphere.

$$\text{Therefore, flux density } \vec{D} = \epsilon \vec{E} = \frac{r \rho}{3} \hat{r}$$

b) A conductor has free electrons. As long as electric field is not zero, the free charge carriers would experience force and drift. When there is no current inside or on the surface of the conductor, the free charges have distributed themselves so the electric field is zero everywhere inside the conductor. Electrostatic field is zero inside a conductor. As inside the conductor the electric field is zero, so no work is done against the electric field to bring a charge particle from one point to another.

$$E = -\left(\frac{dV}{dr}\right) = 0$$

where E is the electrostatic potential, V is the potential difference between two points on a conductor and r is the distance between the two points.

Since $E=0$, therefore the potential V inside the surface is constant.

Because there is no potential difference between any two points inside the conductor, the electrostatic potential is constant throughout the volume of the conductor.

$$\Delta V = V_1 - V_2 = 0$$

$$\Rightarrow V_1 = V_2$$



Fig. 2

This proves that the potential at every point is same inside the conductor.

c) It is given that $C_1 = C_2 = C_3 = C$ (say)

Then their equivalent capacitance in series $C_s = \frac{C}{3} = 1 \mu F$

$$\Rightarrow C = 3 \mu F$$

$\therefore$ Net capacitance in parallel arrangement $C_p = 3C = 3 \times 3 \mu F = 9 \mu F$

Let the capacitor combination be connected, in both configurations, to a given source of voltage V . Then net energy stored in parallel combination

$$u_p = \frac{1}{2} C_p V^2 \Rightarrow \frac{u_p}{u_s} = \frac{C_p}{C_s} = \frac{1 \mu F}{9 \mu F} = \frac{1}{9}$$

d) The total number of electric field lines passing a given area in a unit time is defined as the electric flux. Similar to the example above, if the plane is normal to the flow of the electric field, the total flux is given as:

$$\phi_p = EA$$

When the same plane is tilted at an angle Θ , the projected area is given as $A \cos \Theta$ and the total flux through this surface is given as:

$$\phi = EA \cos \Theta$$

Where,

- E is the magnitude of the electric field
- A is the area of the surface through which the electric flux is to be calculated
- Θ is the angle made by the plane and the axis parallel to the direction of flow of the electric field

The SI unit of electric flux is voltimeters (V m).

2. A very long cylindrical object carries charge distribution proportional to the distance from the axis (r). If the cylinder is of radius a , then find the electric field both at $r > a$ and $r < a$, by the application of Gauss' law in electrostatics.

[Model Question]

Answer:

Let ρ be the volume density of charge of the cylinder having radius a . Let us choose two cylindrical surfaces of length l and radii x and $x+dx$. Since $\vec{E}$ is parallel to the end faces, the component of $\vec{E}$ normal to the end faces is zero. For cylindrical symmetry at all points of the curved surface, $|\vec{E}| = \text{Constant}$ for $r > a$

We choose a Gaussian surface of radius r . By using Gauss's law we have

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0} \rightarrow E 2\pi r l = \frac{1}{\epsilon_0} \int_0^a 2\pi x dx l \times \rho$$

$$E 2\pi r l = \frac{1}{\epsilon_0} \int_0^a 2\pi x dx l \times kx$$

$$\rightarrow E 2\pi r l = \frac{2\pi l k}{\epsilon_0} \int_0^a x^2 dx$$

{ k is proportionality constant as before}

$$\rightarrow E = \frac{ka^3}{3\epsilon_0 r} \quad \text{for } r < a$$

Same way we can choose a Gaussian surface as stated before and this surface is internal to the cylinder. So electrical flux through the Gaussian surface is

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \rightarrow E 2\pi r l = \frac{1}{\epsilon_0} \int_0^r 2\pi x dx l \times \rho$$

$$E 2\pi r l = \frac{1}{\epsilon_0} \int_0^r 2\pi x dx l \times kx$$

$$\rightarrow E 2\pi r l = \frac{2\pi l k}{\epsilon_0} \int_0^r x^2 dx \quad \{k \text{ is proportionality constant as before}\} \rightarrow E = \frac{kr^2}{3\epsilon_0}$$

which is the required electrical field and direction is radially outward.

3. State Gauss's law in electrostatics.

[Model Question]

Answer:

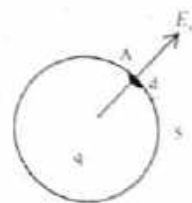
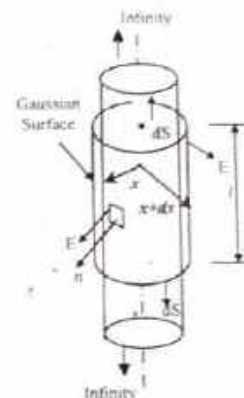
Gauss's law statement: The net outward electric flux (i.e., integral of the normal component of the electric field) over any closed surface of any shape drawn in an electric

field is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

Mathematically it can be represented as $\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} q$ where

$\oint_S$ indicates the surface integral over whole of the closed surface and q is net charge enclosed by the surface.

From Gauss's law we have $\oint_S \vec{E} \cdot \hat{n} dS = \frac{1}{\epsilon_0} q$



Considering the sphere of radius r , and elementary area dS as shown in the figure. Electric field at A due to charge q at the center is E . The direction of E is along $\hat{r}$ i.e., normal to dS . Hence $\vec{E} \cdot d\vec{S} = E dS$

From above Gauss theorem we can write $E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} q$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

If another charge q' is placed on A then force on the charge q'

$$\vec{F} = \vec{E}q' = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2} \hat{r} \text{ which is coulomb's law.}$$

4. A capacitor is made with two infinitely long cylindrical conductors of radii a and b ($a > b$), with vacuum in the intervening space. If the internal cylinder is kept at ground and the outer cylinder has a charge density σ then solve Laplace equation to find the electrostatic potential in the space between the cylinders. [Model Question]

Answer:

Since the region is free from charge so we can apply Laplace's equation as $\nabla^2 V = 0$.

In cylindrical form we have Laplace's equation as $\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 0$

Solution of the above differential equation as $V = C \ln r + D$

at $r = b$ $V = 0$, $\therefore 0 = C \ln b + D$ or, $D = -C \ln b$ $\therefore V = C \ln \left(\frac{r}{b} \right)$

at $r = a$ $V = V_0$ So, $V_0 = C \ln \left(\frac{a}{b} \right) \Rightarrow C = \frac{V_0}{\ln \left(\frac{a}{b} \right)}$ $\therefore V = \frac{V_0}{\ln \left(\frac{a}{b} \right)} \ln \left(\frac{r}{b} \right)$ (i)

Using Gauss law $\int \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \rightarrow 2\pi r l E = \frac{2\pi a l}{\epsilon_0} \sigma \rightarrow E = \frac{a\sigma}{\epsilon_0 r}$

$$\text{So, } V_0 = \int \frac{a\sigma}{\epsilon_0 r} dr = \frac{a\sigma}{\epsilon_0} \ln \left(\frac{b}{a} \right)$$

On substituting the value of V_0 in (i) we have

$$V = \frac{a\sigma}{\epsilon_0} \frac{\ln \left(\frac{b}{a} \right)}{\ln \left(\frac{a}{b} \right)} \ln \left(\frac{r}{b} \right)$$

or, $V = -\frac{a\sigma}{\epsilon_0} \ln \left(\frac{r}{b} \right)$ which is the required potential.



5. (a) Use Gauss' law to calculate the electric field between infinite extent parallel plate capacitor carrying charge density σ and mutual separation d .
(b) Verify whether the potential function $V(x, y)$ satisfy Laplace's equation or not. Find also the charge density. [Model Question]

Answer:

a) We have from Gauss's law $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} = \frac{\sigma S}{\epsilon_0}$

For one conducting plate $2ES = \frac{\sigma S}{\epsilon_0}$ or, $E = \frac{\sigma}{2\epsilon_0}$

For the case of a charged conducting plate, charge is distributed over the surface of conducting plane of uniform thickness and of infinite size with uniform charge density σ . So electric field will be the sum of the electric fields arising from two surfaces of the conducting plate.

So the electric field will be $E = 2 \times \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \rightarrow \vec{E} = \frac{\sigma}{\epsilon_0} \hat{i}$ (along X direction say)

So, electrical field is independent of distance of separation of plates.

b) Using $\vec{E} = -\nabla V = -\left(\frac{d}{dx} V \hat{i} + \frac{d}{dy} V \hat{j} + \frac{d}{dz} V \hat{k} \right)$ we have

$$\text{So, } -\left(\frac{d}{dx} V \hat{i} + \frac{d}{dy} V \hat{j} + \frac{d}{dz} V \hat{k} \right) = \frac{\sigma}{\epsilon_0} \hat{i} \rightarrow \frac{dV}{dx} = -\frac{\sigma}{\epsilon_0} \rightarrow V = -\frac{\sigma}{\epsilon_0} x$$

So $\nabla^2 V = 0$ which satisfy Laplace's equation.

So the charge density is zero.

6. a) Write down Laplace's equation. Show that the potential function $x^2 - y^2 + z^2$ satisfies Laplace's equation.

b) Show that when a dielectric is placed in an electric field, the field within the dielectric becomes weaker than the original field. [Model Question]

Answer:

a) If the region of space is charge free i.e., $\rho = 0$, then in that region potential V will be $\nabla^2 V = 0$ which is Laplace's equation.

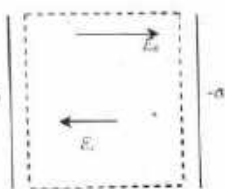
As $V = (x^2 - y^2 + z^2)$ now we have $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 2 - 2 + 0 = 0$ which indicates that V satisfies Laplace's equation.

b) Let the vacuum inside the charged plate is replaced by medium of dielectric constant ϵ . The dielectric will be polarized. That is each molecule of the dipole will acquire dipole moment and will be oriented along the electric field. As a result, the surfaces of the

dielectric will acquire induced opposite bound charges of charge density σ_i which is in general less than σ . The corresponding free and bound charges are q_0 and q_i . Also $q_i < q$. The induced electric

field within the plates of the capacitor will be $E_i = \frac{\sigma_i}{\epsilon_0}$. It

will be directed opposite to $\vec{E}_0$. Therefore the net electric field within the dielectric medium will be $E = E_0 - E_i$. So electric field on the dielectric E will be less than the original field E_0 .



7. (a) In a region of space the electric field is given by $E = 8\hat{i} + 4\hat{j} + 3\hat{k}$. Calculate the electric flux through a surface area 100 unit in XY plane.
(b) Show that $\phi = 1/r$ is a solution of Laplace's equation. [Model Question]

Answer:

a) As the surface area is in xy plane so the unit vector of the surface area will be its outward drawn vector of xy plane i.e., along Z axis i.e., $\hat{k}$
So $d\vec{S} = \hat{k}dS = 100\hat{k}$ hence electric flux $\vec{E} \cdot d\vec{S} = (8\hat{i} + 4\hat{j} + 3\hat{k}) \cdot 100\hat{k} = 300$ units.

$$\text{b) } \nabla^2 = \nabla \cdot \nabla = \nabla \cdot \frac{d}{dr} \hat{r} \quad \text{Hence } \nabla^2 f(r) = \nabla \cdot \left\{ \frac{d}{dr} (f(r)) \right\} = \nabla \cdot \frac{df(r)}{dr} \hat{r}$$

$$\therefore \nabla^2 f(r) = \nabla \cdot \frac{df(r)}{dr} \hat{r} = \nabla \cdot \left(\frac{df(r)}{dr} \frac{\vec{r}}{r} \right) = \nabla \cdot \left(\frac{1}{r} \frac{df(r)}{dr} \right) \vec{r}$$

Comparing standard formula $\nabla \cdot (\phi \vec{r}) = \phi \nabla \cdot \vec{r} + \vec{r} \cdot \nabla \phi$ with the above equation we have scalar function $\phi = \frac{1}{r} \frac{df(r)}{dr}$ and $\vec{r} = \vec{r}$. So we can write

$$\nabla^2 f(r) = \frac{1}{r} \frac{df(r)}{dr} \nabla \cdot \vec{r} + \vec{r} \cdot \nabla \left(\frac{1}{r} \frac{df(r)}{dr} \right) \quad [\because \nabla \cdot \vec{r} = 3]$$

$$= \frac{3}{r} \frac{df(r)}{dr} + \frac{d}{dr} \left[\frac{1}{r} \frac{df(r)}{dr} \right] \vec{r} \cdot \vec{r}$$

$$\nabla^2 f(r) = \frac{3}{r} \frac{df(r)}{dr} - \frac{1}{r^2} \frac{df(r)}{dr} \vec{r} \cdot \vec{r} + \frac{1}{r} \frac{d^2 f(r)}{dr^2} \vec{r} \cdot \vec{r}$$

$$\nabla^2 f(r) = \frac{3}{r} \frac{df(r)}{dr} - \frac{1}{r^2} \frac{df(r)}{dr} \vec{r} \cdot \vec{r} + \frac{1}{r} \frac{d^2 f(r)}{dr^2} \vec{r} \cdot \vec{r}$$

$$\nabla^2 f(r) = \frac{3}{r} \frac{df(r)}{dr} - \frac{1}{r} \frac{df(r)}{dr} + \frac{1}{r} \frac{d^2 f(r)}{dr^2}$$

$$\nabla^2 f(r) = \frac{2}{r} \frac{df(r)}{dr} + \frac{1}{r} \frac{d^2 f(r)}{dr^2}$$

Now substituting $f(r) = \frac{1}{r}$ in the above identity we have,

$$\nabla^2 \left(\frac{1}{r} \right) = \frac{2}{r} \frac{d}{dr} \left(\frac{1}{r} \right) + \frac{d^2}{dr^2} \left(\frac{1}{r} \right) = \frac{2}{r} \left(-\frac{1}{r^2} \right) + 2r^{-3} = 0$$

So it satisfies Laplace's equation.

8. The electrostatic potential at a point $P(\vec{r})$ is given by $V(\vec{r}) = \frac{1}{r}$. Show that it satisfies Laplace's equation $\nabla^2 V = 0$. [Model Question]

Answer:

$$\nabla^2 V(r) = \nabla \cdot \nabla \left(\frac{1}{r} \right) = \nabla \cdot \frac{d}{dr} \left(\frac{1}{r} \right) \vec{r} = \nabla \cdot (-r^{-2}) \vec{r}$$

$$= \nabla \cdot \left[-\frac{\vec{r}}{r^2} \right] = -\frac{1}{r^2} \nabla \cdot \vec{r} + \vec{r} \cdot \nabla \left[-r^{-2} \right]$$

$$= -\frac{3}{r^2} + \vec{r} \cdot \frac{d}{dr} [-r^{-2}] \hat{r} \quad [\because \nabla \cdot \vec{r} = 3]$$

$$\text{or, } \nabla^2 V(r) = -\frac{3}{r^2} + \vec{r} \cdot [-2r^{-3}] \vec{r} = -\frac{3}{r^2} + \frac{3}{r^2} = 0 \text{ which is Laplace's equation.}$$

Hence proved.

9. What is meant by polarization of a dielectric medium? [Model Question]

Answer:

When a dielectric is placed in an external electric field, the centre of positive charge of the constituent atoms or molecules is displaced along the applied field direction and that of the centre of negative charge is displaced in the opposite direction. Thus a dipole is produced. The process of producing electric dipoles which are originated along the field direction is called polarization in dielectrics.

10. The potential in a medium is given by $\phi(r) = \frac{qe^{-\frac{r}{\lambda}}}{4\pi\epsilon_0 r}$.

(i) Obtain the corresponding electric field,

(ii) Find the charge density that may produce the potential mention above.

[Model Question]

Answer:

$$\text{i) Given } \phi(r) = \frac{qe^{-\frac{r}{\lambda}}}{4\pi\epsilon_0 r} = A \frac{e^{-\frac{r}{\lambda}}}{r} \text{ where } A = \frac{q}{4\pi\epsilon_0}$$

$$\text{Electric field } \vec{E}(r) = -\frac{d\phi(r)}{dr} \hat{r} = A \left[\frac{-\frac{1}{\lambda} r e^{-r/\lambda} - e^{-r/\lambda}}{r^2} \right] \hat{r} = A \left[\frac{r+\lambda}{r^2 \lambda} \right] e^{-r/\lambda} \hat{r}$$

$$\vec{E}(r) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} + \frac{1}{\lambda} \right] e^{-r/\lambda} \frac{\vec{r}}{r^2}$$

ii) Taking derivative on both sides of the electric field expression we have

$$\rho = -\epsilon_0 \nabla^2 \phi = -\epsilon_0 \nabla \cdot \vec{E}(r)$$

$$\rho = \frac{q}{4\pi} \nabla \cdot \left[e^{-r/\lambda} \frac{\vec{r}}{r^3} + \frac{1}{\lambda} e^{-r/\lambda} \frac{\vec{r}}{r^2} \right]$$

$$\rho = \frac{q}{4\pi} \left[\nabla \cdot \left(e^{-r/\lambda} \frac{\vec{r}}{r^3} \right) + e^{-r/\lambda} \left(\nabla \cdot \frac{\vec{r}}{r^2} \right) + \frac{1}{\lambda} \nabla \cdot \left(e^{-r/\lambda} \frac{\vec{r}}{r^2} \right) + \frac{1}{\lambda} e^{-r/\lambda} \left(\nabla \cdot \frac{\vec{r}}{r^2} \right) \right]$$

$$\text{or, } \rho = \frac{q}{4\pi} \left[-\frac{1}{\lambda} e^{-r/\lambda} \frac{\vec{r}}{r^3} + e^{-r/\lambda} 4\pi \delta(\vec{r}) - \frac{1}{\lambda^2} e^{-r/\lambda} \frac{1}{r} + \frac{1}{\lambda} e^{-r/\lambda} \frac{1}{r^2} \right]$$

[where $\delta(\vec{r})$ is dirac delta function]

$$\text{or, } \rho = q \left[\delta(\vec{r}) - \frac{1}{4\pi\lambda^2 r} \right] e^{-r/\lambda}$$

Now using poisson's equation we have $\frac{d^2 \phi(r)}{dr^2} = -\frac{\rho}{\epsilon_0}$ so charge density will be

$$\rho = -\frac{q}{4\pi\lambda^2} \left[\frac{1}{r} - \frac{2\lambda^2}{r^3} \right] e^{-r/\lambda} \text{ which is the required expression of charge density.}$$

11. Check whether the electrostatic field given by $\vec{E} = ax^2y^2(\hat{i} + \hat{j})$ is conservative or not. [Model Question]

Answer:

A field is conservative if its curl is zero i.e., $\vec{\nabla} \times \vec{E} = 0$

$$\text{Now, } \vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy^3 & ax^2y^2 & 0 \end{vmatrix} = \hat{k}(2axy^2 - 3axy^2) = -axy^2 \hat{k}$$

Since $\vec{\nabla} \times \vec{E} \neq 0$, so $\vec{E} = ax^2y^2(\hat{i} + \hat{j})$ is not conservative.

12. (a) The potential field at any point in free space is given by $V = 5x^2y + 3yz^2 + 6xz$ Volt, where x, y, z are in meters. Calculate the volume charge density at point (2, 5, 3)m.

(b) Find out the relation between electric flux density (D), electric field (E) and polarization vector (F) in dielectric medium. [Model Question]

Answer:

a) Using Poisson's equation $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ we have

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = (16y)$. So $\frac{\rho}{\epsilon_0} = -16 \times 5 = -80$ or, $\rho = -80\epsilon_0$ which is the required charge density at (2, 5, 3)

b) Let the vacuum inside the charged plate is replaced by medium of dielectric constant k . The dielectric will be polarized. That is each molecule of the dipole will acquire dipole moment and will be oriented along the electric field. As a result, the surfaces of the dielectric will acquire opposite charge density σ_p which is in general less than σ_0 . That is $\sigma_p < \sigma_0$. Here σ_p is called induced charge density or bound charge density. The corresponding free and bound charges are q_0 and q_p . Also $q_p < q$. The induced electric

field within the plates of the capacitor will be $E_p = \frac{\sigma_p}{\epsilon_0}$. It will be directed opposite to

$\vec{E}_0$. Therefore the net electric field within the dielectric medium will be

$$\vec{E} = \vec{E}_0 - \vec{E}_p = \frac{\sigma_0}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0} \dots (1)$$

Polarization vector, $\vec{P}$ is equal to the bound charge per unit area or equal to the surface density of bound charges (because surface density is charge per unit area).

$$\text{Thus } \vec{P} = \frac{q_p}{A} = \sigma_p$$

$\vec{P}$ is also defined as the electric dipole moment of material per unit volume.

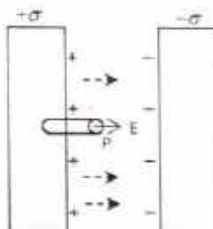
$\vec{P} = n\vec{p}$ where n is number of molecules per unit volume.

Displacement vector, $\vec{D}$ is equal to the free charge per unit area or equal to the surface density of free charges, thus $\vec{D} = \frac{q_0}{A} = \sigma_0$

From (1) we have $\epsilon_0 \vec{E} = \sigma_0 - \sigma_p$. Hence $\epsilon_0 \vec{E} = \vec{D} - \vec{P}$ or $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ which is the required relation.

13. Derive an expression for the electric field between two infinite extent parallel plate capacitors carrying charge density σ and mutual separation d . Draw the necessary diagram. [Model Question]

Answer:



Let us consider two infinite plane parallel sheets as shown. Let us first consider a point P lying anywhere in the region between the two plates with charge densities $+\sigma$ and $-\sigma$. We draw a normal cylinder of cross sectional area S through P having one of its ends in the conducting plate. Then we have from Gauss's law (as in the case in the infinite plane sheet)

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \text{ or, } ES = \frac{\sigma S}{\epsilon_0} \text{ hence } E = \frac{\sigma}{\epsilon}$$

(if we assume permittivity of a capacitor be ϵ)

So the field between the two plates is uniform and independent of their separation.

But any point lying outside the plate field will be zero as per the Gauss's law (no charge enclosed within)

14. a) Derive the relation between electric field intensity and electric potential. [Model Question]

Answer:

A charge in an electrical field intensity $\vec{E}$ experiences a force given by $\vec{F} = q\vec{E}$. If the charge moves, then the field does work if the field has a component parallel to the displacement. If $\vec{E}$ is uniform and has a component in the direction of the displacement, then the work done is $W = F \Delta x = qE \Delta x$.

If $\vec{E}$ makes an angle θ with respect to the displacement, then this can be written as $W = F \Delta x \cos \theta = qE \Delta x \cos \theta$.

The electrical force is conservative, so we can associate a potential energy with this force. If the work done by $\vec{E}$ is positive, then the potential energy decreases; if it is negative, then the potential energy increases. Specifically, $\Delta PE = -W = -qE \Delta x$.

The electric potential difference ΔV between two points A and B is defined as the electric potential energy difference of a charge q between these two points divided by the charge.

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q} \text{ (unit = J/C = volt = V)}$$

$$\text{Since } \Delta PE = -qE \Delta x \text{ (uniform E), then } \Delta V = -E \Delta x \text{ or, } E = -\frac{\Delta V}{\Delta x} = -\frac{dV}{dx}$$

$$\text{For three dimensional case } E_x = -\frac{dV}{dx} \text{ or, } E_y = -\frac{dV}{dy} \text{ or, } E_z = -\frac{dV}{dz}$$

$$\text{As } \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \text{ So } \vec{E} = -\frac{dV}{dx} \hat{i} - \frac{dV}{dy} \hat{j} - \frac{dV}{dz} \hat{k}$$

Hence $\vec{E} = -\nabla V$ which is the required relation.

14. (b) The potential in XY plane is given by

$$V = (x^2 + y^2)^{\frac{1}{2}}$$

Calculate the electric field at the point (3, 4).

[Model Question]

Answer:

$$\text{Electric field will be } \vec{E} = -\nabla V = -\frac{\partial}{\partial x}(x^2 + y^2)^{\frac{1}{2}} \hat{i} - \frac{\partial}{\partial y}(x^2 + y^2)^{\frac{1}{2}} \hat{j}$$

$$\vec{E} = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2x \hat{i} + \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2y \hat{j}$$

$$\vec{E} = \frac{x}{(x^2 + y^2)^{\frac{3}{2}}} \hat{i} + \frac{y}{(x^2 + y^2)^{\frac{3}{2}}} \hat{j}$$

$$\text{So electric field at (3, 4) is } \vec{E} = \frac{3\hat{i} + 4\hat{j}}{125}$$

14. (c) Write down Laplace's equation in cylindrical coordinates and find the solution. [Model Question]

Answer:

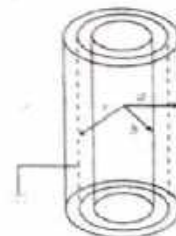
Laplace's equation in cylindrical coordinate can be represented as

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \text{ where } V \text{ is the potential.}$$

Let us consider a cylindrical capacitor of inner and outer radii a and b respectively. Let, the inner cylinder be at a potential V_0 and outer cylinder is grounded i.e., at $r = b$, $V = 0$.

To solve the Laplace's equation $\nabla^2 V = 0$ in cylindrical coordinate system, let us consider any point P inside a cylinder at a distance r from the axis of the cylinder. In this case the variation of potential V is considered only along the radial direction only.

Hence, the corresponding Laplace's equation for potential V in cylindrical coordinates can be written as,



$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = 0 \quad \text{or} \quad \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) = 0$$

Integrating we get,

$$\rho \frac{\partial V}{\partial \rho} = \text{constant} = A \text{ (say)} \quad \text{or} \quad \frac{\partial V}{\partial \rho} = \frac{A}{\rho}$$

Integrating again we get,

$$V = A \ln \rho + B \quad \dots (2)$$

where B is another constant of integration.

This is the solution of Laplace's equation in cylindrical coordinate considering the variation of V along the radial direction i.e., along ρ .

From (2) we can write,

$$0 = A \ln b + B \quad \text{or} \quad B = -A \ln b$$

$$\therefore V = A \ln \rho - A \ln b = A \ln \frac{\rho}{b}$$

when $\rho = a$, $V = V_0$

$$\text{From (2) we get, } V_0 = A \ln \frac{a}{b} \quad \text{or} \quad A = \frac{V_0}{\ln \frac{a}{b}}$$

$$\text{From (2) substituting the value of } A \text{ we get, } V = \frac{V_0 \ln \frac{\rho}{b}}{\ln \frac{a}{b}}$$

Above equation gives the value of potential at a point inside the cylindrical capacitor for which $a > \rho > b$.

14. (d) Why electric field inside a dielectric decreases due to polarization?

[Model Question]

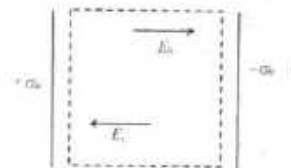
Answer:

Let the vacuum inside the charged plate is replaced by medium of dielectric constant k . The dielectric will be polarized. That is each molecule of the dipole will acquire dipole moment and will be oriented along the electric field. As a result, the surfaces of the dielectric will acquire induced opposite bound charges of charge density σ_i which is in general less than σ . The corresponding free and bound charges are q_0 and q_i . Also

$q_i < q$. The induced electric field within the plates of the capacitor will be $E_i = \frac{\sigma_i}{\epsilon_0}$. It

will be directed opposite to $\vec{E}_0$. Therefore the net electric field within the dielectric

medium will be $E = E_0 - E_i$. So electric field on the dielectric E will be less than the original field E_0 .



As we know surface charge density due to polarised charges is equal to P .

$$\therefore \text{Effective field } E = \frac{\sigma - P}{\epsilon_0}$$

$$\text{Again we know } \vec{P} = \text{const} \times \vec{E} = \epsilon_0 \chi \vec{E}$$

$$\therefore E = \frac{\sigma}{\epsilon_0} - \chi E \quad \therefore E = \frac{\sigma}{\epsilon_0 (1 + \chi)}$$

In a particular capacitor $\frac{\sigma}{\epsilon_0}$ is constant. So, effective field is reduced by a factor $\frac{1}{1 + \chi}$ inside a dielectric.

15. Using Gauss' law find out the electric field due to a charge q uniformly distributed over a sphere of radius R (i) at an external point, (ii) at an internal point and (iii) at the surface of the sphere. Show the variation graphically.

[Model Question]

Answer:

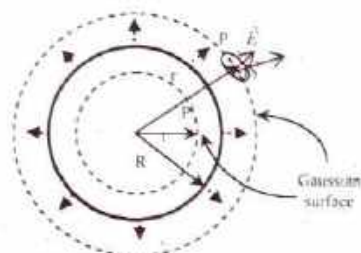
(i) **Outside the sphere:** Let the point P be at a distance r from the center of the sphere so that $r > R$. To find the electric field at P we draw a sphere of radius r concentric with the charged sphere and passing through the point P to represent Gaussian surface.

Total electric flux over the spherical surface of radius r will be $\int \vec{E} \cdot d\vec{S}$

$$\therefore \int \vec{E} \cdot d\vec{S} = E \int dS = E \cdot 4\pi r^2$$

$$\text{As per Gauss theorem, } \int \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \rightarrow E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{or} \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$



The value of E is the same as if the whole charge were concentrated at the center.

Further total charge of the sphere $q = \frac{4}{3}\pi R^3 \rho$ where ρ is volume density of charge.

Hence
$$E = \frac{1}{4\pi\epsilon_0 r^2} \cdot \frac{4}{3}\pi R^3 \rho = \frac{R^3 \rho}{3\epsilon_0 r^2}$$

Vectorially we can write
$$\vec{E} = \frac{R^3 \rho}{3\epsilon_0 r^2} \hat{r}$$

(ii) **On the surface:** For a point just on the surface of the sphere, the whole charge q , as before can be supposed to be concentrated at the center.

Hence Electric Field
$$\vec{E} = \frac{1}{4\pi\epsilon_0 R^2} \hat{r} \quad \text{or,} \quad \vec{E} = \frac{R^3 \rho}{3\epsilon_0 R^2} = \frac{R\rho}{3\epsilon_0}$$

(iii) **Inside the sphere:** Let the point P be inside the sphere at a distance r from the center of the sphere so that $r < R$. We again choose the Gaussian surface as a sphere of radius r with the center O . Let ρ be the volume charge density of the sphere and as before (previous Example).

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^3}$$

and net charge within the Gaussian surface of radius $r = \frac{4}{3}\pi r^3 \rho = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} q = q \frac{r^3}{R^3}$

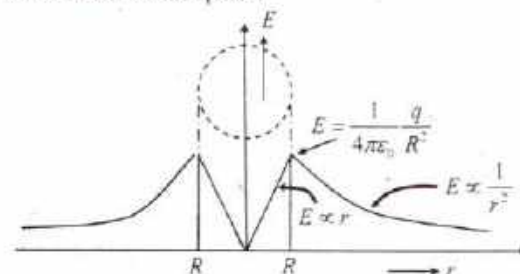
Total electric flux through the Gaussian surface
$$\oint \vec{E} \cdot d\vec{S} = E \int dS = E \cdot 4\pi r^2$$

As per Gauss theorem

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \frac{r^3}{R^3} \rightarrow E \cdot 4\pi r^2 = \frac{q}{\epsilon_0} \frac{r^3}{R^3} \rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3}$$

So vectorially
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} \hat{r} \quad \text{or,} \quad \vec{E} = \frac{\rho r}{3\epsilon_0} \hat{r}$$

The electric field at any point inside a uniformly charged sphere is directly proportional to its distance from the center of the sphere.



Chapter 4: Current Electricity

Chapter at a glance:

4.1 Kirchhoff's Laws for Steady Currents

First rule: It states that at any junction of the circuit, the sum of incoming currents is equal to sum of the outgoing currents. For example, in the figure, O is the junction of five conductors (rather wires). The currents I_1 and I_3 are flowing towards junction O, on the other hand the remaining currents I_2, I_4 and I_5 are leaving the junction O. Hence from the first rule $I_1 + I_3 = I_2 + I_4 + I_5$

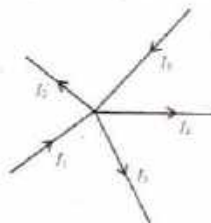


Fig 1: Kirchhoff's first rule

Second rule: This rule states that starting from any point of the circuit if one returns to the same point after travelling along a closed loop, the algebraic sum of the potential differences across each branch of the path must be zero.

Illustration: The figure shows the loop ABCDA of a wire mesh circuit. In the branch AB, there is a cell having e.m.f. E and internal resistance r . The resistances of the branches BC, CD and DA are R_1 , R_2 , and R_3 respectively. Now if the current in the loop is I , then applying the sign convention, the Kirchhoff's second rule gives along ABCDA: $E - rI - R_1I - R_2I - R_3I = 0$ again along ABCBA: $R_3I + R_2I + R_1I + rI - E = 0$

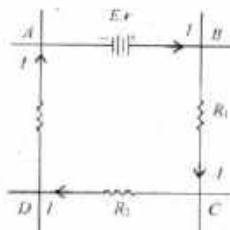


Fig 2: Kirchhoff's second rule

Hence, along both the paths $E = R_1I + R_2I + R_3I + rI$.

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. Introducing the constant of proportionality, the resistance, one arrives at the usual mathematical equation that describes this relationship:

$$I = \frac{V}{R}$$

where I is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms.

4.2 Specific Resistance

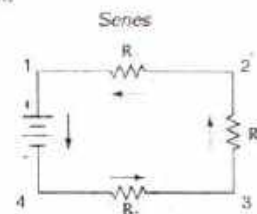
Specific resistance of a material is the resistance of the material having unit length and unit cross-sectional area.

The S.I unit of it is ohm-m.

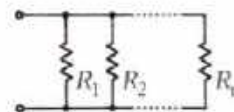
The resistance of a material is dependent on the following factors,

1. The cross-sectional area of the material
2. The length of the material
3. The specific resistance of the material

Series and Parallel Connection:



If $R_1, R_2, \dots, R_n$ resistances are in series combination then the equivalent resistance is $R_{eq} = R_1 + R_2 + \dots + R_n$



If $R_1, R_2, \dots, R_n$ resistances are in parallel combination then the equivalent resistance is

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

Ammeter is a device to measure the current through a circuit.

Voltmeter is a device to measure the voltage difference between two nodes in a circuit.

Voltmeter and ammeter can be converted into each other by connecting suitable resistances in parallel or in series connection.

4.3 Seebeck Effect

The Seebeck effect is a phenomenon in which a temperature difference between two dissimilar electrical substances produces a voltage difference between the two substances.

When heat is applied to one of the two materials, heated electrons flow toward the cooler one. If the pair is connected through an electrical circuit, current flows through the external circuit.

The voltage produced by Seebeck effect is small, usually only a few microvolts per kelvin of temperature difference at the junction. If the temperature difference is large enough, some Seebeck-effect devices can produce a few millivolts (thousandths of a volt). Numerous such devices can be connected in series to increase the output voltage or in parallel to increase the maximum deliverable current. Large arrays of Seebeck-effect devices can provide useful, small-scale electrical power if a large temperature difference is maintained across the junctions.

The Seebeck effect is responsible for the behavior of thermocouples, which are used to approximately measure temperature differences or to actuate electronic switches that can turn large systems on and off. This capability is employed in thermoelectric cooling technology. Commonly used thermocouple metal combinations include constantan/copper, constantan/iron, constantan/chromel and constantan/alumel.

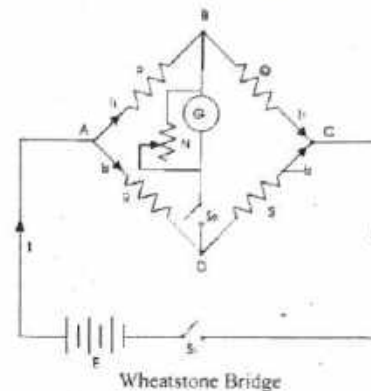
4.4 The Peltier Effect

The Peltier effect is a temperature difference created by applying a voltage between two electrodes connected to a sample of semiconductor material. This phenomenon can be useful when it is necessary to transfer heat from one medium to another on a small scale. In a Peltier-effect device, the electrodes are typically made of a metal with excellent electrical conductivity. The semiconductor material between the electrodes creates two junctions between dissimilar materials, which, in turn, creates a pair of thermocouple voltage is applied to the electrodes to force electrical current through the semiconductor, thermal energy flows in the direction of the charge carriers.

Peltier-effect devices are used for thermoelectric cooling in electronic equipment and computers when more conventional cooling methods are impractical.

Wheatstone Bridge:

For measuring accurately any resistance **Wheatstone Bridge** is widely used. There are two known resistors, one variable resistor and one unknown resistor connected in bridge. By adjusting the variable resistor the current through the Galvanometer is made zero. When the current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance. In this way the value of unknown resistance can easily be measured by using a **Wheatstone Bridge**.



The theory of Wheatstone bridge is used in the Meter Bridge and P.O. Box.

When current flows through a conductor then some amount of heat is generated in the conductor. The amount of heat energy generated can be obtained from Joule's law.

Joule's law: It states the relation between heat generated by an electric current flowing through a conductor.

$$Q = I^2 R \cdot t$$

Where Q is the amount of heat, I is the electric current flowing through a conductor, R is the amount of electric resistance present in the conductor, and t is the amount of time for what the current is flowing through the conductor.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

- The specific resistance of a copper conductor depends on [WBSCTE 2011]
 (a) length (b) cross-sectional area
 (c) mass (d) none of these

Answer: (d)

- The resistances 6Ω and 3Ω are connected in parallel and the combination is connected in series with a 4Ω resistance. A battery of emf $6V$ is connected across the combination. Current passing through the battery is [WBSCTE 2011]
 (a) $1A$ (b) $>1A$ (c) $<1A$ (d) none of these

Answer: (a)

3. Two resistances of 6Ω and 3Ω are connected in parallel and the combination is connected across a 6 volt battery. The total power consumed is - [WBSCTE 2012]
(a) 12 watt (b) 18 watt (c) 2 watt (d) 36 watt

Answer: (b)

4. The resistance of a wire of length 170cm and diameter 0.3mm is 4Ω . The specific resistance of the material of the wire is - [WBSCTE 2013]
(a) $1.66 \times 10^{-4} \Omega m$ (b) $1.66 \times 10^{-3} \Omega m$ (c) $1.66 \times 10^{-5} \Omega m$ (d) none of these

Answer: (d)

5. When current is passed through a junction of two dissimilar metals, heat is evolved or absorbed at the junction. This process is called - [WBSCTE 2014]
(a) Seebeck effect (b) Joule effect
(c) Peltier effect (d) Thomson effect

Answer: (c)

6. The length of a conductor is doubled and its radius is halved, its specific resistance is- [WBSCTE 2015]
(a) unchanged (b) halved (c) doubled (d) quadrupled

Answer: (a)

7. The electric current in a lamp decreases by 5%. The power decreases by - [WBSCTE 2015]
(a) 5% (b) 10% (c) 15% (d) 20%

Answer: (a)

8. Two resistances of 6Ω and 3Ω are connected in parallel and the combination is connected across a 6 V battery. The total power consumed is - [WBSCTE 2016]
(a) 12 watt (b) 18 watt (c) 2 watt (d) 36 watt

Answer: (b)

9. Nichrome wire is used as the heating element because it has - [WBSCTE 2016]
(a) Low specific resistance (b) low melting point
(c) high specific resistance (d) high conductivity

Answer: (a)

10. The ratio of resistances of two bulbs of powers 40W and 60W, designed to operate at 110V and 220V respectively, is - [WBSCTE 2017]
(a) 3:8 (b) 3:4 (c) 3:2 (d) 3:1

Answer: (a)

11. The specific resistance of a conductor depends upon - [WBSCTE 2018]
(a) length of the conductor
(b) area of cross-section of the conductor
(c) temperature of the conductor
(d) none of these

Answer: (c)

12. A bulb rated 60W-240V is connected to a 160V main. The current through the bulb is [WBSCTE 2019]
(a) 1/2 A (b) 1/3 A (c) 1/6 A (d) 1/12 A

Answer: (a)

13. If The length of a conducting wire be made half and the area of cross section be made double, the resistance becomes [WBSCTE 2022]
(a) four times (b) two times
(c) halved (d) one-fourth

Answer: (a)

14. The specific resistance of a copper conductor depends on [WBSCTE 2022]
(a) Length (b) area of cross-section
(c) temperature (d) all of these

Answer: (c)

15. The SI unit of resistivity is [WBSCTE 2022]
(a) ohm.m^{-1} (b) volt.m^{-1}
(c) volt.m (d) ohm.m

Answer: (d)

16. Two resistances of 6Ω and 8Ω are connected in parallel and the combination is connected across a 24 volt battery. The total power consumed is [WBSCTE 2022]
(a) 48 watt (b) 84 watt (c) 168 watt (d) 36 watt

Answer: (d)

B. Fill in the blanks with the alternatives given in the bracket:

17. In an electric lamp P_0 = Power consumed when V_0 = Voltage and P = Power consumed when V = Voltage. $P = \left(\frac{V}{V_0}\right)^2 P_0$ ($P = \left(\frac{V}{V_0}\right)^2 P_0$ or $P = \left(\frac{V_0}{V}\right)^2 P_0$) [WBSCTE 2011]

18. There are two bulbs of 60W and 25W. When they are connected to the mains in series 25W lamp will be brighter (25W lamp will be brighter / 60W lamp will be brighter.) [WBSCTE 2012]

Long Answer Type Questions

1. State and explain Kirchhoff's laws for steady currents.

[WBSCTE 2005, 2006, 2008]

OR,

Write down the expression of the thermo-emf (E) developed in a thermocouple in terms of the temperature difference (θ) between its junctions. Draw E vs θ curve and show the positions of neutral temperature and inversion temperature.

[WBSCTE 2017]

OR,

State Kirchhoff's rules for an electric network.

[WBSCTE 2022]

Answer: Refer to Chapter at a Glance 4.1.

2. A 50 mV voltmeter of resistance 500 Ω is to be converted in to -

- an ammeter reading up to 1 ampere,
- a voltmeter reading up to 5 volt. Circuit diagram for both is essential.

[WBSCTE 2005, 2013]

Answer:

(i) For converting a voltmeter in to an ammeter we have to connect an resistance in parallel combination with the voltmeter.

Let the resistance of the parallel combination is $R\Omega$ the voltage across the voltmeter is 50 mV & resistance of the voltmeter is 500 Ω .

$$\text{So, effective resistance of the combination} = 500 \parallel R = \frac{500 R}{500 + R}$$

Total current through it will be 1 ampere

So, voltage across it,

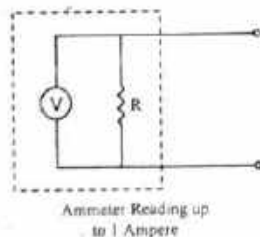
$$\frac{1 \times 500 R}{500 + R} = 50 \times 10^{-3}$$

$$\text{or, } \frac{500 R}{500 + R} = 0.05$$

$$\text{or, } 500 R = 25 + 0.05 R$$

$$\text{or, } 499.95 R = 25$$

$$\text{or, } R = \frac{25}{499.95} \approx 0.05 \Omega$$



(ii) For converting a voltmeter into a voltmeter of higher reading, we have to connect a resistance R in series with it.

Let the resistance is R .

The voltage across the voltmeter is 50 mV and resistance of it is 500 Ω .

$$\text{So, current through it} = \frac{50 \text{ mV}}{500} = 0.1 \text{ mA}$$

After connecting a resistance in series the equivalent resistance is $= (R + 500)\Omega$

So, voltage across it $= (R + 500) \times 0.1 \text{ m} = (R + 500) \times 0.1 \times 10^{-3} = 5$.

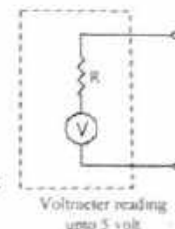
$$\text{or, } R + 500 = \frac{5}{0.1 \times 10^{-3}}$$

$$\text{or, } R + 500 = 50000$$

$$\text{or, } R = 50000 - 500$$

$$\text{or, } R = 49.5 \text{ k}\Omega$$

So, 49.5 $\text{k}\Omega$ is to be connected in series with the voltmeter to convert it to a voltmeter of reading upto 5 volt.



3. (a) Distinguish between peltier effect and Joule effect

[WBSCTE 2005, 2006, 2007, 2013, 2014]

OR,

Write down two important differences between Joule effect and Peltier effect.

[WBSCTE 2017]

(b) The resistance of a coil of copper wire is 10 Ω at 30 $^{\circ}\text{C}$. Determine the resistance of the coil at 100 $^{\circ}\text{C}$. Given the temperature coefficient of resistance of copper $= 4.2 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$

[WBSCTE 2005]

Answer:

(a)

Peltier effect	Joule heating effect
It is a reversible process.	It is an Irreversible process
It takes place only at the junctions	It takes place throughout the conductor
Heat is either evolved or absorbed	Heat is only evolved
Two different materials are required in it	Single material is sufficient for it

(b) Let R_0 is the resistance of the wire at 0 $^{\circ}\text{C}$

then the resistance at 30 $^{\circ}\text{C}$ is $R_{30} = R_0(1 + \alpha \times 30)$

here $R_{30} = 10 \Omega$

the value of α is $4.2 \times 10^{-3} \text{ }^{\circ}\text{C}^{-1}$ (given)

$$\text{so, } R_0 = \frac{R_{30}}{(1 + 4.2 \times 10^{-3} \times 30)} = \frac{10}{(1 + 4.2 \times 10^{-3} \times 30)} = 9.999$$

$$\text{so resistance at } 100^{\circ}\text{C} = R_0(1 + \alpha \times 100) = 9.999(1 + 4.2 \times 10^{-3} \times 100) = 10.000 \text{ ohm}$$

4. (a) State clearly the balanced condition of a Wheatstone Bridge. A circuit diagram with explanation of all the symbols is essential.

[WBSCTE 2006, 2011, 2012, 2013]

OR,

Draw a circuit diagram for a Wheatstone bridge. For a Wheatstone bridge, use Kirchhoff's laws to obtain its balance condition.

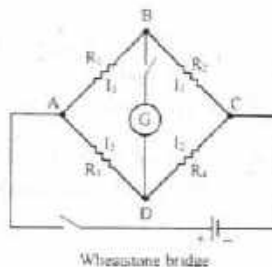
[WBSCTE 2022]

(b) A galvanometer of resistance 50 ohm given full scale deflection when a current of $500\mu\text{A}$ passes through it. How can this be converted to – (i) a voltmeter reading upto 10V (ii) an ammeter reading up to 1 ampere. Circuit diagram for both is essential. [WBSCTE 2011]

Answer:

(a) Wheat stone bridge is an electrical circuit. In wheat-stone bridge four resistances R_1 , R_2 , R_3 and R_4 are connected end to end with each other to form a closed loop. A sensitive galvanometer "G" is connected between their junctions as shown. The circuit is provided with two keys ' K_1 ' and ' K_2 '.

Generally wheat-stone bridge is used to determine unknown resistances.



Balanced Wheat-Stone Bridge

When ' K_1 ' is connected, no current passes through the galvanometer because K_2 is disconnected. When ' K_1 ' and ' K_2 ' are connected and no current flows through the galvanometer, the condition of bridge is called "Balanced condition". It is possible only when the potential difference between the terminals of galvanometer is zero or potential of point 'B' = potential of point 'D' i.e.

$$V_B - V_D = 0$$

$$\text{or, } V_B = V_D$$

(b) i) When a galvanometer is converted into a voltmeter, a high resistance (R) is to be added in series.

Maximum current through the galvanometer = $500\mu\text{A} = 0.0005\text{A}$

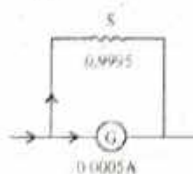
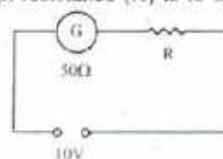
$$\therefore 0.0005(R + 5) = 10$$

$$\text{or, } R + 5 = \frac{10}{0.0005} = 20,000$$

$$\text{or, } R = 19950\Omega$$

ii) To convert galvanometer into an ammeter a low resistance shunt (s) is to be introduced parallel to the galvanometer.

As maximum current through the galvanometer = $500\mu\text{A} = 0.0005\text{A}$



POLY-AP

$$\therefore \text{current through shunt (s)} = 1 - 0.0005 = 0.9995\text{A}$$

Since potential difference across galvanometer and shunt same.

$$\therefore 0.9995S = 0.0005 \times 50$$

$$S = 0.025\Omega$$

$\therefore$ Shunt resistance of 0.025Ω is to be inserted.

5. (a) Define specific resistance of a material. Obtain its SI unit.

[WBSCTE 2007, 2008, 2012]

(b) Write down the relation between resistance of a conductor and temperature.

Define temperature coefficient of resistance. Under what condition the resistance of conductor may be zero? What is it called then?

[WBSCTE 2008, 2014]

Answer:

(a) Refer to Chapter at a Glance 4.2.

(b) The relation between the temperature and the resistance is, $R_t = R_0(1 + \alpha t)$

where R_t is the resistance at $t^\circ\text{C}$ and R_0 is the resistance at 0°C and α is the temperature of resistance.

Temperature coefficient of resistance: It is the fractional increase in resistance per degree centigrade rise in temperature. Unit of it is per degree centigrade.

If the temperature coefficient of resistance is negative quantity then at a specific temperature the resistance of the material will be zero. at that temperature the material will not show any resistance to the current flow then it is called super conductivity.

6. An electric kettle is rated 1 KW, 220V. Using this Kettle, 20 minutes will be required to heat 2 kg of water from 25°C to 100°C . What percentage of heat developed is utilized for heating water. [WBSCTE 2006, 2008]

Answer:

Heat energy required to heat 2kg of water from 25°C to 100°C is $= 2 \times (100-25) \times 4200 = 630\text{KJ}$

Power supplied to the kettle in 20 minutes $= 20 \times 60 \times 1000 = 1200\text{KJ}$

So the percentage of heat developed is utilized for heating water $= \frac{630}{1200} \times 100 = 52.5\%$

7. Explain Seeback effect and Peltier effect.

[WBSCTE 2008, 2011, 2012]

Answer:

See back effect: Refer to Chapter at a Glance 4.3.

The Peltier effect: Refer to Chapter at a Glance 4.4.

POLY-AP

8. (a) A heating element of a boiler rated at 220V gives of 580 KJ of heat in 10 minutes. What is the resistance of the element?

(b) Show that the resistance of a wire of a particular material and given volume will depend inversely as the square of the cross-sectional area of the wire.

[WBSCTE 2012]

Answer:

a) Time = 10 minutes = $10 \times 60 = 600$ seconds.

Amount of heat transfer = 580 KJ = 580000J

So power = $580000J/600 = 967W$

Let the resistance is r .

So, $(220)^2/r = 967$

or, $r = (220)^2/967 = 50 \text{ ohm}$

So the resistance of the element is 50ohm.

b) Assume, the length of wire = l

Radius of wire = r

Volume of wire = $\pi r^2 l$

Area of wire = πr^2

According to, specific resistance

$$\rho = R \times \frac{A}{l} = R \times \frac{\pi r^2}{l} = R \times \frac{\pi r^2 l}{(\pi r^2)^2} = R \times \frac{1}{A} \quad [\text{Volume=Const.}]$$

$$\therefore \rho \propto \frac{1}{A^2} \quad [\because R = \text{constant}] \quad (\text{Proved}).$$

The above answer is confusing

$$\text{Easier and straight approach } R = \frac{\rho l}{A} = \frac{\rho V}{A^2}$$

9. (a) The ratios between the lengths, diameters and resistivities of two wires are each equal to 1:2. If the thinner wire has been the resistance 20Ω , find the resistance of the other wire.

[WBSCTE 2013]

(b) Give two practical applications of thermoelectricity.

[WBSCTE 2013]

Answer:

a) Let the lengths of two wires are l_1 and l_2

Diameters of two wires are d_1 and d_2

Resistivities of two wires are ρ_1 and ρ_2

Resistances of two wires are R_1 and R_2

$$\therefore R_1 = \rho_1 \frac{l_1}{\pi \left(\frac{d_1}{2}\right)^2} = \rho_1 \frac{4l_1}{\pi d_1^2} \quad \dots (i)$$

$$R_2 = \rho_2 \frac{l_2}{\pi \left(\frac{d_2}{2}\right)^2} = \rho_2 \frac{4l_2}{\pi d_2^2} \quad \dots (ii)$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_1 4l_1}{\pi d_1^2} \times \frac{\pi d_2^2}{\rho_2 4l_2} = \left(\frac{\rho_1}{\rho_2}\right) \frac{l_1}{l_2} \left(\frac{d_2}{d_1}\right)^2$$

$$\text{Again, it is known, } \frac{\rho_1}{\rho_2} = \frac{l_1}{l_2} = \frac{d_2}{d_1} = \frac{1}{2}$$

$$\therefore \frac{R_1}{R_2} = \frac{1}{2} \cdot \frac{1}{2} \cdot (2)^2 = 1$$

$$\therefore R_1 = R_2$$

$\therefore$ If the resistance of the thinner wire is 20Ω then the resistance of the another wire is also 20Ω . Ans.

b) **Thermoelectric generator:** Thermo-couples can be used to generate electric power using Seebeck effect, in remote areas. This can be done by heating one junction of a thermo-couple in a flame and keeping the other junction in air at room temperature.

Thermoelectric Refrigerator: It is based on the Peltier effect. If a current is passed through a thermo-couple, heat is absorbed (or removed) at one junction and is evolved at the other junction of the thermo-couple.

10. (a) Define Watt-hour.

[WBSCTE 2014]

(b) An electric bulb is rated 60W at 220V, what is meant by the statement?

Answer:

a) Watt-hour is defined as the energy dissipated in a circuit of power 1 watt in time 1 hour.

b) A 60 Watt 220V electric bulb means that it will consume 60 Watts when the voltage across it is 220V.

In such a case the current flowing through it will be given by $I = P/V = 60/220 = 3/11$ Amperes. And the resistance of the filament will be $P/I^2 = (60 \times 220/60 \times 220/60) = 806.67 \text{ ohms}$.

11. (a) State Ohm's law. Point out the limitations of this law.

[WBSCTE 2014]

(b) Define resistivity of a material. Obtain its SI unit.

[WBSCTE 2014, 2022]

Answer:

a) Refer to Chapter at a Glance 4.1.

b) **Electrical resistivity** (also known as **resistivity**, **specific electrical resistance**, or **volume resistivity**) is an intrinsic property that quantifies how strongly a given material opposes the flow of electric current. A low resistivity indicates a material that

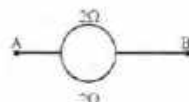
readily allows the movement of electric charge. Resistivity is commonly represented by the Greek letter ρ (rho). The SI unit of electrical resistivity is the ohm-metre ($\Omega\text{-m}$) although other units like ohm-centimetre ($\Omega\text{-cm}$) are also in use. As an example, if a $1\text{ m} \times 1\text{ m} \times 1\text{ m}$ solid cube of material has sheet contacts on two opposite faces, and the resistance between these contacts is $1\ \Omega$, then the resistivity of the material is $1\ \Omega\text{-m}$.

12. (a) What is super conductivity? [WBSCTE 2015]
 (b) A wire of resistance $4\ \Omega$ is bent in the form of a circle. What is the effective resistance between the ends of the diameter?
 (c) Explain how you can convert a galvanometer into an Ammeter.

Answer:

a) If the temperature coefficient of resistance is negative quantity then at a specific temperature the resistance of the material will be zero, at that temperature the material will not show any resistance to the current flow then it is called super conductivity.

b)



c. The effective resistance between the ends of the diameter is $= \frac{2 \times 2}{4} = 1\ \Omega$

c) To convert galvanometer into an ammeter a low resistance shunt (s) is to be introduced parallel to the galvanometer.

13. (a) State Joule's law in current electricity.
 (b) Show graphically how the thermo-e.m.f. changes with temperature at the hot junction keeping the other junction at 0°C . Define neutral temperature and temperature of inversion. [WBSCTE 2015]

Answer:

a) According to Joule's law, if a current of I ampere flows through a conductor of resistance R ohm for t second, then the amount of work required (W.D.) to maintain the current flow will be given by

W.D. = $I^2 R t$ joule = $V I t$ joule as $R = V/I = P t$ joule as power $P = V \times I$

$$= \frac{V^2 t}{R} \text{ joule, as } I = \frac{V}{R}$$

Heat equivalent of this work may be expressed as $H = \frac{W.D.}{J}$

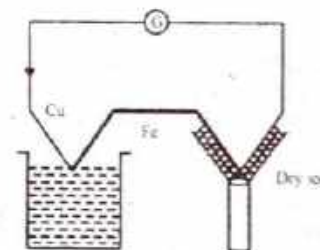
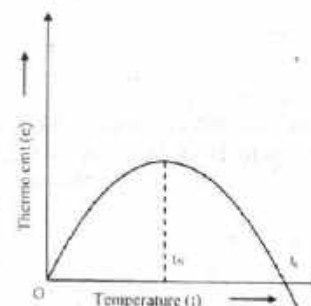
where J is mechanical equivalent of heat whose value is $4186 \text{ joule/kcal} \approx 4200 \text{ joule/kcal}$.

$$\text{Hence, } H = \frac{I^2 R t}{4200} \text{ kcal}$$

b) To study the variation of thermo emf with temperature let us take Cu - Fe thermo couple. One junction is kept in dry ice (0°C) and the other in heated sand bath. When the temperature of sand is gradually increased continuously, the thermo-emf changes. A typical curve showing the variation of thermo emf with temperature (t) is shown in Figure below. The curve shows that the thermo emf increases with increase of temperature of hot junction, attains a maximum value at a temperature (t_n) say, after which it begins to decrease and becomes zero at a temperature (t_i). When the temperature is further increased. The thermo-emf changes direction.

The thermo-emf of a number of thermo-couples is found to obey the relation

$$e = \alpha t + \frac{1}{2} \beta t^2 \text{ when cold junction is at } 0^\circ\text{C}.$$



Neutral Temperature and Inversion Temperature

Neutral temperature: The neutral temperature is the temperature of hot junction at which the thermo emf is maximum. It is denoted by t_n .

Inversion temperature: The inversion temperature is the temperature of hot junction at which thermo emf becomes zero and reverse the direction. It is denoted by t_i .

If t_0 is temperature of cold junction, then inversion temperature $t_i = 2t_n - t_0$

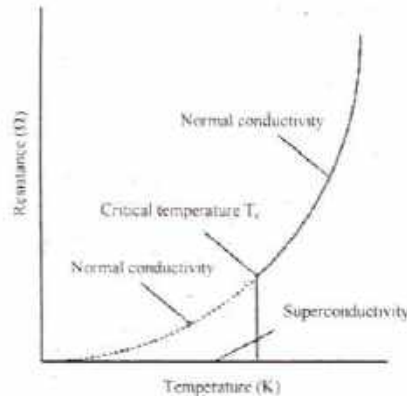
if $t_0 = 0$, then $t_i = 2t_n$

14. (a) What is superconductivity?
 (b) Draw the graph between the resistivity and the temperature for a superconductor. [WBSCTE 2016]

Answer:

a) Refer to Question No. 12(a) of Long Answer Type Questions.

b)



15. (a) What is non-ohmic conductor? Give an example of it.
 (b) The resistance of a 20 mV voltmeter is 1000 ohm. How can you convert this voltmeter into an ammeter which can measure up to 2.0 A. Draw the necessary circuit diagram. [WBSCTE 2017]

Answer:

(a) Non-Ohmic conductors do not follow Ohm's law and have their own characteristics. There are a number of examples of non-Ohmic conductors; including bulb filaments and semiconductors like diodes and transistors. Let's take the diode. A diode provides a near constant voltage drop even if you vary the current, so it does not follow Ohm's law. The opposite happens in a light bulb filament; even as you increase the voltage significantly, it only allows a certain amount of current to pass through.

(b) Since the internal resistance of a voltmeter is much greater than ammeter, for conversion to ammeter we need to decrease the voltmeter's internal resistance by adding appropriate shunt i.e. resistance in parallel to the meter. To calculate the shunt resistance, we need to know the resistance of the voltmeter. That is given 1000 ohm as resistance and

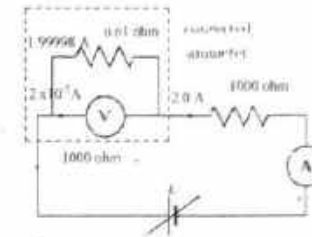
$V_0 = 20\text{mV}$

$$I_m = \frac{V_0}{R_m} = \frac{20 \times 10^{-3}}{1000} = 2 \times 10^{-5} \text{ A}$$

$$I_0 = 2.0 \text{ A}$$

$$\therefore I_{sh} = I_0 - I_m = 2.0 - (2.0 \times 10^{-5}) = 1.99998 \text{ A}$$

$$R_{sh} = \frac{V_0}{I_{sh}} = \frac{20 \times 10^{-3}}{1.99998} = 0.01 \text{ ohm}$$



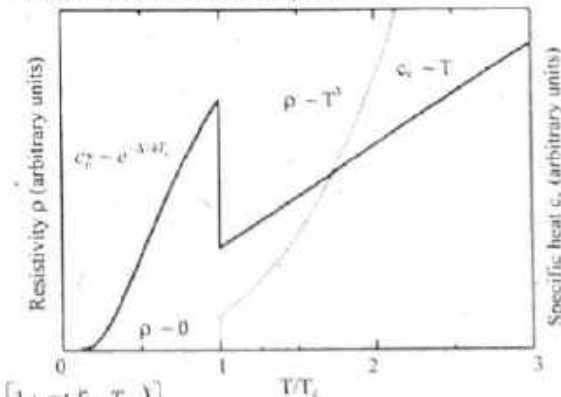
16. (a) Define temperature coefficient of resistance of a substance. What do you mean by the term 'critical temperature' in respect to the phenomenon of superconductivity?
 (b) The resistance of a coil at 30 °C is 15 ohm. If the resistance of the coil at 100°C be 18 ohm, calculate the value of the temperature coefficient of resistance of the coil. [WBSCTE 2018]

Answer:

a) 1st part: Refer to Question No. 5.b) (2nd Part) of Long Answer Type Questions.

2nd part:

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic flux fields occurring in certain materials, called **superconductors**, when cooled below a characteristic **critical temperature**.



$$b) R = R_{ref} [1 + \alpha (T - T_{ref})]$$

R = conductor resistance at temperature T^* (100°C) = 18Ω

R_{ref} = conductor resistance at reference temperature (30°C) = 15Ω

α = Temperature coefficient of resistance of the coil

$$\therefore 18 = 15[1 + \alpha(100 - 30)] = 15[1 + 70\alpha]$$

$$\Rightarrow \frac{18}{15} - 1 = 70\alpha$$

$$\Rightarrow \frac{3}{15} = 70\alpha$$

$$\Rightarrow \alpha = \frac{3}{15 \times 70} = 0.0028/^{\circ}\text{C}$$

17. (a) What is Seebeck effect? Write down one importance of Seebeck series.
 (b) An electric bulb is rated 60 W at 220 V. Calculate the resistance of the bulb. If the bulb be connected across a 200 V supply calculate the current through the bulb and the electric energy consumed by the bulb in one hour. [WBSCTE 2018]

Answer:

a) Refer to Article No. 4.3 of Chapter at a Glance.

b) $P = 60 \text{ W}$, $V = 220 \text{ V}$

$$P = \frac{V^2}{R}$$

$$\Rightarrow R = \frac{V^2}{P} = \frac{(220)^2}{60} = 806.67 \Omega$$

When the bulb is connected across 200V, the resistance remains the same.

$$\therefore P = \frac{V^2}{R} = \frac{(200)^2}{806.67} = 49.59 \text{ W}$$

$$\therefore \text{Energy consumed/hour} = 49.59 \text{ W}$$

$$\therefore \text{Current, } I = \frac{P}{V} = \frac{49.59}{200} = 0.25 \text{ A}$$

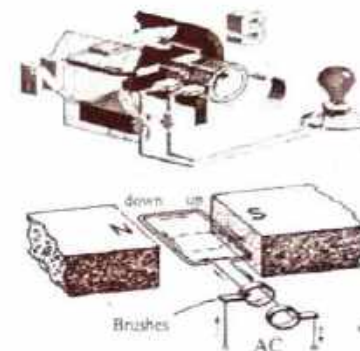
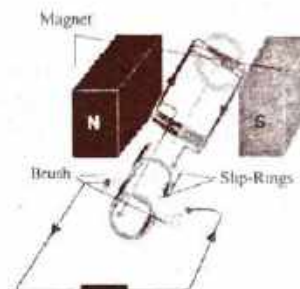
18. Briefly state the principle of generation of alternating current and name the basic components of an AC generators. [WBSCTE 2018]

Principle:

A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as D.C. generators.

Alternating voltage may be generated by rotating a coil in the magnetic field or by rotating a magnetic field within a stationary coil. The value of the voltage generated depends on-

- the number of turns in the coil.
- strength of the field.
- the speed at which the coil or magnetic field rotates.



19. Calculate the resistance of a wire of 1.5 m long and area of cross-section 0.4 mm^2 whose resistivity is $46 \times 10^{-8} \text{ ohm metre}$. [WBSCTE 2019]

Answer:

Length of the wire (ℓ) = 1.5 m

Area of the cross-section (a) = 0.4 mm^2

Resistivity (ρ) = $46 \times 10^{-8} \text{ ohm metre}$

$$\text{Resistance } (R) = \rho \frac{\ell}{a} = 46 \times 10^{-8} \times \frac{1.5}{0.4} = 1.725 \times 10^{-6} \text{ ohm}$$

20. Write the difference between Joule Effect and Peltier Effect. [WBSCTE 2019]

Answer: Refer to Questions No. 3(a) of Long Answer Type Questions.

21. (a) Which one is fundamental-resistance or resistivity? Explain.

[WBSCTE 2019]

Answer: Refer to Questions No. 11(b) of Long Answer Type Questions.

21. (b) A source of emf 9 volt and internal resistance 1 ohm is connected to an external resistance 8 ohm. Find - (i) current drawn from the source, (ii) lost voltage, (iii) potential difference between terminals of the source?

[WBSCTE 2019]

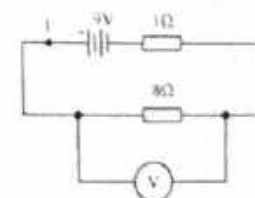
Answer:

$$I = \frac{V}{R}$$

$$R = 8 + 1 = 9 \Omega$$

$$(i) I = \frac{9}{9} = 1 \text{ A}$$

$$(ii) V' = IR = 1 \times 8 = 8 \text{ V}$$



(iii) So 9V emf cell actually supplies 8V to the external circuit.

22. A wire of 15Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3V battery. Find the current drawn from the battery. [WBSCTE 2022]

Answer:

When a resistor is stretched to double in original length the new resistance becomes 4 times its original resistance become $R \propto L^2$

$$\therefore \text{New resistance} = 4R = 4 \times 15 = 60\Omega$$

Resistance of each part = 30Ω

i.e. $R_1 = 30\Omega$ & $R_2 = 30\Omega$

When they are connected in parallel, the effective resistance,

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{30 \times 30}{30 + 30} = \frac{1}{15} \quad \left[\because \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \right]$$

$$\therefore R_p = 15\Omega$$

$$\text{Current, } I = \frac{V}{R} = \frac{3.0}{15} = 0.2 \text{ Amp}$$

23. What is Carey Foster Bridge? Write down the applications and advantages of Carey foster Bridge. [Model Question]

Answer:

The bridge circuit that can calculate medium resistances or can compare and measure the two large/equal resistance values with small variations is known as Carey foster bridge. It is the modified form of Wheatstone's bridge circuit. It is also referred to as the method of small resistances.

Carey Foster Bridge Principle:

The Carey foster bridge principle is simple and similar to Wheatstone's bridge working principle. It works on the principle of null detection. That means the ratios of the resistances will be equal and the galvanometer records zero where there is no current flow.

The bridge circuit is balanced when there is no current flow through the galvanometer. At an unbalanced condition, the current flows through the galvanometer and the reading is recorded by observing the deflection.

The advantages of Carey foster bridge are:

- The complexity of the bridge circuit is reduced because there is no need for additional equipment except the slide wire and the resistances.
- It can be utilized as the meter bridge where the slide wire length can be increased by connecting resistances in series. Hence the accuracy of the bridge circuit is increased.
- Construction is simple and easy to design.
- The components used in the circuit are not complex.

Applications of Carey Foster Bridge

The applications of Carey foster bridge are:

- It is used to calculate the values of medium resistances.
- It is used to compare the approximate values of equal resistances.
- It is used to measure the value of the specific resistance of the slide wire, and also used in light detector circuits.
- It is used to measure the intensity of light, pressure, or strain. Since it is a modified form of Wheatstone's bridge.

24. Write down the definition and its units: (i) Electric current, (ii) Resistance, (iii) Specific conductance. [Model Question]

Answer:

(i) Electric current:

Electric current is the rate of flow of electrons in a conductor.

The SI unit of electric current is the Ampere.

(ii) Resistance:

Resistance is known that the directed movement of electrons constitutes a current flow.

The SI unit of Resistance is Ohm.

(iii) Specific conductance:

Specific conductance is the ability of a substance to conduct electricity. It is the reciprocal of the specific resistance. Conductance means the flow of current through the liquid conductor.

Unit of specific conductance = Seimen cm^{-1}

25. Write down the properties of electric current. [Model Question]

Answer:

Electric current is a significant quantity in electronic circuits. The different properties of the electric current are:

- The electric current is the result of the flow of electrons. A wire is said to convey a current of one ampere when the charge moves through it at the rate of one coulomb each second.
- At the point when an electrical potential distinction is applied across the metallic wire, the approximately appended free electrons begin moving towards the positive terminal of the cell appearing in the figure beneath. This consistent progression of electrons comprises the electrical flow. The progression of flows in the wire is from the negative terminal of the cell to the positive terminal through the outside circuit.
- As per the electron hypothesis, when the potential contrast is applied across the transmitter some matter moves through the circuit which establishes the electric flow.

Chapter 5: Electromagnetism

Chapter at a glance:

5.1 Magnetic Flux

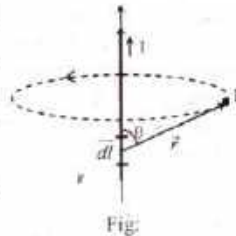
The **magnetic flux** (often denoted Φ or Φ_B) through a surface is the component of the $\vec{B}$ field passing through that surface. The SI unit of magnetic flux is the weber (Wb).

5.2 Magnetic Flux Density

Magnetic flux density at any point in a magnetic field is the magnetic flux linked with an imaginary unit area drawn about that point perpendicular to the field.

5.3 Biot-Savart's Law

- The magnitude dB of the magnetic field $\vec{dB}$ is proportional to the current i and to the length dl of the element $\vec{dl}$.
- dB is inversely proportional to the square of the distance r of the point P from $\vec{dl}$.
- The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ i.e., along $(\vec{dl} \times \vec{r})$.



Thus in vector notation, $\vec{dB} \propto i \frac{\vec{dl} \times \vec{r}}{r^3}$

5.3.1 Magnetic Field for Infinitely Long Straight Current Conductor

$$H = I / 2\pi \text{ A/m}^2$$

Where I is the current through the wire.

5.3.2 Magnetic Field on the Axis of The Circular Coil

$$H_a = \frac{NI}{2r} \text{ A/m}^2$$

Where I is the current through the wire, N is the number of turn, and r is radius of the coil.

5.3.3 Magnetic Field Infinitely Long Current Solenoid

$$H \approx \frac{NI}{2l} \times 2 = \frac{NI}{l} \text{ A/m}^2$$

Where I is the current through the wire, N is the number of turn, and l is the length of the solenoid.

5.3.4 Force on a Current Carrying Conductor Placed in a Magnetic Field

$$\vec{F} = I \int \vec{dl} \times \vec{B}$$

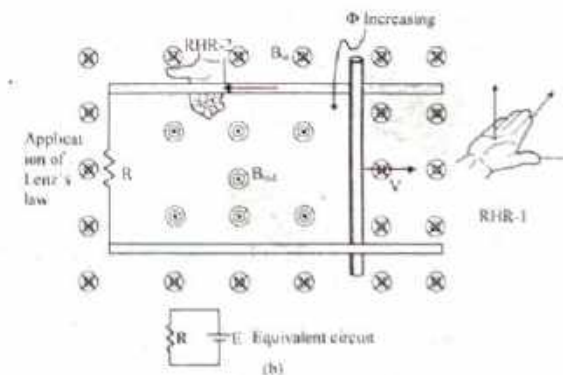
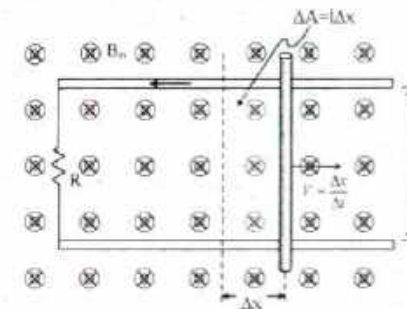
Where $\vec{F}$ is the force, I is the current through the conductor, $\vec{B}$ is the magnetic field and l is the length of the conductor.

5.4 Application of Magnetic effect of electric current – Galvanometer

A galvanometer is a type of sensitive ammeter. It is an instrument for detecting electric current. Here a pointer deflects in response to electric current flowing through its coil in a magnetic field. By noticing the deflection, we can measure the current.

5.4.1 Motional e.m.f

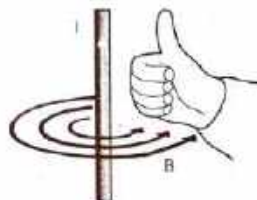
It is the e.m.f produced in a conductor due to the motion of a conductor.



A motional emf = BLv is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them.

5.4.2 Fleming's Right Hand Rule

An electric current passes through a straight wire. Here, the thumb points in the direction of the conventional current (from positive to negative), and the fingers point in the direction of the magnetic lines of flux.



5.5 Law's of Electromagnetic Induction

1st Law: Whenever there is a change in the magnetic flux linked with loop (or coil), an instantaneous e.m.f is induced in it.

2nd Law: The magnitude of the e.m.f induced in the loop (or coil) is proportional to the time rate of change of the magnetic flux linked with it.

3rd Law: The induced current is always in such a direction that it opposes the action that produced it.

5.6 Lenz's Law

The induced current is always in such a direction that it opposes the action that produced it.

5.7 Electrical Generator

An Electrical Generator is a device that produces an Electromotive Force (e.m.f.) by changing the number of Magnetic Flux Lines (Lines of Force), Φ , passing through a Wire Coil. Figure is one type of Generators. When the Coil is rotated between the Poles of the Magnet by cranking the handle, an AC Voltage Waveform is produced.



Fig.: Electrical Generator

Operation principle of a Generator is based on Electromagnetic Induction, which is defined by Faraday's Law, which states: $E_{\text{emf}} = -N \frac{d\phi}{dt}$

5.8 Self Induction

Self induction is that phenomenon in which a change in electric current in a coil produces an induced emf in the coil itself.

Self induced emf in a coil is directly proportional to the rate of change of electric current in the coil.

$$\text{emf} \propto -\frac{\Delta I}{\Delta t} \quad \text{or, } \text{emf} = -L \frac{\Delta I}{\Delta t}$$

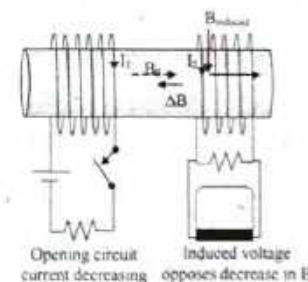
Where, L = self inductance of the coil.

It is denoted by 'L' and it depends upon the physical characteristics of the coil.

Unit of self inductance is Henry.

5.9 Mutual Induction

When an emf is introduced in a coil because of the change in the current in a coupled coil, the effect is called mutual inductance. The emf is described by Faraday's law and it's direction is always opposed the change in the magnetic field produced in it by the coupled coil. The induced emf in coil 1 is due to self inductance L .



The induced emf in coil 2 caused by the change in current I_1 can be expressed as

$$\text{Emf}_2 = -N_2 A \frac{\Delta B}{\Delta t} = -M \frac{\Delta I_1}{\Delta t}$$

The mutual inductance M can be defined as the proportionality between the emf generated in coil 2 to the change in current in coil 1 which produced it.

In S.I it's unit is Henry.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. If a conductor moves a magnetic field, the direction of induced E.M.F. is obtained from – [WBSCTE 2006, 2007, 2012, 2022]
 (a) Laplace's law (b) Fleming's left hand rule
 (c) Fleming's right hand rule (d) none of these

Answer: (b)

2. The induced e.m.f. in a conductor is – [WBSCTE 2013]
 (a) directly proportional to the rate of change of flux
 (b) inversely proportional to the rate of change of flux
 (c) directly proportional to the total flux associated with the conductor
 (d) none of these

Answer: (a)

3. When the current in a solenoid changes from 8A to 2A in 3×10^{-2} s, the emf induced in the coil is 2V. The self inductance of the coil in millinery is [WBSCTE 2014]
 (a) 1 (b) 5 (c) 20 (d) 10

Answer: (d)

4. Lenz's Law is based on which conservation law? [WBSCTE 2015]
 (a) Charge (b) mass (c) momentum (d) energy

Answer: (d)

5. The magnetic field at any point due to an infinitely long straight current carrying wire is directly proportional to – [WBSCTE 2015]
 (a) distance of the point from the wire
 (b) diameter of the wire
 (c) resistance of the wire
 (d) the magnitude of the current in the wire

Answer: (d)

6. Lenz's law is a consequence of – [WBSCTE 2016]
 (a) Conservation of linear momentum (b) Conservation of angular momentum
 (c) Conservation of energy (d) Conservation of mass

Answer: (c)

Electromagnetism

7. A conductor moves in a magnetic field. The direction of the induced e.m.f. is obtained from – [WBSCTE 2016]
 (a) Laplace's law (b) Fleming's left hand rule
 (c) Fleming's right hand rule (d) None of these

Answer: (b)

8. The S.I unit of magnetic flux density is – [WBSCTE 2016, 2017]
 (a) Am^{-1} (b) Oersted (c) Gauss (d) Tesla

Answer: (d)

9. Magnetic field due to a long straight current carrying conductor is – [WBSCTE 2015]
 (a) $\left(\frac{\mu_0}{4\pi}\right) \frac{2I}{r}$ (b) $\left(\frac{\mu_0}{4\pi}\right) \frac{I}{r}$ (c) $\left(\frac{\mu_0}{2\pi}\right) \frac{2I}{r}$ (d) None of these

Answer: (a)

10. A straight conductor of length 50 cm is placed in a uniform magnetic field of 1.5 T with its length parallel to the direction of the field. If the current through the conductor be 1.0A, the force on the conductor is – [WBSCTE 2017]
 (a) 0.75N (b) 7.5N (c) 75.0N (d) zero.

Answer: (a)

11. An air-filled very long solenoid, of length L and total number of turns N is carrying a current I. The magnetic field well inside the solenoid is (μ_0 = permeability of air) [WBSCTE 2018]
 (a) $B = \mu_0 N I$ (b) $B = \mu_0 N I L$
 (c) $B = (\mu_0 N I)/L$ (d) $B = (\mu_0 L I)/N$

Answer: (c)

12. The SI unit of self- inductance is Henry which can also be expressed as – [WBSCTE 2018]
 (a) volt s amp (b) volt s amp^{-1}
 (c) volt s^{-1} amp (d) volt s amp^{-2}

Answer: (b)

13. At inversion temperature thermoelectric EMF will be [WBSCTE 2019]
 (a) maximum (b) infinite (c) zero (d) 1

Answer: (b)

14. Lenz's law is based on which conservation law? [WBSCTE 2019]
 (a) Charge (b) Mass (c) Momentum (d) Energy

Answer: (a)

15. The self-inductance of a straight wire is
(a) zero (b) infinity (c) very high (d) very low
[WBSCTE 2019]
Answer: (a)

16. A moving charge produces
(a) an electric field only
(b) a magnetic field only
(c) both electric and magnetic field
(d) either electric field or magnetic field
[WBSCTE 2022]
Answer: (c)

17. The SI unit magnetic flux is
(a) weber (b) gauss (c) oersted (d) none of these
[WBSCTE 2022]
Answer: (a)

18. The induced emf in a 1 millihenry inductor in which the current changes from 5A to 3A in 10^{-3} s is
(a) 2×10^{-6} V (b) 8×10^{-6} V (c) 2V (d) 8V
[WBSCTE 2022]
Answer: (c)

19. A source of emf 2 volt sends 0.2 amp current to an external circuit for 5s. The energy used by the source is
(a) 0.2 joule (b) 0.4 joule (c) 4.0 joule (d) 2.0 joule
[WBSCTE 2022]
Answer: (c)

20. The integral form of Biot-Savart's Law is ----
(a) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$ (b) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{r^3}$
(c) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{r^2}$ (d) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{r}$
[Model Question]
Answer: (b)

21. The divergence of a magnetic flux density is ---
(a) 0 (b) 1 (c) -1 (d) none of these
[Model Question]
Answer: (a)

22. The force on a charge q which is at rest in a magnetic field $\vec{B}$ is equal to-
(a) $\vec{F} = q\vec{v} \times \vec{B}$ (b) $\vec{F} = \frac{\vec{B} \times \vec{v}}{q}$ (c) $\vec{F} = 0$ (d) $\vec{F} = 1$
[Model Question]
Answer: (b)

B. Fill in the blanks in the following statements:

23. SI unit of magnetic flux is tesla. ($Am^{-1}/Tesla$) [WBSCTE 2011]
24. The unit of specific resistance in S.I. is ohm-m [Model Question]

Long Answer Type Questions

1. State Lenz's law [WBSCTE 2005, 2006, 2007]
Answer: Refer to Chapter at a Glance 5.6.
2. (a) State and explain Biot – Savart law for the determination of the magnetic field due to a current element. [WBSCTE 2009, 2013, 2014]
(b) Explain – (i) magnetic flux, (ii) magnetic flux density. State the respective units in SI. [WBSCTE 2009, 2012, 2014]
(c) State Faraday's laws of electromagnetic induction. What is Lenz's law? [WBSCTE 2009, 2012]

Answer:
(a) Refer to Chapter at a Glance 5.3.
(b) Refer to Chapter at a Glance 5.2.
(c) Refer to Chapter at a Glance 5.5 and 5.6.

3. (a) Define self-inductance.
(b) Write down the expression for magnetic field at the centre of a circular coil current conductor. [WBSCTE 2013]

Answer:
(a) Self – inductance is the inherent inductance of a circuit, given by the ratio of the electromotive force produced in the circuit to the rate of change of current producing it. It is usually expressed in henries. Symbol L also called coefficient of self-induction. Consider a single conducting circuit around which a current I is flowing. This current generates a magnetic field B which gives rise to a magnetic flux ϕ linking the circuit. We expect the flux ϕ to be directly proportional to the current I , given the linear nature of the laws of magneto-statics, and the definition of magnetic flux. Thus, we can write $\phi = LI$, where the constant of proportionality L is called the *self inductance* of the circuit. It is measured in units of henries.

- (b) The expression for the magnetic field at the centre of a circular coil current conductor is,

$$B = \frac{\mu_0 n I}{2r}$$

Where μ_0 is the absolute permeability of free space.

n is the number of turns in the coil.

I is the amount of current carrying by the conductor.

r is the radius of conductor coil.

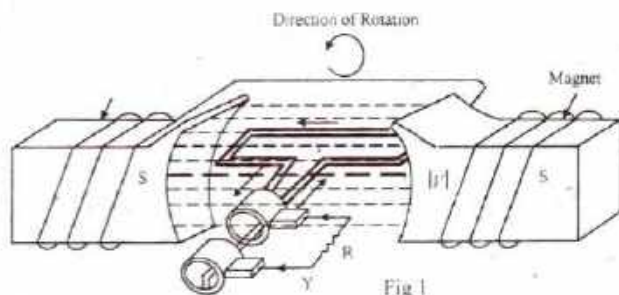
4. (a) What are self & mutual inductances? Write their SI units.

(b) State and explain the principle of generation of AC. What are rms and average value of an alternating emf? [WBSCTE 2014]

Answer:

(a) Refer to Chapter at a Glance 5.8 and 5.9.

(b) The elementary AC generator (Figure 1) consists of a conductor or loop of wire in a magnetic field that is produced by an electromagnet. The two ends of the loop are connected to slip rings, and they are in contact with two brushes. When the loop rotates it cuts magnetic lines of force, first in one direction and then the other.



Development of a Sine-Wave Output

Step 1:

At the instant the loop is in the vertical position (Figure 2, 0°), the coil sides are moving parallel to the field and do not cut magnetic lines of force. In this instant, there is no voltage induced in the loop.

Step 2:

As the coil rotates in a counter-clockwise direction, the coil sides will cut the magnetic lines of force in opposite direction. The direction of the induced voltages depends on the direction of movement of the coil. The potential drop across resistor R will cause a current to flow through it.

Step 3:

The voltage and hence current continues to increase until it reaches a maximum value when the coil is perpendicular to the magnetic lines of force (Figure 2, 90°) and is cutting the greatest number of magnetic lines of force.

Step 4:

As the coil continues to turn, the voltage and current induced decrease until they reach zero, where the coil is again in the horizontal position (Figure 2, 180°).

Step 5:

In the other half revolution, an equal voltage is produced except that the polarity is reversed (Figure 2, 270° , 360°). The current flow through R is in the opposite direction.

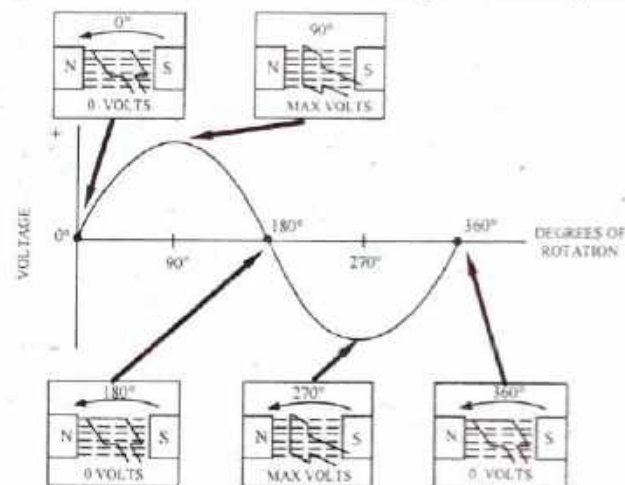


Fig: 2 Developing a Sine-Wave Voltage

The periodic reversal of polarity results in the generation of a voltage, as shown in Figure 2. The rotation of the coil through 360° results in an AC sine wave output.

Average Value

$$\begin{aligned} V_{avg} &= \frac{1}{\pi} \int_0^\pi V_m \sin \omega t d(\omega t) = \frac{1}{\pi} (-V_m \cos \omega t)_0^\pi \\ &= \frac{-V_m}{\pi} (0 - 1 - 1) = \frac{2}{\pi} V_m = 0.637 V_m \end{aligned}$$

$$\text{Similarly, } I_{avg} = \frac{1}{\pi} \int_0^\pi I_m \sin \omega t d(\omega t) = 0.637 I_m$$

RMS Value

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \omega t d(\omega t)} = V_m \sqrt{\frac{1}{4\pi} \int_0^{2\pi} (1 - \cos 2\omega t) d(\omega t)} \\ &= V_m \sqrt{\frac{1}{4\pi} \int_0^{2\pi} d(\omega t) - \frac{1}{4\pi} \int_0^{2\pi} \cos 2\omega t d(\omega t)} \end{aligned}$$

$$= V_m \sqrt{\frac{1}{2} - \frac{1}{8\pi} [\sin 2\omega t]_0^{2\pi}}$$

$$= V_m / \sqrt{2} = 0.707 V_m$$

Similarly,

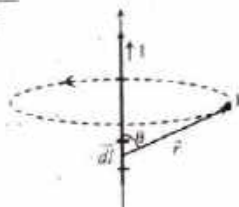
$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2 \omega t d(\omega t)} = 0.707 I_m$$

5. (a) What is the magnitude of force on a current carrying conductor placed in a magnetic field?
 (b) State Fleming's left hand rule
 (c) The magnetic flux linked with a coil is varying according to the relation $\phi = 4 \cos(100\pi t)$, where ϕ & t measured in Weber and second respectively. Find the expression for induced alternating voltage in the coil. Also calculate the frequency of the alternating voltage. [WBSCTE 2015]

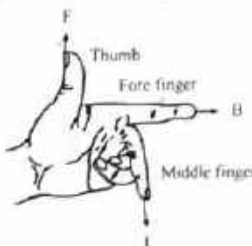
Answer:

a) The magnetic field of induction dB at any point P , at a distance r from the elementary length of current carrying wire is, $dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}$

In vector notation, $\vec{dB} \propto \vec{dl} \times \vec{r}$



b) Fleming's Left Hand Rule may be stated as -
 If the forefinger, the middle finger and the thumb of the left hand are stretched mutually at right angles to one another such that the forefinger points in the direction of the magnetic field ($\vec{B}$) and the middle finger in the direction of the current (I) then the thumb will point in the direction of the force (F) on the conductor.



c) Induced Alternating voltage

$$e = \frac{d\phi}{dt} = 400\pi \sin(100\pi t)$$

$$\therefore \omega = 100\pi$$

$$\text{or, } 2\pi f = 100\pi$$

$$\therefore \text{Frequency (f)} = \frac{100}{2} = 50 \text{ Hz}$$

6. (a) State Faraday's law of electromagnetic induction. [WBSCTE 2015]
 (b) Why is 220 volt A.C more dangerous than 220 D.C.?
 (c) A glass rod of length l moves with velocity V perpendicular to a uniform magnetic field B . What is the induced e.m.f. in the rod?

Answer:

a) 1st Law: Whenever there is a change in the magnetic flux linked with loop (or coil), an instantaneous e.m.f is induced in it.

2nd Law: The magnitude of the e.m.f induced in the loop (or coil) is proportional to the time rate of change of the magnetic flux linked with it.

b) The voltage of domestic alternating current supply is 220 volt. As alternating current meters read root mean square values; that is root mean square value of domestic alternating current supply is 220 volt. Therefore peak value of voltage,

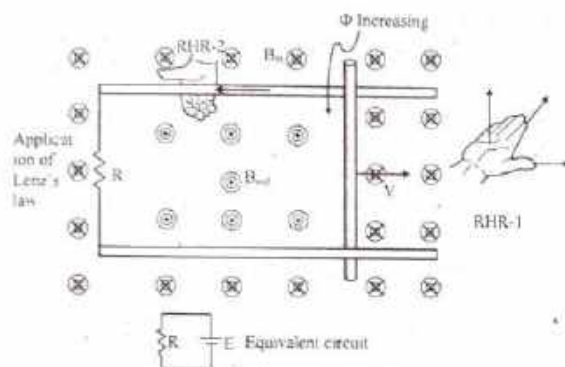
$$V_0 = \sqrt{2} V_{rms} = 1.414 \times 220 \approx 311 \text{ volt}$$

As alternating voltage is $V = V_0 \sin \omega t$ and $\sin \omega t$ varies with time having values between +1 and -1; therefore the voltage of domestic a.c. supply varies between +311 V to -311 V; while the voltage of 220 volt direct current (dc) supply remains 220 volt. That is why a 220 V a.c. is more dangerous than a 220 V d.c.

c) E.M.F is produced at the ends of a moving conductor in a magnetic field. The potential difference between the ends of a conductor of length l moving in a uniform magnetic field B perpendicular to the plane of paper inwards with a velocity v is given by- $V = vBL$

If v is in m/sec and B is in weber/m² (or newton/ampere-metre) and L is in metre then e.m.f will be in volts.

If θ is the angle between the velocity v of the conductor and the magnetic field B then e.m.f. induced between its ends will be, $V = vBL \sin \theta$.



7. (a) We do not experience any electrostatic field near a current carrying conductor inspite of flow of huge number of electrons through it. Explain in brief.

(b) What is the work done by the magnetic field on moving charge?

[WBSCTE 2016]

Answer:

a) If we consider a current carrying conductor, every instant an electron enters the conductor another electron will be leaving the conductor. This current carrying conductor will not be charged. It would not have any net positive and negative charge. We know that, a dipole has zero net charge but it does have electric field around it. So, if net charge is zero, then it doesn't mean there is no electric field. So that we do not experience any electrostatic field near a current carrying conductor.

b) If we place a moving charge in the magnetic field then work done will be

$$dw = \vec{F} \cdot d\vec{s} = F ds \cos \theta$$

8. (a) State Biot-Savart's law for the magnetic field due to a small current element.

(b) Write down the expression for the magnetic field developed inside a long solenoid carrying a current.

(c) Which rule gives the direction of the force on a current carrying conductor when placed in a magnetic field? State it.

[WBSCTE 2017]

Answer:

(a) Refer to Chapter at a Glance 5.3.

(b) Let, a long solenoid of n turns per unit length carries a current I . So, the magnetic field generated inside the solenoid is uniform and is expressed as:

$$B_{sol} = \mu_0 n I$$

(c) Fleming's left-hand rule gives the direction of the force on a current carrying conductor when placed in a magnetic field.

2nd part: Refer to Long Answer Type Question No. 5(b).

9. (a) State the laws of electromagnetic induction.

(b) Define coefficient of mutual inductance of a pair of coils. Write down its SI unit.

[WBSCTE 2017]

Answer:

(a) Refer to Chapter at a Glance 5.5.

(b) The coefficient of mutual induction of two coils is defined as the emf induced in one coil when the rate of change of current through the other coil is unity.

The S.I. unit of coefficient of mutual inductance is Henry.

10. (a) Two long parallel conductors carrying currents in the same direction (like currents) attract each other-explain.

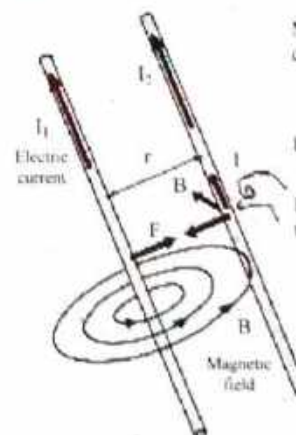
(b) Lenz's law in electromagnetic induction is consistent with the principle of conservation of energy-explain.

(c) Define self-inductance of a coil in terms of magnetic flux.

[WBSCTE 2018]

Answer:

a) Once the magnetic field has been calculated, the magnetic force expression can be used to calculate the force. The direction is obtained from the right hand rule. Note that two wires carrying current in the same direction attract each other, and they repel if the currents are opposite in direction.



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

- (b) Refer to Question No. 1 of Long Answer Type Questions.
(c) Refer to Question No. 3.a) of Long Answer Type Questions.

11. State Fleming's right hand rule.

[WBSCTE 2018]

Answer:

Refer to Question No. 5.b) of Long Answer Type Questions.

12. (a) State Lenz's law. Show that this law is accordance with the law of conservation of energy.

[WBSCTE 2019]

Answer:

1st Part: Refer to Article No. 5.6 of Chapter at a Glance.

2nd Part:

The energy is converted into electrical energy and then dissipated as heat in the loop by joule heating produced by the induced current.

12. (b) Why magnetic field do not apply force on a static charge? [WBSCTE 2019]

Answer:

The force on an electric charge moving in a magnetic field is given by Lorentz force $F = qV \times B$, where V is the velocity of the charge and B is the magnetic induction field. The force is zero if $V = 0$. There is no force on a static charge due to magnetic induction. A static charge gives rise to only electrostatic field.

13. Define mutual inductance. Write down its SI unit.

[WBSCTE 2019]

Answer:

Refer to Article No. 5.9 of Chapter at a Glance.

14. State Biot-Savart's law related to the production of magnetic field due to a small current element.

[WBSCTE 2019]

Answer:

Refer to Article No. 5.3 of Chapter at a Glance.

15. (a) State Biot-Savart's law in vector form expressing the magnetic field due to an element carrying current at a distance from the element. A circular coil of radius R current I . Derive the expression for the magnetic field due to this coil at its centre. What is the effect of increasing the number of turns on magnetic field produced to a circular coil?

(b) State the Faraday's law of electromagnetic induction.

(c) A galvanometer of resistance 50Ω gives full scale deflection when a current of $500 \mu A$ passes through it. It is to be converted into a voltmeter reading up to $10V$. Explain how this can be done? Circuit diagram is essential.

[WBSCTE 2022]

POLY-AP

Answer:

(a) 1st Part: Refer to Question No. 2.a) of Long Answer Type Questions.

2nd Part:

The expression for the magnetic field at the centre of a circular coil current conductor is,

$$B = \frac{\mu_0 n I}{2R}$$

Where μ_0 is the absolute permeability of free space.

n is the number of turns in the circular coil.

I is the amount of current carrying by the conductor.

R is the radius of circular coil.

3rd Part:

By increasing the number of turns in the coil, the strength of the magnetic field increases.

(b) Refer to Question No. 6.a) of Long Answer Type Questions.

(c) Galvanometer resistance is 50Ω .

and full scale deflection current = $500 \mu A = 500 \times 10^{-6}$ Amp.

Voltmeter reading = 10 volt.

$$\begin{aligned} \therefore \text{Resistance of galvanometer, } R &= \frac{V}{I_{\text{full scale}}} \\ &= \frac{10}{500 \times 10^{-6}} \\ &= 20,000 \Omega \end{aligned}$$



$$\begin{aligned} \therefore \text{Additional resistance in series required is, } R_s &= R - R_{\text{galvanometer}} \\ &= 20,000 - 50 = 19,950 \Omega \end{aligned}$$

16. The power expended in a resistance of 20 ohm connected to a dynamo is 2 KW . On changing the resistance to 42 ohm , the power expended becomes 1.05 KW . Determine the EMF of the dynamo and its internal resistance. Circuit diagram is essential. [Model Question]

Answer:

We know, power $\Rightarrow P = EI = I^2 R$

Again, $E = I(R + r)$

$$\therefore I = \frac{E}{R + r}$$

$$\therefore P = \left(\frac{E}{R + r} \right)^2 R \quad \left[\text{where, } E = \text{EMF, } r = \text{Internal Resistance} \right]$$

1st case:

$$2 \times 10^3 = \frac{E^2}{(20+r)^2} \times 20$$

$$\text{or, } \frac{2 \times 10^3}{20} (20+r)^2 = E^2$$

$$\text{or, } 100(20+r)^2 = E^2 \quad [\text{Here, } R=20\Omega]$$

$$\text{or, } E = 200 + 10r$$

$$\text{or, } E - 10r = 200 \dots\dots\dots (1)$$

2nd case:

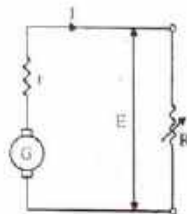
$$1.05 \times 10^5 = \frac{E^2}{(42+r)^2} \times 42$$

$$\text{or, } \frac{1.05 \times 10^5}{42} (42+r)^2 = E^2$$

$$\text{or, } 25(42+r)^2 = E^2 \quad [\text{Here, } R=42\Omega]$$

$$\text{or, } E - 5r = 210 \dots\dots\dots (2)$$

By solving equation (1) and (2), we get
 $E = 220V$.



17. A straight segment of wire of 0.002 m carries a current of 4 A. Calculate the magnitude of the magnetic field of induction produced by it at a distance of 1.6 m in a direction perpendicular to itself. [Model Question]

Answer:

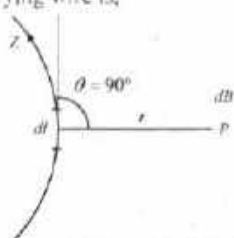
The law of Biot-Savart states that, we can find the magnetic field of induction dB at any point P , at a distance r from the elementary length of current carrying wire is,

$$dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}$$

Here, $\theta = 90^\circ$

$$dB = \frac{\mu_0 I}{4\pi r^2} dl = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{4 \times 0.002}{(1.6)^2}$$

$$= 3.125 \times 10^{-10} T$$



$$dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}, \theta = 90^\circ$$

$$dB = \frac{\mu_0 I}{4\pi r^2} dl = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{4 \times 0.002}{(1.6)^2} = 3.125 \times 10^{-10} T$$

18. A horizontal overhead power line carries a current of 60 A in east to west direction. What is the magnitude of the magnetic field of induction due to the current at a point 1.2 m below the line? [Model Question]

Answer:

Magnetic field at any point due to current flowing through an infinitely long conductor is,

$$B = \frac{\mu_0 2I}{4\pi r}$$

Here current $I = 60 A$, $r = 1.2 m$

$$\therefore B = \frac{\mu_0 2I}{4\pi r} = 10^{-7} \times \frac{2 \times 60}{1.2} = 1 \times 10^{-5} T$$

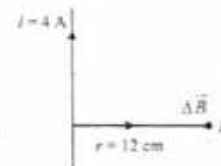
19. A straight wire carries a current of 4 A. Find the magnitude and the direction of magnetic field of induction at a point 12 cm away from the wire. [Model Question]

Answer:

Here, $I = 4 A$, $r = 12 cm = 0.12 m$

$$\text{Hence, } B = \frac{\mu_0 2I}{4\pi r} = \frac{10^{-7} \times 2 \times 4}{0.12} = 6.67 \times 10^{-6} T$$

The direction of magnetic field of induction will be perpendicular to the plane of paper and in inward direction.



20. Calculate the magnetic field of induction due to long wire carrying a current of 10 A at a distance of 5 cm from it. [Model Question]

Answer:

Here, radius, $r = 5 cm = 0.05 m$, $I = 10 A$, $\mu_0 = 4\pi \times 10^{-7} Wb \cdot A^{-1} \cdot m^{-1}$

Here, from Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$$\text{or, } B \oint dl = \mu_0 I$$

$$\text{or, } B 2\pi r = \mu_0 I$$

$$\text{or, } B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 4 \times 10^{-5} T$$

21. Calculate the force per unit length on a straight wire carrying current 3 A due to another long parallel wire with current 4 A at a separation of 0.3 m. Find also the nature of force for antiparallel direction of current through these wires. [Model Question]

Answer:

Here, $I_1 = 3 A$, $I_2 = 4 A$, $r = 0.3 m$

So, the force per unit length acting on the wire carrying a current 3 A will be,

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3 \times 4}{2\pi \times 0.3} = 8 \times 10^{-6} N$$

The force will be repulsive for antiparallel current.

22. A short conductor with length 5 cm is placed parallel to a conductor of length 1.5 m. Both of them carry a current of 3 A and 2 A respectively in the same direction. Find the nature and magnitude of the force experienced by the long conductor for their separation of 3 cm. [Model Question]

Answer:

Since, the two conductors carry currents in the same direction, so the resultant force will be attractive.

Here, current through the short conductor $I_1 = 3$ A

Its length $l_1 = 5$ cm = 0.05 m.

Hence, the magnetic field produced by short conductor at a distance of 0.03 m is,

$$B = \frac{\mu_0 I_1}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.03} = 2 \times 10^{-5} \text{ Wb} \cdot \text{m}^{-2}$$

So, the force experienced by the long conductor will be,

$$F = B I_2 l_2 = 2 \times 10^{-5} \times 2 \times 1.5 = 6 \times 10^{-6} \text{ N}$$

where $I_2 = 2$ A and $l_2 = 1.5$ m

23. (a) Find magnetic field at a point $(1, 1, 1)$ if vector potential at that position is $\vec{A} = (10x^2 + y^2 - z^2)\vec{j}$. [Model Question]

Answer:

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & (10x^2 + y^2 - z^2) & 0 \end{vmatrix} = 2\vec{i} + 20\vec{k}$$

- (b) A very long cylinder of radius a carries current I , which is uniformly distributed over the cross-section of the cylinder. If the current flows along the axis of the cylinder then find out the electric field in the conductor, the magnetic field just outside the conductor and the Poynting vector at the surface of the cylinder. You may assume that the conductivity of the material is σ . [Model Question]

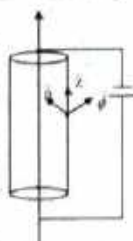
Answer:

Using ampere's circuital law we have the magnetic field H as

$$\oint \vec{H} \cdot d\vec{l} = I \rightarrow H 2\pi r = I \rightarrow H = \frac{I}{2\pi r} \text{ at } r = a \text{ field is } \vec{H} = \frac{I}{2\pi a} \vec{\phi} \text{ and}$$

direction of the field be tangential to the surface. We know electric field and current density is related with $\vec{J} = \sigma \vec{E}$. (let the direction be along z axis as shown)

As pointing vector is $\vec{S} = \vec{E} \times \vec{H}$. So on substituting $\vec{E}$ and $\vec{H}$ we have



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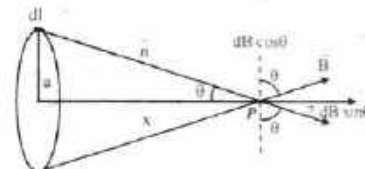
$\vec{S} = \sigma \vec{E} \times \vec{H}$ where $\vec{H}$ is perpendicular to electric and magnetic field and acting inward to the conductor as shown.

- (c) Calculate the magnetic field at the centre of a circular loop carrying current I . [Model Question]

Answer:

Let us consider the center of the current carrying loop of radius a be at the origin and its axis be along the x direction as shown in the figure. According to Biot-Savart law the magnetic field at the point P due to element of current $d\vec{l}$ will be $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^2}$

At the point P the magnetic field vectors are resolved into horizontal and vertical components as shown.



Perpendicular components on both sides of the circular loop will be cancelled and horizontal components will be added up for two halves of the loop. So net horizontal component

$$dB' = 2dB \sin \theta = 2 \times \frac{\mu_0}{4\pi} \frac{I d\ell}{r^2} \sin \theta \text{ as } r^2 = x^2 + a^2$$

$$\text{As } \sin \theta = \frac{a}{r} = \frac{a}{(x^2 + a^2)^{1/2}}$$

$$dB' = 2 \frac{\mu_0}{4\pi} \frac{I d\ell a}{(x^2 + a^2)^{3/2}}$$

$$\rightarrow B' = \int dB' = \frac{I a}{(x^2 + a^2)^{3/2}} \int_0^{2\pi} d\ell = \frac{\mu_0}{4\pi} \frac{2\pi I a^2}{(x^2 + a^2)^{3/2}} \text{ at centre } x = 0$$

$$B' = \frac{\mu_0 2\pi I}{4\pi a} \text{ or } B = \frac{\mu_0 I}{2a}$$

If there are n number of turns of conducting wire in the loop the net magnetic field at the

$$\text{centre will be } B = \frac{\mu_0 n I}{2a}$$

24. (a) Write Lorentz force equation and explain the symbols used.
(b) Deduce an expression of the force experienced by a current element placed in a magnetic field.

[Model Question]

Answer:

(a) If in a region both electric field $\vec{E}$ and magnetic field $\vec{B}$ are present, then a moving charge q experiences a force given by $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

This force is known as Lorentz force. So, Lorentz force has two components.

(b) Let us consider a current element placed in a magnetic field $\vec{B}$. If $\vec{v}$ be the velocity of the charge carriers moving along the wire, then force on each charge carrier is $q(\vec{v} \times \vec{B})$. Let n be the number of charge carriers per unit volume and if S be the cross sectional area of the conductor, then the total number of charge carriers in an elemental current element of length, $d\vec{l}$ of the said conductor is $nSdl$.

So the total magnetic force on the element $d\vec{l}$ is $d\vec{F} = nSdlq(\vec{v} \times \vec{B}) = I(d\vec{l} \times \vec{B})$ where $I = nSvq$

Case 1: For finite length of the conductor, the above force will be $d\vec{F} = I \int (d\vec{l} \times \vec{B})$

Case 2: If $\vec{B}$ is perpendicular to $\vec{l}$ the above equation reduces to $dF = IdlB$

Case 3: For a closed loop $d\vec{F} = I \oint d\vec{l} \times \vec{B} = 0$ $\therefore \oint d\vec{l} = 0$

25. Explain the types of magnetic materials.

[Model Question]

Answer:

There are three major kinds of magnetic behaviour:

i) Diamagnetic materials:

These materials are barely magnetized when placed in a magnetic field. Magnetic dipoles in these substances tend to align in opposition to the applied field. In effect, they produce an internal magnetic field that opposes the applied field and the substance tends to repel the external field around it.

Example: Gold, water, mercury and even animals.

ii) Paramagnetic materials:

In these materials the magnetic dipoles in the Magnetic Materials tend to align along the applied magnetic field and thus reinforcing the applied magnetic field. Such substances are attracted by a magnet if it applies a sufficiently strong field. It must be noted that such materials are still feeble magnetized and the magnetization disappears as soon as the external field is removed. The magnetization (M) of such materials was discovered by Madam Curie and is dependent on the external magnetic field (B) and temperature T as:

$$M = C \frac{B}{T} \quad [\text{Where } C = \text{Curie constant}]$$

Example: Liquid oxygen, sodium, platinum, salts of iron and nickel.

iii) Ferromagnetic materials:

We are most familiar with these materials as they exhibit the strongest magnetic behaviour. Magnetic dipoles in these materials are arranged into domains where the arrangements of individual magnetic dipoles are essentially perfect that can produce strong magnetic fields. Normally, these domains are usually randomly arranged and thus the magnetic field of each domain is cancelled by another and the entire material does not show any magnetic behaviour.

Example: Iron, cobalt, neodymium.

26. Explain the construction and diagram of a moving coil galvanometer.

[Model Question]

Answer:

A moving coil galvanometer is an instrument which is used to measure electric currents. It is a sensitive electromagnetic device which can measure low currents even of the order of a few microamperes.

Moving-coil galvanometers are mainly divided into two types:

- Suspended coil galvanometer
- Pivoted-coil or Weston galvanometer

Moving Coil Galvanometer Principle:

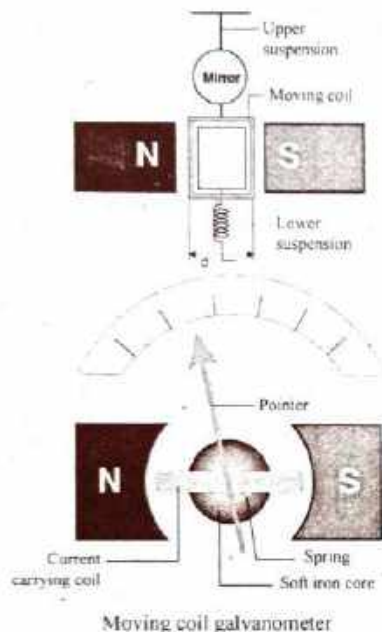
A current-carrying coil when placed in an external magnetic field experiences magnetic torque. The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.

Moving Coil Galvanometer Construction and Diagram

The moving coil galvanometer is made up of a rectangular coil that has many turns and it is usually made of thinly insulated or fine copper wire that is wound on a metallic frame. The coil is free to rotate about a fixed axis. A phosphor-bronze strip that is connected to a movable torsion head is used to suspend the coil in a uniform radial magnetic field.

Essential properties of the material used for suspension of the coil are conductivity and a low value of the torsional constant. A cylindrical soft iron core is symmetrically positioned inside the coil to improve the strength of the magnetic field and to make the field radial. The lower part of the coil is attached to a phosphor-bronze spring having a small number of turns. The other end of the spring is connected to binding screws.

The spring is used to produce a counter torque which balances the magnetic torque and hence helps in producing a steady angular deflection. A plane mirror which is attached to the suspension wire, along with a lamp and scale arrangement, is used to measure the deflection of the coil. Zero-point of the scale is at the centre.



Chapter 6: Semiconductor Physics

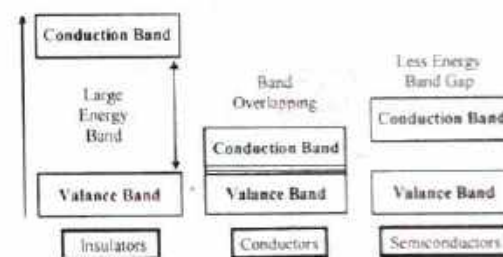
Chapter at a glance:

6.1 Semiconductor Physics

The range of energies that an electron may possess in an atom is known as the energy band.

There are mainly three types of energy bands in a material, they are-

- i) Valence Band, ii) Conduction Band, iii) Forbidden Band



6.2 Insulators

- The materials in which the conduction band and valence bands are separated by a wide energy gap which is of the value $\approx 15 \text{ eV}$.
- A wide energy gap means that a large amount of energy is required, to free the electrons, by moving them from the valence band into the conduction band.
- The valence electrons of an insulator do not have enough energy to jump in to the conduction band, therefore insulators do not have an ability to conduct current. So the insulators have very high resistivity (or extremely low conductivity) at room temperatures.

6.3 Conductors

The materials in which conduction and valence bands overlap as shown in figure are called conductors.

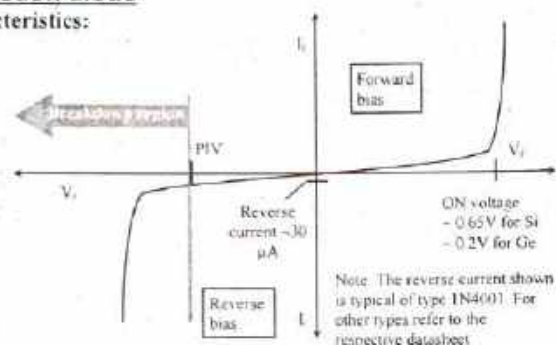
- The overlapping indicates a large number of electrons available for conduction.
- Hence the application of a small amount of voltage results a large amount of current.

6.4 Semiconductors

- The materials, in which the conduction and valence bands are separated by a small energy gap ($< 3 \text{ eV}$) are called semiconductors.
- Silicon and germanium are the commonly used semiconductors.
- A small energy gap means that a small amount of energy is required to free the electrons by moving them from the valence band in to the conduction band.

6.5 P-N junction diode

Diode Characteristics:



6.6 Half Wave Rectifier

This is the simplest rectifier arrangement, as shown in Fig. (a).

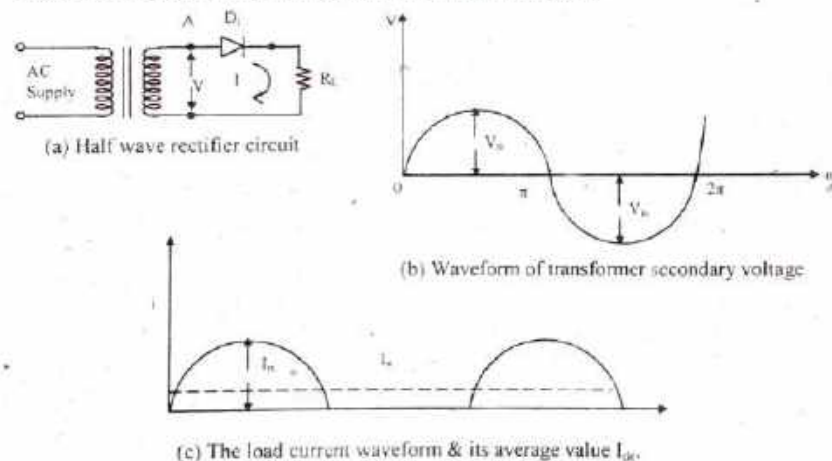


Fig.: Half-wave rectifier input voltage and load current waveforms

The secondary voltage of the transformer (being sinusoidal) can be expressed as

$$v_s = V_m \sin \omega t = V \sin \alpha$$

where $\alpha = \omega t$

This is shown in Fig. (b), where V_m is the peak value of voltage.

During the positive half cycle, when the voltage of the point A is positive with respect to point B, the diode D_1 is forward biased and will conduct. This current, which flows through the load resistance R_L , the diode (having forward resistance R_f) and secondary winding of the transformer (having resistance R_s) can be expressed as

$$i = I_m \sin \alpha \text{ for } 0 \leq \alpha \leq \pi$$

$$\text{where } I_m = \frac{V_m}{R_f + R_s + R_L}$$

(This assumes that the cut-in voltage of the diode is negligible compared to V_m).

During the negative half cycle, when the voltage of the point A is negative with respect to point B, the diode D_1 is reverse biased and will not conduct. Therefore, the current

$$i = 0 \text{ for } \pi \leq \alpha \leq 2\pi$$

The waveforms of current i during both the half cycles are shown in Fig (c). The current, though not a good d.c. is unidirectional and has an average value which is non-zero (actually positive in this case).

6.7 Full Wave Rectifier

Full Half Wave Rectifier

The figure shows a full-wave rectifier circuit. The transformer secondary has a centre-tap and each half give a peak voltage of V_m . In each half there is one diode i.e., D_1 and D_2 .

The load resistance R_L is common to both halves.

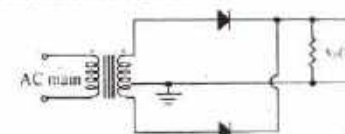


Fig.: Full-wave rectifier circuit

This can be seen to comprise of two half-wave circuits. On the positive half cycle, when the point A is positive with respect to B, the diode, D_1 conducts and current i_1 flows through R_L . During this half cycle, the point C is negative with respect to point B and hence the diode D_2 does not conduct. Therefore $i_2 = 0$.

On the negative half cycle, the point C is positive with respect to point B. Hence the diode D_2 conducts and current i_2 flows through R_L . During this half cycle, the point A is negative with respect to point B and hence the diode D_1 does not conduct. Therefore $i_1 = 0$.

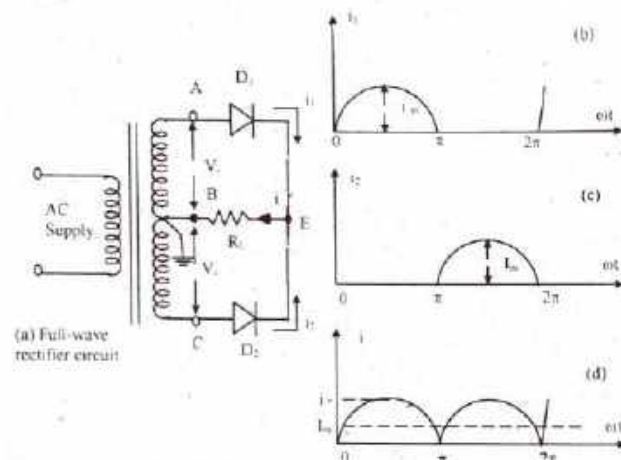


Fig.: Full-wave rectifier and the current waveforms

Fig. (b) and (c) shows the waveforms of currents i_1 and i_2 . Since both i_1 and i_2 flow through the load R_L , the total current i through R_L is $i = i_1 + i_2$, which is obtained by adding the two waveform and is shown (d).

$$i_m = I_0 \sin \omega t, 0 < \omega t < \pi = -I_0 \sin \omega t, \pi < \omega t < 2\pi$$

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. In a junction diode the holes are due to: [WBSCTE 2014]
 (a) protons (b) extra electrons (c) neutrons (d) missing electrons

Answer: (d)

2. When a forward bias is applied to p-n junction, it [WBSCTE 2014]
 (a) raises the potential barrier
 (b) reduces the majority carrier current to zero
 (c) lowers in potential barrier
 (d) widens the depletion layer

Answer: (c)

3. A P-type Semiconductor is – [WBSCTE 2015]
 (a) negatively charged (b) positively charged
 (c) neutral (d) none of these

Answer: (b)

4. Holography produces – [WBSCTE 2016]
 (a) Real image (b) Virtual image
 (c) both (a) & (b) (d) none of these

Answer: (b)

5. In which conductor holes are majority carrier – [WBSCTE 2016]
 (a) p-type semiconductor (b) n-type semiconductor
 (c) bad conductor (d) metal

Answer: (a)

6. The pumping method used in He – Ne laser is – [WBSCTE 2016]
 (a) Optical method (b) Gas – dynamic pumping
 (c) chemical pumping (d) electrical pumping

Answer: (a)

7. If the inversion temperature is θ_i , neutral temperature is θ_n and the temperature of the cold junction is θ_c , then – [WBSCTE 2016]
 (a) $\theta_c + \theta_i = 0$ (b) $\theta_i - \theta_c = 2\theta_n$ (c) $\theta_c + \theta_i = 2\theta_n$ (d) $\theta_i - \theta_c = 2\theta_n$

Answer: (c)

8. If a very small amount of boron is added to a pure germanium crystal at room temperature, it becomes, – [WBSCTE 2017]
 (a) an n-type semiconductor (b) a p-type semiconductor
 (c) an insulator (d) a good conductor

Answer: (a)

9. The value of the energy band gap for germanium crystal is – [WBSCTE 2018]
 (a) 1.1 eV (b) 0.11 eV
 (c) 0.068 eV (d) 0.68 eV

Answer: (d)

10. If the forward bias voltage across a p-n junction be increased, the width of the depletion region [WBSCTE 2022]
 (a) decrease
 (b) increase
 (c) remains same
 (d) increases proportionally to the applied voltage

Answer: (a)

11. In germanium crystal, the energy band gap is
(a) 0.068eV (b) 0.8eV (c) 68eV

[WBSCTE 2022]
(d) 0.68eV

Answer: (d)

12. The output of a full wave rectifier is

[WBSCTE 2022]

- (a) an AC voltage (b) a constant DC voltage
(c) zero (d) a pulsating unidirectional voltage

Answer: (b)

13. For obtaining a p-type semiconductor, doping is to be done with impurity material which is

[WBSCTE 2022]

- (a) tetravalent (b) pentavalent
(c) trivalent (d) none of these

Answer: (c)

14. The lasing action is based on

[WBSCTE 2022]

- (a) Stimulated emission (b) Spontaneous emission
(c) absorption (d) none of these

Answer: (a)

B. Fill in the blanks in the following statements:

15. Solar photo voltaic cell is a source of non-conventional energy(non-conventional / conventional) [WBSCTE 2006, 2007]

16. Solar photovoltaic cell is a source of non-conventional energy (conventional/non-conventional energy) [WBSCTE 2010]

17. In the Antimony (Sb) – Bismuth (Bi) Couple, the current will flow from Sb to Bi through the cold (cold / hot) junction in case of Seebeck effect. [WBSCTE 2013]

C. State whether the following statements are True or False:

18. n-type extrinsic semiconductor is obtained by doping Al with Si. [WBSCTE 2011]

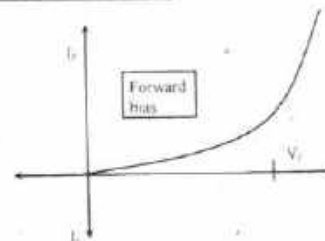
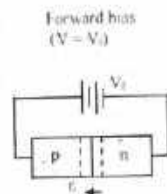
Answer: False

D. Answer the following questions:

19. Draw the necessary circuit diagram for studying the forward bias characteristic curve of a p-n junction diode. Also draw the characteristic curve. [WBSCTE 2017]

Answer:

In forward bias, the p-type is connected with the positive terminal and the n-type is connected with the negative terminal. So that the holes in the p-type region and the electrons in the n-type region are pushed toward the junction. This reduces the width of the depletion zone.



Long Answer Type Questions

1. (a) Give the circuit diagram using a bridge rectifier for full wave rectification indicating the polarity of the d.c. voltage developed across the resistive load. [WBSCTE 2005, 2006, 2010, 2012]

- (b) Distinguish conductor, semiconductor and insulator in terms of energy band diagram. [WBSCTE 2005, 2007, 2008, 2011]

OR,

Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams. [WBSCTE 2022]

Answer:

a)

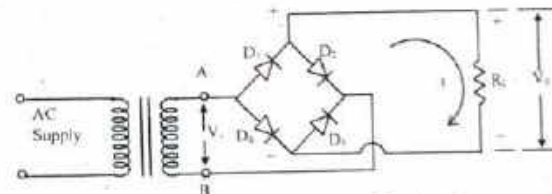


Fig.: Bridge rectifier circuit

- b) Refer to Chapter at a Glance 6.1.

2. Distinguish between P-type N-type semiconductors. [WBSCTE 2006, 2012]

OR,

Write down two important differences between n-type and p-type semiconductor. [WBSCTE 2017]

Answer:

P-Type Semiconductor	N-Type Semiconductor
i) It can be formed by adding trivalent material to the semiconductor.	i) It can be formed by adding pentavalent material to the semiconductor.
ii) Fermi-level is near the valence band.	ii) Fermi-level is near the conduction band.
iii) Holes(h ⁺) are the majority current carrier.	iii) Electrons(e ⁻) are the majority current carrier.
iv) Resistivity is higher.	iv) Resistivity is lower.

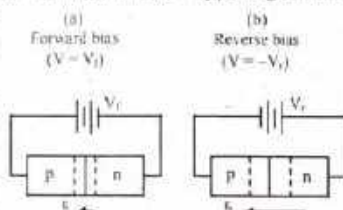
3. (a) What is meant by forward biasing and reverse biasing of a p-n junction?
 (b) Show clearly the differences in the band structure of a metal, an insulator and a semiconductor? [WBSCTE 2013]

Answer:

a) In forward bias (Fig a), the p-type is connected with the positive terminal and the n-type is connected with the negative terminal. So that the holes in the P-type region and the electrons in the N-type region are pushed toward the junction. This reduces the width of the depletion zone.

In reverse bias (Fig b), the P-type region is connected to the negative terminal of the battery and the N-type region to the positive terminal. If a diode is reverse-biased, the voltage at the cathode is higher than that at the anode. Therefore, no current will flow until the diode breaks down.

Because the p-type material is now connected to the negative terminal of the power supply, the 'holes' in the P-type material are pulled away from the junction, and it causes increase in the width of the depletion zone.



b) Refer to Chapter at a Glance 6.1.

4. (a) What is an intrinsic semiconductor?
 (b) A beam of constant intensity and increasing frequency is incident on a photosensitive surface. Explain what happens in this case and sketch the graph of stopping potential and photoelectric current. [WBSCTE 2013]

Answer:

(a) A perfect semiconductor crystal with no impurities or lattice defects is called an **intrinsic semiconductor**. In such material there are no charge carriers at 0 K, since the valence band is filled with electrons and the conduction band is empty. At higher temperatures electron-hole pairs are generated as valence band electrons are excited thermally across the band gap to the conduction band. These EHPs are the only charge carriers in intrinsic material.

Since the electrons and holes are created in pairs, thus for *intrinsic material*

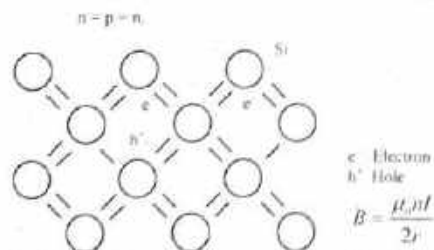


Fig: Electron - hole pairs in the covalent bonding model of the Si crystal

At a given temperature there is a certain concentration of electron-hole pairs n_i . Obviously, if a steady state carrier concentration is maintained, there must be *recombination* of EHPs at the same rate at which they are generated.

$$i.e., r_i = g_i$$

(b) We know $h\nu = W + E = eV_0 + E$

Where h is plank constant.

ν is the frequency of the incident light

W is work function

E is the charge of an electron.

V_0 is the stopping potential.

E is the maximum energy of the emitted ion.

$$eV_0 = (h\nu - h\nu_0) = h(\nu - \nu_0)$$

ν_0 is fixed for a metal.

So as ν increases, V_0 increases.

5. What are extrinsic semiconductors? What kind of impurity is chosen to make n-type and p-type semiconductors? Give examples. [WBSCTE 2014]

Answer:

1st Part:

An **extrinsic semiconductor** is a semiconductor that has been *doped*, that is, into which a doping agent has been introduced, giving it different electrical properties than the intrinsic (pure) semiconductor.

Doping involves adding dopant atoms to an intrinsic semiconductor, which changes the electron and hole carrier concentrations of the semiconductor at thermal equilibrium. Dominant carrier concentrations in an extrinsic semiconductor classify it as either an n-type or p-type semiconductor. The electrical properties of extrinsic semiconductors make them essential components of many electronic devices.

2nd Part:

A common dopant for n-type silicon semiconductors is phosphorus.

A common p-type dopant for silicon semiconductors is boron.

	Intrinsic semiconductor	Donor atoms	Acceptor atoms
Group IV semiconductors	Silicon, Germanium	Phosphorus, Arsenic	Boron, Aluminium
Group III-V semiconductors	Aluminum phosphide, Aluminum arsenide, Gallium arsenide, Gallium nitride	Selenium, Tellurium, Silicon, Germanium	Beryllium, Zinc, Cadmium, Silicon, Germanium

6. (a) Explain with circuit diagram the action of a full wave bridge rectifier.
(b) Draw the characteristic curve of a p-n junction diode in the forward and reverse bias.
(c) What are the minority carriers in a n type semiconductor? [WBSCTE 2015]

Answer:

(a) Refer to Chapter at a Glance 6.7.

(b) Refer to Chapter at a Glance 6.5.

(c) The minority carriers in a n type semiconductor are holes.

7. (a) Explain full wave rectification by bridge rectifier (circuit diagram, input and output wave forms are essential).
(b) Explain the term 'depletion region' in p-n junction. [WBSCTE 2018]

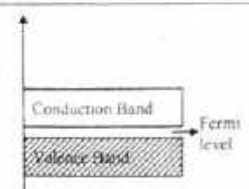
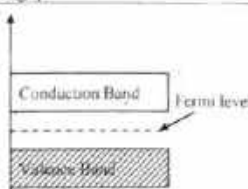
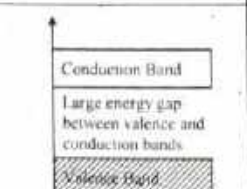
Answer:

(a) Refer to Question No. 1.a) of Long Answer Type Questions.

(b) Depletion region or depletion layer is a region in a P-N junction diode where no mobile charge carriers are present. Depletion layer acts like a barrier that opposes the flow of electrons from n-side and holes from p-side.

8. a) Distinguish conductors, semiconductors and insulators in terms of energy band diagram. [WBSCTE 2019]

Answer:

Conductors	Semi-conductors	Insulator
Conductors like metals; the valence band overlaps the conduction band.	Semiconductors have a small gap between the valence and conduction bands that thermal or other excitations can bridge the gap.	The valence bands are separated by a large gap from the conduction band.
		

8. (b) What kind of impurity is chosen to make n-type and p-type semiconductors? Give examples. [WBSCTE 2019]

Answer:

n-type semiconductor is created by adding pentavalent impurities like Phosphorous, Arsenic, Antimony or Bismuth. p-type semiconductor is created by adding trivalent impurities like Gallium and Indium.

9. (a) Distinguish between an intrinsic semiconductor and p-type semiconductor.
(b) With the help of circuit diagrams, distinguish between forward biasing and reverse biasing of a p-n junction diode.
(c) Draw a labelled diagram of a full wave rectifier circuit. Show the input-output waveforms.
(d) Draw the circuit diagram of an illuminated photodiode in reverse bias. How is photodiode used to measure light intensity. [WBSCTE 2022]

Answer:

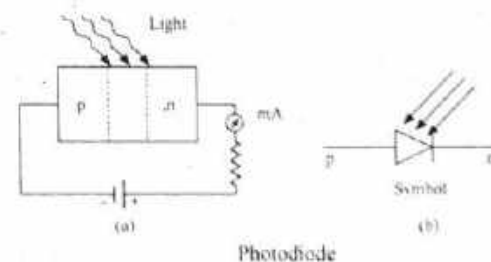
(a) Distinguish between an intrinsic semiconductor and p-type semiconductor:

- (i) An intrinsic semiconductor is a pure semiconductor where as a p-type semiconductor is a semiconductor doped with trivalent impurity atoms like boron or gallium.
(ii) An intrinsic semiconductor has same number of holes and electrons while in a p-type semiconductor the number of holes is greater than the number of electrons.

(b) Refer to Question No. 3.a) of Long Answer Type Questions.

(c) Refer to Article No. 6.7 of Chapter at a Glance.

(d) When p-n junction is reversed biased with no current, a very small reverse saturated current flows across the junction called the dark current. When the junction is illuminated with light, electron-hole pairs are created at the junction, due to which additional current begins to flow across the junction; the current is solely due to minority charge carriers.



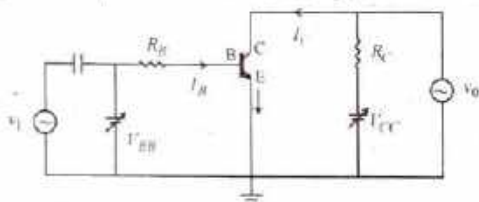
10. (a) State two advantages of LED lamps over conventional incandescent lamps.
(b) Draw a circuit diagram of n-p-n transistor amplifier in CE configuration.
(c) Why GaAs is most commonly used in making of a solar cell? [WBSCTE 2022]

Answer:

(a) The two advantages of LED lamps over conventional incandescent lamps are:

- LEDs have longer life as compared to incandescent lamps.
- LED requires low operational voltage and less power as compared to incandescent lamps. LEDs also have fast on-off switching capabilities.

(b) The circuit diagram for n-p-n transistor in CE configuration is as shown below:



The transistor acts as an amplifier when the emitter-base junction is forward biased and the collector-emitter junction is reverse biased with a high voltage V_{CE} .

(c) Ga As (gallium arsenide) is most commonly used in making of a solar cell because it absorbs relatively more energy from the incident solar radiations being of relatively higher absorption coefficient.

Chapter 7: Modern Physics

Chapter at a glance:

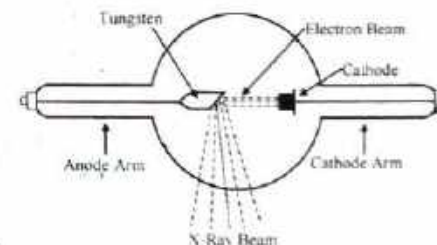
7.1 X-rays

Production of X-rays by Coolidge X-ray tube:

The characteristic features of the Coolidge tube are its high vacuum and its use of a heated filament as the source of electrons. There is so little gas inside the tube that it is not involved in the production of x-ray, unlike the situation with cold cathode gas discharge tubes.

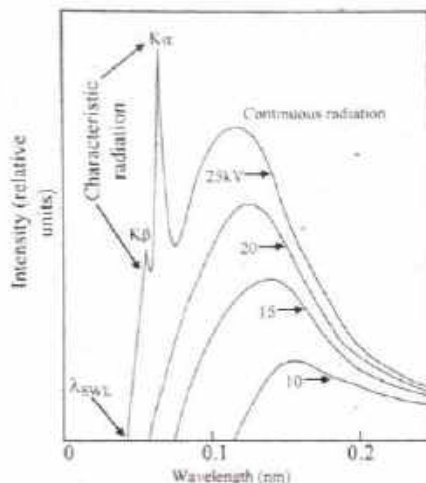
The operation of the Coolidge tube is as follows.

As the cathode filament is heated, it emits electrons. The hotter the filament gets, the greater the emission of electrons. These electrons are accelerated towards the positively charged anode and when the electrons strike the anode, they change direction and emit x-rays with a continuous range of energies. The maximum energy of the x-rays is the same as the kinetic energy of the electrons striking the anode. In addition to the x-rays produced at the focal spot of the anode, some undesirable x-rays (stray radiation) are produced by electrons striking other tube components.



The key advantages of the Coolidge tube are its stability, and the fact that the intensity and energy of the x-rays can be controlled independently. Increasing the current to the cathode increases its temperature. This increases the number of electrons emitted by the cathode, and as a result, the intensity of the x-rays. Increasing the high voltage potential difference between the anode and the cathode increases the velocity of the electrons striking the anode, and this increases the energy of the emitted x-rays. Decreasing the current or the high voltage would have the opposite effects. The high degree of control over the tube output meant that the early radiologists could do with one Coolidge tube what before had required a stable of finicky cold cathode tubes. As a bonus, the Coolidge tube could function almost indefinitely unless broken or badly abused.

X-Ray spectrum:



Continuous X-ray spectrum is the part where the graph is a smooth one, characteristic spectrum is where the spectrum has an abrupt rise (and fall). It can be observed that the continuous spectrum has a **minimum wavelength** (λ_{min}). This minimum value occurs when all of the kinetic energy of an electron is converted into X-ray energy.

The **minimum wavelength** (λ_{min} W L) is given by:

$$hf = h(c/\lambda_{min}) = 1/2mv^2$$

hf is the energy of the beam X-ray, c is the speed of light, m is the mass of an electron and v is the speed of the electron.

Properties of X-Rays:

1. X-Rays are electromagnetic radiations having a wavelength between 10-0.01 Å
2. In free space X-Rays moves in a straight line.
3. It's speed is same as light (1,86,000 miles/sec)
4. They are invisible to the eyes
5. X-rays cannot be reflected or refracted or deflected by the magnetic or electric field.
6. They show the properties of interference, diffraction and reflection as of visible light.
7. They have electric and magnetic field components to the right angle of the propagation.
8. They do not require any medium for propagation, i.e. they can propagate in the space.

9. They can penetrate liquids, solid and gases. The degree of penetration depends on the material.
10. They can be absorbed by liquids, solid and gases. The degree of absorption depends on the material.
11. X-rays interact with materials they penetrate and cause ionization.
12. When X-ray fall on certain materials they emit visible light, which is known as fluorescence.
13. They also show heating effect (i.e. falling X-ray on some material makes the material heated.)
14. X-rays are capable of producing image on a photographic film.
15. The excitation property of X-rays is used in the treatment of malignant lesions.

Application of X-rays:

X-rays are used in,

1. Material testing
2. X-ray imaging in the medical field.
3. Finding any crack in any manufactured product.
- X-ray crystallography in which the pattern produced by the diffraction of X-rays through the closely spaced lattice of atoms in a crystal is recorded and then analysed to reveal the nature of that lattice.
- X-ray astronomy, which is an observational branch of astronomy, which deals with the study of X-ray emission from celestial objects.
- X-ray microscopic analysis, which uses electromagnetic radiation in the soft X-ray band to produce images of very small objects.
- Industrial CT (computed tomography) is a process which uses X-ray equipment to produce three-dimensional representations of components both externally and internally. This is accomplished through computer processing of projection images of the scanned object in many directions.
- Course of painting, or by later restorers. Many pigments such as lead white show well in X-ray photographs.
- Airport security luggage scanners use X-rays for inspecting the interior of luggage for security threats before loading on aircraft.
- Border control truck scanners use X-rays for inspecting the interior of trucks.

7.2 Working Principle of Laser

In order to realize the basic working principle of lasing action a statistical quantum process understanding is required. From Bohr's theory we know that while dropping from excited state to ground state an electron can emit energy in form of photon. Likewise atomic (atom will be used to include molecules & ions) transition can also take place between energy states, which is purely a statistical one. Now difficulty arises, in order to locate a particular atom responsible for transition between different states. Only quantum mechanical probability calculation helps us to calculate the rate of radiative transition from an assembly of statistically large number of atoms.

Before going to the details of the atomic transition amongst energy states let us categorise the possible methods by which we can excite an atom from energy state to a higher one. By collision between two different atoms where kinetic energy of one is transferred to other to move it to the higher excited state.

By absorption of photons whose energy is just the right amount of energy i.e., which can raise an atom to higher energy level.

By applying an electrical field atomic ions will be accelerated till kinetic energies are sufficient to excite atoms they collide with underlined causes act as stimuli.

Most important part of laser is that the excited atoms etc about which we are talking about may be of a solid, a liquid or a gas,

which are present in these substances in a high proportion. A block diagram of laser operation is given.

To understand how the active medium (solid, liquid, gas) amplifies light we need to focus our attention on the following process.

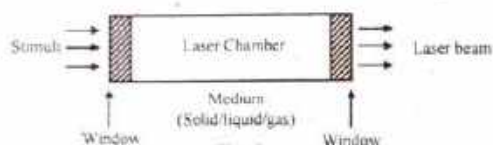


Fig. 1

7.2.1 Stimulated Absorption

An atom residing in lower energy state E_1 may absorb the incident photon and jump to the excited state E_2 as depicted in the figure 2. The transition is known as stimulated absorption or induced absorption or simply as absorption. Corresponding to each transition made by an atom one photon disappears from the incident beam. The number of absorption transitions occurring in the material at any instant will be proportional to the number of atoms in the lower state E_1 and the number of photons in the incident beam.

The absorption of such photon is termed as **resonant process**. To satisfy the resonant process only photons of particular frequency will be absorbed.

Excited state with higher energy is inherently unstable because of a natural tendency of atoms to seek out the lowest energy configuration. Therefore, excited atoms do not stay in the excited state for a relatively longer time [life time of an atom in the excited state is 10^{-18} sec] but tend to return to the lower state by giving up the excess energy $h\nu = E_2 - E_1$ in the form of spontaneous emission or stimulated emission.

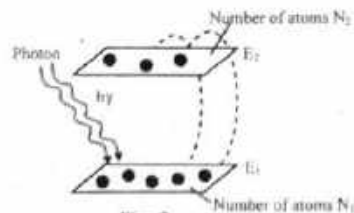


Fig. 2

7.2.2 Spontaneous Emission

The excited atom in the state E_2 may return to the lower state E_1 on its own out of the natural tendency to attain minimum potential energy condition. During the transition the excess energy is released as a photon of energy $h\nu = E_2 - E_1$. This type of process in which photon emission occurs without any external impetus is called spontaneous emission. It can be represented as $A^* \rightarrow A + h\nu$. The process of spontaneous emission is essentially probabilistic, that is the atom has some probability for making the transition, and it is not amenable for control from outside. The instant of the transition, direction of emission of photon, the phase of the photon, the polarization state of the photon are all random quantities.

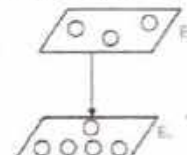


Fig. 3

There will not exist any correlation among parameters of the innumerable photons emitted spontaneously by the assembly of atoms of the medium. Therefore the light generated by the medium will be incoherent. The light from the conventional sources originates in spontaneous emission process and is incoherent. It contains a superposition of many waves of random phases. The net intensity of such incoherent waves is proportional to the number of radiating atoms. The light is radiated in the form of short duration wave trains emitted in all direction and the intensity goes on decreasing as the wave trains travel away from the source. Each of them bears no consistent phase relationship with each other nor do they share a common polarization plane. As a consequence, there is no compounding of the individual waves.

7.2.3 Stimulated Emission

An atom in the excited state need not wait for spontaneous emission to occur. There exists an alternative mechanism by which an excited atom can make a downward transition and emit light. A photon of energy $h\nu = E_2 - E_1$ can induce the excited atom to make a downward transition releasing the energy in the form of a photon. Thus, the interaction of a photon with an excited atom triggers the excited atom to drop to the lower energy state giving up another photon. The phenomenon of forced emission of photon is called induced or stimulated emission. The existence of this mechanism was predicted by Einstein in 1916.

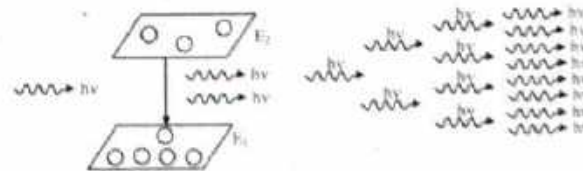


Fig. 4

The rate of stimulated emission depends on

1. The number of atoms in the excited state.
2. the Intensity of the external radiation.

As the direction of propagation, the frequency, phase and state of polarisation of stimulated emission in the same as the incident photon it is highly coherent.

On interaction of a photon of energy $h\nu = E_2 - E_1$ with an atom at higher excited state induce excited atom to make downward transition releasing the excess energy in the form of photon. We may think of in such a way that single photon after interaction with the atom at the excited state emits another photon. These two photons in their path producing a total four photons and again in similar fashion they produce eight photons and so on i.e., multiplication of photon takes place resulting the amplification of light.

Necessary condition: For this chain process to sustain the essential condition is that the number of atoms in the excited state must be greater than that in the ground state that is $N_2 > N_1$. The highly populated energy state is called the population inversion. So population inversion is the prerequisite for lasing action.

7.2.4 Population Inversion

So far we have learnt that for lasing action stimulated emission process is essential to ensure a strong lasing beam with high percentage stimulated emission is required. To get high percentage of stimulated emission process a technique is adopted so that more number of atoms should be in the higher excited state than that at the ground state. This situation is known as population inversion.

Before understanding population inversion we must clear about population. Basically the number of atoms present in a particular energy state is called as population. At a thermal equilibrium, the no. of atoms present in a particular energy state is determined by Maxwell-Boltzmann statistics i.e.,

$$N_2 = N_0 \exp\left(-\frac{E_2}{KT}\right) \quad \dots(1)$$

Where $N_2 \Rightarrow$ no. of atoms present in the energy state $N_0 \Rightarrow$ no. of atoms present in ground state $K \Rightarrow$ Boltzmann constant, $T \Rightarrow$ absolute temperature. Similarly for energy state, E_1 the no. of atoms

$$N_1 = N_0 \exp\left(-\frac{E_1}{KT}\right) \quad \dots(2)$$

$$\text{So, } N_0 = N_1 \exp\left(\frac{E_1}{KT}\right) \quad \dots(3)$$

Putting N_0 in the above expression we have,

$$N_2 = N_1 \exp\left(\frac{E_1}{KT}\right) \exp\left(-\frac{E_2}{KT}\right) \quad \dots(4)$$

$$N_2 = N_1 \exp\left(\frac{E_1 - E_2}{KT}\right) \quad \dots(5)$$

Since $E_1 < E_2$ hence $N_2 < N_1$

i.e. at thermal equilibrium no. of atoms present in the upper energy state is lower than that of the lower energy state. We can define, population inversion is an artificial situation established by generation of large number of atoms in the higher energy state. As an example how population inversion achieved is shown in the following figure.

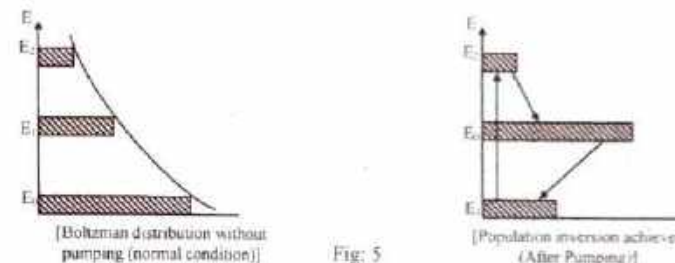


Fig: 5

A three level lasing scheme is shown in the above diagram. Fig 5(a) shows the number of atomic distribution when no pumping is employed. Fig 5(b) shows the distribution function changed i.e., population inversion when pumping is employed. Let us assume E_1 is a metastable state (life time of atoms in this state is 10^{-4} sec.). Excited atoms from E_2 level (short lived of life time 10^{-8} sec.) decay fast to level E_1 and atoms are accumulated at E_1 . Thus intense pumping from E_0 to E_2 and then rapid decay to E_1 it is possible to achieve non-equilibrium distribution of atoms. Then E_1 is more populated than E_0 when lasing transition takes place between E_1 and E_0 .

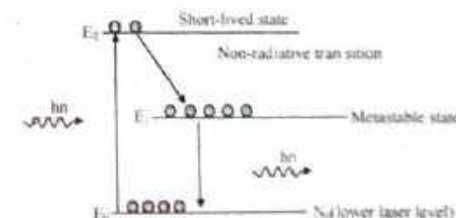


Fig: 6

It is not possible to maintain continuously, E_1 more populated than E_0 due to depopulation of E_1 . Such system works in pulsed mode only (in Ruby laser).

Holography: Holography is a technique which enables three-dimensional images to be made. It involves the use of a laser, interference, diffraction, light intensity recording and suitable illumination of the recording. The image changes as the position and orientation of the viewing system changes in exactly the same way as if the object were still present, thus making the image appear three-dimensional.

7.3 Bohr's atomic model

Neils Bohr made the following postulates—

- In an atom, the electrons revolve around the nucleus in certain definite circular paths called orbits, or shells.
- Each shell or orbit corresponds to a definite energy. These circular orbits are also known as energy levels or energy shells. The orbits or energy levels are characterized by the numbered 1, 2, 3, 4..... or K, L, M, N.....etc.
- The electrons can move from a lower energy level to a higher energy level or from a higher energy level to a lower energy level by absorbing or emitting the appropriate energy respectively.



Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The wavelength of most energetic x-rays emitted when a metal target is bombarded by 40 keV electrons is approximately [WBSCTE 2014]
 (a) 300°A (b) 10°A (c) 4°A (d) 0.31°A

Answer: (c)

2. The characteristic X-ray depends on— [WBSCTE 2015]
 (a) nature of the target material (b) velocity of electron
 (c) number of electrons striking the target (d) none of these

Answer: (a)

3. The energy of a photon of characteristic X-ray from a Coolidge tube comes from [WBSCTE 2016]
 (a) the kinetic energy of the free electrons of the target
 (b) the kinetic energy of ions of the target
 (c) the kinetic energy of the striking electron
 (d) an atomic transition in the target

Answer: (b)

4. The pumping method used in He – Ne laser is – [WBSCTE 2016]
 (a) Optical method (b) Gas – dynamic pumping
 (c) chemical pumping (d) electrical pumping

Answer: (a)

5. Which of the following statement is true?— [WBSCTE 2017]
 (a) Frequency of X-rays is slightly less than that of visible light
 (b) Frequency of X-rays is much less than that of visible light
 (c) Frequency of X-rays is greater than that of visible light
 (d) Frequency of X-rays is equal to that of visible light

Answer: (c)

6. The penetrating power of X-rays produced by an X-ray tube can be increased by — [WBSCTE 2018]
 (a) increasing the filament current (b) increasing the tube voltage
 (c) decreasing the filament current (d) decreasing the tube voltage

Answer: (b)

7. Laser radiation is [WBSCTE 2019]
 (a) highly monochromatic (b) partially monochromatic
 (c) white light (d) dichromatic

Answer: (a)

8. In an X-ray tube, the intensity of the emitted X-ray beam is increased by [WBSCTE 2019]
 (a) increasing the filament current (b) decreasing the filament current
 (c) increasing the target potential (d) decreasing the target potential

Answer: (a)

9. LASER is the acronym for [WBSCTE 2022]
 (a) Light Amplification by Stimulated Absorption of Radiation
 (b) Light Amplification by Stimulated Emission of Radiation
 (c) Light Amplification by Spontaneous Emission of Radiation
 (d) None of these

Answer: (b)

10. X-rays are deflected by [WBSCTE 2022]
 (a) electric field
 (b) magnetic field
 (c) both magnetic field and electric field
 (d) neither electric field nor magnetic field

Answer: (d)

11. If the potential difference applied to an X-ray tube be doubled, the cut off (minimum) wavelength [WBSCTE 2022]
 (a) becomes halved (b) becomes doubled
 (c) becomes four time (d) becomes same

Answer: (a)

12. The intensity of X-rays produced by an X-ray tube can be increased by [WBSCTE 2022]
- increasing the tube voltage
 - decreasing the tube voltage
 - decreasing the filament current
 - increasing filament current

Answer: (a)

13. In He-Ne laser pumping is done by [WBSCTE 2022]
- optical method
 - Thermo-chemical reaction
 - atom-atom inelastic collision through electrical discharge
 - none of these

Answer: (a)

Long Answer Type Questions

1. (a) What is LASER? Differentiate between ordinary light and laser.
(b) Define spontaneous and stimulated emission?
(c) What is population inversion? [WBSCTE 2014]

Answer:

(a) MASERS (Microwave Amplification by Stimulated Emission of Radiation) in the optical range called LASER (Light Amplification by Stimulated Emission of Radiation).

(b) Refer to Chapter at a Glance 7.2.2 & 7.2.3.

(c) Refer to Chapter at a Glance 7.2.4.

2. (a) Write two properties of LASER? [WBSCTE 2015]
(b) What is hologram?
(c) An X-ray tube operates a potential difference V . What is the minimum wavelength of X-rays? Mention two important uses of X-rays.

Answer:

(a) Laser light is monochromatic, directional, and coherent.

Monochromatic

The light emitted from a laser is monochromatic, that is, it is of one wavelength (color). In contrast, ordinary white light is a combination of many different wavelengths (colors). Directional

Lasers emit light that is highly directional. Laser light is emitted as a relatively narrow beam in a specific direction. Ordinary light, such as coming from the sun, a light bulb, or a candle, is emitted in many directions away from the source.

(b) A hologram (pronounced HOL-o-gram) is a three-dimensional image, created with photographic projection. The term is taken from the Greek words holos (whole) and gramma (message). Unlike 3-D or virtual reality on a two-dimensional computer display, a hologram is a truly three-dimensional and free-standing image that does not simulate spatial depth or require a special viewing device. Theoretically, holograms could someday be transmitted electronically to a special display device in your home and business.

The theory of holography was developed by Dennis Gabor in 1947. The development of laser technology made holography possible.

$$(c) eV = hf_{\max} = hc / \lambda_{\min}$$

where V is the accelerating voltage for the tube, $f_{\max}$ is maximum x-ray frequency, h is Planck's constant, and e is the charge of the electron.

X-rays are used in,

- Material testing
 - X-ray imaging in the medical field.
 - Finding any crack in any manufactured product.
- X-ray crystallography in which the pattern produced by the diffraction of X-rays through the closely spaced lattice of atoms in a crystal is recorded and then analysed to reveal the nature of that lattice.

3. (a) Write the properties of X-ray.
(b) How much voltage be applied in X-ray tube such that the minimum wavelength of the emitted X-ray will be 1 angstrom?
(c) Explain graphically the continuous and characteristics spectra indicating $\lambda_{\min}$. [WBSCTE 2015]

Answer:

(a) Refer to Chapter at a Glance 7.1.

$$(b) \text{ We know that } \lambda_{\min} = \frac{hc}{eV} \approx \frac{12400}{V} \text{ \AA}$$

So from the above equation we can say if the applied voltage is 12400 v then emitted X-Ray will be 1 angstrom.

(c) The x-ray photon of shortest wavelength (highest frequency) is emitted when the incident electron loses all of its kinetic energy in a single encounter. In this case $E' = 0$ and hence

$$h\nu_{\max} = \frac{ch}{\lambda_{\min}} = E \text{ \& where } E = eV$$

where $V \rightarrow$ accelerating voltage. From above relation we have $\frac{ch}{\lambda_{\min}} = eV$ So that

$$\lambda_{\min} = \frac{ch}{eV} \text{ \AA}$$

$$\text{the value of } \frac{hc}{eV} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times V} = \frac{12.420 \times 10^{-7}}{V} = \frac{12420}{V} \text{ \AA}$$

So $\lambda_{\min}$ is inversely proportional to accelerating voltage.

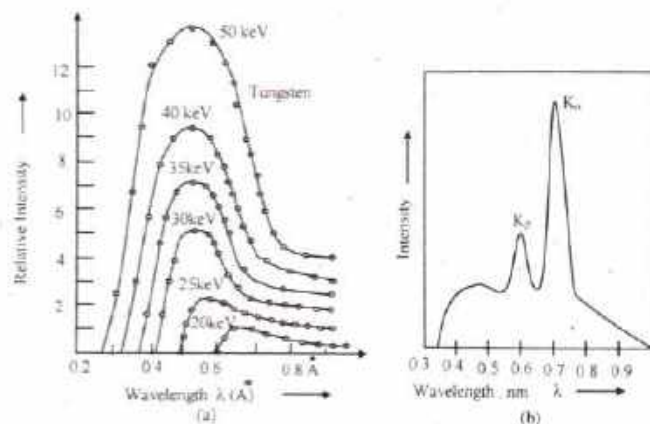


Fig 1: X-ray spectrum: (a) continuous spectrum emitted by a tungsten target. Note that the short wavelength limits shift to lower wavelength for increasing accelerating potentials; (b) the characteristic line spectrum emitted by a molybdenum target. K_{α} and K_{β} lines are seen superposed on the continuous spectrum.

Thus we see that quantum theory provides an easy and convincing explanation for the existence of short wave length limit. If we set $h = 0$ we get $\lambda_{\min} = 0$. This means that the existence of $\lambda_{\min}$ is a quantum mechanical phenomenon. The emission of X-ray from deceleration electron is called Bremsstrahlung process. This process is sometimes called inverse photoelectric effect. In a photoelectric process, photon is absorbed; its energy and momentum are transferred to electron. In this process a photon is created, its energy and momentum are derived from the electron-nuclear collision.

4. (a) Explain the term population inversion & optical pumping?
 (b) Explain the working of He - Ne Laser.
 (c) Mention some important application of LASER.

[WBSCTE 2016]

Answer:

a) Population Inversion: Refer to Chapter at a Glance 7.2.4.

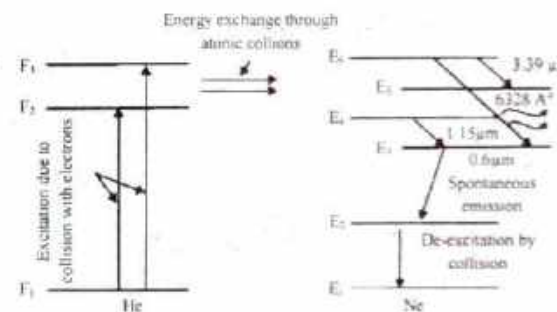
2nd Part:

Optical pumping is a process in which light is used to raise (or "pump") electrons from a lower energy level in an atom or molecule to a higher one. It is commonly used in laser construction, to pump the active laser medium so as to achieve population inversion. The technique was developed by 1966 Nobel Prize winner Alfred Kastler in the early 1950s.

(b) Working of Helium Neon Laser

In the earlier articles, I have explained the construction of helium neon laser. Today let us discuss the working of helium neon laser:

Pumping of He atoms: When electric discharge is passed through the gas mixture of He and Ne, electrons are accelerated down the discharge tube in which mixture of He-Ne is placed. These accelerated electrons collide with helium atoms and excite them to higher energy levels (let us say F2 and F3).



These levels happen to be Metastable and thus the He atoms spend a sufficient amount of time there before getting de-excited.

(c) Applications of laser includes:

- laser cutting
- laser welding
- barcode readers
- OLED display
- CD/DVD reader & writer
- Holography

5. a) Write down two important applications of X-rays.

- b) Write short notes on – (i) spontaneous emission, and (ii) stimulated emission.
 [WBSCTE 2017]

Answer:

(a) Refer to Chapter at a Glance 7.1.

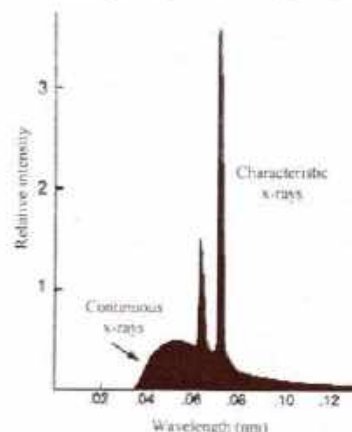
(b) Refer to Chapter at a Glance 7.2.2 & 7.2.3.

6. (a) Write down the basic properties of LASER beam which make it different from ordinary light.
(b) Draw the curve showing the variation of intensity with wavelength of X-rays obtained from X-ray tube and mark cut-off wavelength ($\lambda_{\min}$), continuous & characteristic X-rays. [WBSCTE 2018]

Answer:

(a) Refer to Question No. 2.a) of Long Answer Type Questions.

(b)



7. Calculate the tube voltage required to be applied to an X-Ray tube to produce X-Rays of minimum wavelength $2\text{\AA}$. [WBSCTE 2019]

Answer:

$$\lambda = 2\text{\AA}$$

$$eV = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-10}} \quad \text{Tube voltage} = 9.945 \times 10^{-16} \text{ eV}$$

8. Write the differences between ordinary light and LASER. [WBSCTE 2019, 2022]

Answer:

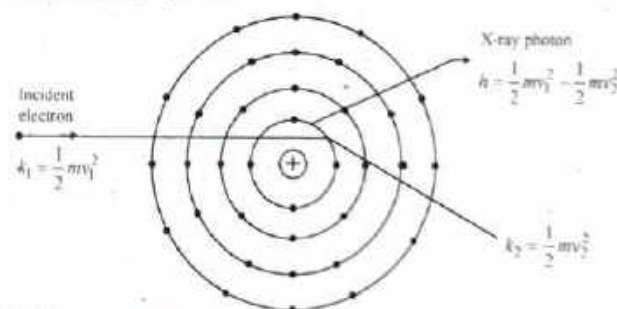
Ordinary Light	Laser Light
i) Ordinary lights are incoherent in nature.	i) Laser lights are highly coherent.
ii) Intensity of ordinary light decreases with distance as it travels in form of short pulses of small length and short duration.	ii) It travels only in one direction, which is parallel to the optic axis.
iii) It is not strictly monochromatic as it travels in form of short pulses of small length and short duration.	iii) It is highly monochromatic as it travels in form of large length and long duration.
iv) Speed of spherical wave front is more hence they spread heavily i.e., divergence is more	iv) Spread of plane wave front is less hence they spread least i.e., their divergence is less.

9. (a) Write a short note on 'population inversion'.
(b) How continuous and characteristic X-rays are produced in Coolidge tube? Explain. [WBSCTE 2022]

Answer:

(a) Refer to Article No. 7.2.4 of Chapter at a Glance.

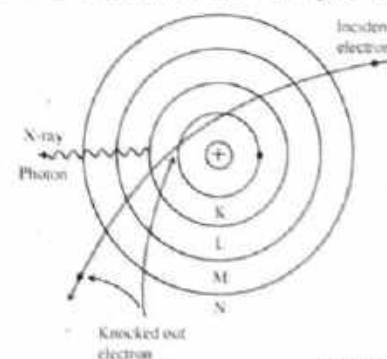
(b) **Continuous X-ray spectrum:** The mechanism of the production of continuous X-ray spectrum is shown in Figure. An electron emitted from the filament in the X-ray tube is accelerated towards the target nucleus with initial kinetic energy say $K_1 = \frac{1}{2}mv_1^2$. The electron with high value of kinetic energy K_1 enters deep inside the target atom. As it approaches the nucleus of the atom, it is strongly attracted by the nucleus and due to this attraction, electron deviates from its original path and thus retarded. So its velocity decreases and consequently its kinetic energy is also decreased. This loss of energy of the electron during retardation appears in the form of X-ray photon. The electron goes on retarding due to the interaction with the nuclei of the target atoms. As a result X-ray photons are emitted till the electron loses all its energy. The X-ray photons so emitted from the continuous X-ray spectrum.



Characteristic X-ray spectrum:

The mechanism of production of characteristic X-ray spectrum is shown in figure. The electrons emitted from cathode of the X-ray tube are accelerated towards the target due to high potential difference applied between the cathode and the target. These high energy electrons penetrate deep into the atoms of the target and knockout electrons from the inner orbits of the atoms.

When an electron is knocked out from an inner orbit or shell, a vacancy or hole is created there, which may be filled up by an electron jumping from higher shell of the atom and in the process energy of the electron



of higher shell of the atom and in the process energy of the process energy of the electron of higher shell is emitted lies in the X-ray region and so it is known as characteristic X-ray photon.

10. Write Bohr's theory of electrons in his atomic model. [Model Question]

Answer:

Niels Bohr applied Planck's quantum theory of radiation to Rutherford's atomic model, which lacked stability. He made the following assumptions, known as postulates of Bohr's model of atom:

- i) An atom has a small positively charged central core called nucleus, where whole mass of the atom is supposed to be concentrated.
- ii) The electron revolves round the nucleus in fixed orbits of definite radii and as long as an electron is in a certain fixed orbit, it does not radiate any energy. These orbits are called stationary orbits and they have definite amount of energy.
- iii) An electron can revolve only in those orbits, in which its angular momentum is an

integral multiple of $\frac{h}{2\pi}$ i.e.,

$$L = mvr = \frac{nh}{2\pi}$$

This is Bohr's quantization condition for angular momentum. Here,

m = mass of the electron,

v = velocity of the electron,

r = radius of the orbit,

h = Planck's constant and

$n = 1, 2, 3, \dots$ = principal quantum number

- iv) The energy is radiated in the form of a photon, only when an electron jumps from an orbit of higher energy to an orbit of lower energy.

If E_1 and E_2 be the energies associated with the orbits of principal quantum numbers n_1 and n_2 respectively ($n_2 > n_1$), then the amount of energy radiated in the form of a photon is given by,

$$h\nu = E_2 - E_1$$

where, ν is the frequency of the emitted radiation.

INTRODUCTION TO IT SYSTEMS

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Chapter 1: Basic Internet Skills & Number System

Chapter at a glance:

1.1 Internet & Intranet

When two or more networks are connected, they become an internetwork, or internet. The most notable internet is called the Internet, a collaboration of more than hundreds of thousands of interconnected networks. Private individuals as well as various organizations such as government agencies, schools, research facilities, corporation, and libraries in more than 100 countries use the internet.

1.2 Number System

A number system is a framework where a set of numbers are represented by numerals in a consistent manner. Ex: Decimal Number System, Binary Number System

Ideally, a Number system will:

- Represent a useful set of numbers (e.g. all whole numbers, integers, or real numbers)
- Give every number represented a unique representation (or at least a standard representation)
- Reflect the algebraic and arithmetic structure of the numbers.

1.3 Decimal to Binary Conversion

The *division method* calculates the least significant bit first.

Let D be a decimal number such that:

D = 84

Converting it into a binary number : where r is 2 .

Steps	Quotient	Remainder
84/2	42	0
42/2	21	0
21/2	10	1
10/2	5	0
5/2	2	1
2/2	1	0
1/2	0	1

The number is : $(84)_{10} = (1010100)_2$

1.4 Logic Gate

Logic gate is an electronic circuit, which makes logic decisions.

- It has one output and one or more inputs.
- The output signal appears only for certain combinations of input signals.

Types of Gates

- OR Gate, AND Gate, NOT Gate, NOR Gate, NAND Gate, EX-OR Gate /XOR Gate / Exclusive OR Gate.
- XNOR Gate / EX-NOR Gate/ Exclusive NOR Gate.

There are just three basic gates- AND gate, OR gate, and NOT gate.

Truth Table: This table gives the output of all possible input combinations.

1.4.1 OR Gate

The OR gate, which is composed of two or more inputs and a single output, performs logical addition:

Symbol of OR gate:		Truth table:		
A		A	B	A+B
B		0	0	0
		0	1	1
		1	0	1
		1	1	1

Logic Equation: $Y=A+B$

1.4.2 AND Gate

The AND gate, which is composed of two or more inputs and a single output, performs logical multiplication

Symbol of AND gate:		Truth table:		
A		A	B	A.B
B		0	0	0
		0	1	0
		1	0	0
		1	1	1

Logic Equation: $Y=A.B$

1.4.3 NOT Gate

The inverter (NOT circuit) performs a basic logic function called inversion or complementation.

Symbol of NOT gate:



Logic Equation: $A = \bar{A}$

Truth table:

A	$\bar{A}$
0	1
1	0

1.4.4 NOR Gate

The NOR gate, which is composed of two or more inputs and a single output, also has a universal property. The term NOR is formed by the concatenation NOT-OR and implies an OR function with an inverted output.

Symbol of NOR gate:



Logic Equation: $Y = \overline{A + B}$

Truth table:

A	B	$\overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

1.4.5 NAND Gate

The NAND, which is composed of two or more inputs and a single output, is a very popular logic element because it may be used as a universal function. The term NAND is formed by the concatenation NOT-AND and implies an AND function with an inverted output.

Symbol of NAND gate:



Logic Equation: $Y = \overline{A \cdot B}$

Truth table:

A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

1.4.6 EX-OR Gate

The exclusive-OR or XOR gate has two or more inputs. In general, an XOR gate gives an output value of 1 when there are an odd number of 1's on the inputs to the gate.

Symbol of the XOR Gate:



Logic Equation: $Y = A \oplus B = \bar{A}B + A\bar{B}$

Truth Table:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

1.4.7 EX-NOR Gate

The exclusive-NOR or XNOR gate has two or more inputs. The output is equivalent to inverting the output from the exclusive-OR gate described above. In general, an XNOR gate gives an output value of 1 when there are an even number of 1's on the inputs to the gate.

Symbol of the XNOR Gate:



Logic Equation: $Y = A \odot B = \overline{A \oplus B} = \bar{A}B + AB$

Truth Table:

A	B	$A \odot B = \overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

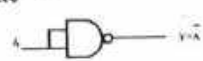
1.5 Universal Gates

By this gate all the Boolean function can be implemented i.e. all the digital system can be implemented.

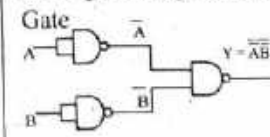
NAND Gate and NOR Gate are two universal gates.

NAND Gate as a Universal Gate

NOT Gate using NAND Gate:

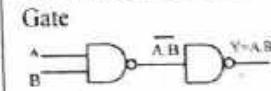


AND gate using NAND Gate:



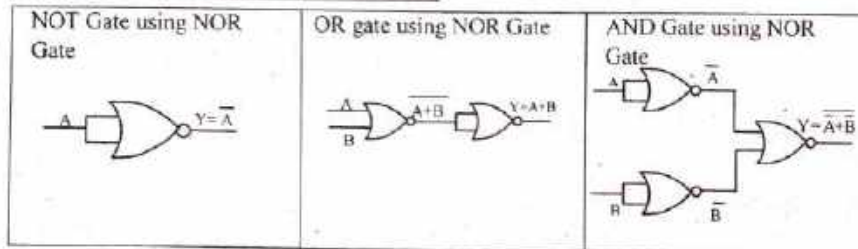
$Y = \overline{\overline{A \cdot B} \cdot \overline{A \cdot B}} = A \cdot B$

OR Gate using NAND Gate:



$Y = \overline{\overline{A \cdot B} \cdot \overline{A \cdot B}} = \overline{\overline{A \cdot B}} = A \cdot B$

1.6 NOR Gate as a Universal Gate



1.7 Boolean Algebra

Boolean algebra is an algebraic logic which deals with logic variable. This logic variable having only two values 1 and 0 alternatively "TRUE" or FALSE".

Basic Definitions

• Boolean constants

Boolean constant have only 2 value i.e., 0 and 1 or True and False.

• Boolean Variable

Boolean variables are same as ordinary algebra i.e. X, Y, Z etc, which represent real numbers. Boolean variable having only two values i.e. 0 and 1.

1.8 Boolean Operation (Boolean Theorems & Identification)

Several operation like AND, OR, NOT, XOR, XNOR etc. which is known as Boolean function.

1.8.1 Commutative law

1. $A+B = B+A$
2. $A.B = B.A$
3. $A \oplus B = B \oplus A$
4. $A \odot B = B \odot A$

A binary operation is **commutative** if changing the order of the operands does not change the end result.

1.8.2 Associative law

1. $A+(B+C) = (A+B)+C$
2. $A.(B.C) = (A.B).C$
3. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

A binary operation is **associative** if the order in which the operations are performed does not matter as long as the sequence of the operands is not changed. That is, rearranging the parentheses in such an expression will not change its value.

Note: NAND & NOR function are not associative.

1.8.3 Distributive law

1. $(B+C).A = A.B+A.C$
2. $A+(B.C) = (A+B).(A+C)$

$a(b+c) = ab + ac$ This is called the **Distributive Law**. **Distributivity** is a property of binary operations that generalizes the **distributive law**.

1.8.4 Double Negation law

$$1. \overline{\overline{A}} = A$$

A double negative occurs when two forms of negation are used in the same value.

1.8.5 Identity law

1. $A.A = A$
2. $A+A = A$
3. $A+\overline{A} = 1$
4. $A.\overline{A} = 0$

1.8.6 Redundancy law

1. $A+A.B = A$
2. $A.(A+B) = A$

1.9 Demorgan's Theorem

Statement 1:

It is state that complement of the sum of variable is equal to the product of the complements of the variable.

Proof: i.e. $\overline{A+B} = \overline{A}.\overline{B}$

A	B	$\overline{A}$	$\overline{B}$	$A+B$	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

From the truth table we see that $\overline{A+B} = \overline{A}.\overline{B}$

Statement 2:

It state that the complement of the product of variable is equal to the sum of the complements of the variable.

Proof: i.e. $\overline{A.B} = \overline{A}+\overline{B}$

A	B	$\overline{A}$	$\overline{B}$	$A.B$	$\overline{A.B}$	$\overline{A}+\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

From the truth table we see that $\overline{A.B} = \overline{A}+\overline{B}$

The various standard forms for logical functions are:-

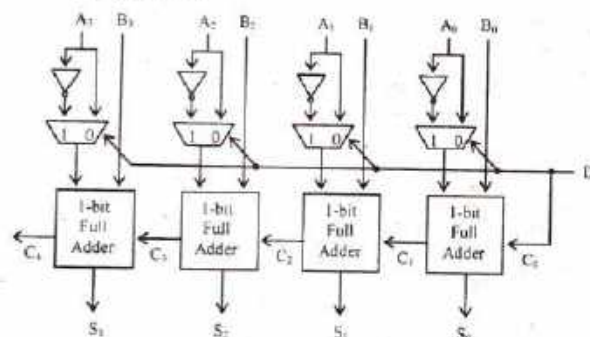
- (i) Standard Sum of Product Form.
- (ii) Standard Product of Sum Form.

1.10 Addition/Subtraction unit block diagram and function

Addition and subtraction are similar algorithms. Taking a look at subtraction, we can see that:

$$a - b = a + (-b)$$

Using this simple relationship, we can see that addition and subtraction can be performed using the same hardware. Using this setup, however, care must be taken to invert the value of the second operand if we are performing subtraction. Note also that in two's-complement arithmetic, the value of the second operand must not only be inverted, but 1 must be added to it. For this reason, when performing subtraction, the carry input into the LSB should be a 1 and not a zero.



Steps for adding and subtracting numbers in signed 2's complement representation with the help of flowcharts

Steps for addition

Step 1: The augend is put in AC and the addend is put in BR.

Step 2: If any of the number is negative, then its 2's complement representation must be considered.

Step 3: The sum is obtained by adding the contents of AC and BR (including their sign-bits) in the parallel adder and complement block.

Step 4: The result is stored in AC. If the XOR of the last two carries is 1, then there is a carryout and the overflow bit V is set to 1 (else it is cleared to 0).

Step 5: If there is an overflow then the carryout bit must be discarded. If the result is negative then its 2's complement must be considered to get the actual positive number and then its negative integer representation is considered as the ultimate result.

Steps for subtraction

Step 1: The minuend is put in AC and the subtrahend is put in BR.

Step 2: If any of the number is negative, then its 2's complement representation must be considered. Also the subtraction operation is accomplished by adding the content of AC

to the 2's complement of B. This is because, taking the 2's complement of B, has the effect of changing a negative number to positive and a positive number to negative.

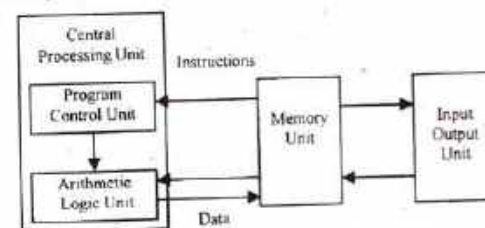
Step 3: The result is obtained by performing the computation in the parallel adder and complementer block.

Step 4: The result is stored in AC. An overflow is checked similarly.

Step 5: The carryout bit must be discarded provided there is an overflow.

1.11 Components of Computer system – Structure of CPU, function of Memory unit and IO unit

- Central Processing Unit (CPU)
- Memory Unit
- I/O (input-output) Unit



Structure and Functions of the Different Units of a Digital Computer

(a) Memory Unit

Its purpose is to store both instructions & data. It is also called the Random-Access Memory (RAM) because the CPU can access any memory location at random.

(b) CPU

It acts as the brain of the computer and performs the bulk of data processing operations in a computer. The two main units of a CPU are the Arithmetic Logic Unit and the Program Control Unit. The important parts of CPU are:

- Arithmetic Logic Unit (ALU):** It performs instructions related to arithmetic operations like ADD, SUB, MUL etc. and logical operations like AND, OR etc.
- Program Control Unit (PCU):** It interprets & sequences instructions i.e., interprets & sequences which instruction in a program is to be executed first.
- Register Sets:** These are collections of registers that store data.

(c) Input-Output (I/O) Unit

This unit provides an efficient mode of communication between the central system (computer) & the outside environment. Through the I/O unit, programs & data must be entered into computer memory for processing & results obtained from computations must be recorded or displayed to the user.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. A hard disc is divided into tracks, which are further sub divided into [WBSCTE 2022]
(a) Clusters (b) Sectors (c) Vectors (d) Heads

Answer: (b)

2. Binary equivalent of 97_{10} is [WBSCTE 2022]
(a) 1100000 (b) 1100001 (c) 1100010 (d) 1100011

Answer: (b)

3. Portion of information processing cycle in which computer acquires data from some source is called [WBSCTE 2022]
(a) Input (b) Output (c) Processing (d) Storage

Answer: (a)

4. The expression $Y = (A+B)(B+C)(C+A)$ shows the _____ operation. [WBSCTE 2022]
(a) AND (b) POS (c) SOP (d) NAND

Answer: (b)

5. Type of software used to create slideshow is [WBSCTE 2022]
(a) Web design software (b) Presentation software
(c) Word-processing software (d) Spreadsheet software

Answer: (b)

6. Smart phone is an example of [WBSCTE 2022]
(a) Personal computer (b) Analog computer
(c) Mainframe computer (d) Super computer

Answer: (d)

7. Top listed super computer is [WBSCTE 2022]

POLY-ITS

- (a) Sunway taihulight (b) Sequoia (c) Cori (d) Mira

Answer: (a)

8. The three main components of a digital computer system are – [Model Question]
(a) Memory, IO, DMA (b) ALU, CPU, Memory
(c) Control Unit, ALU, Register (d) CPU, Memory, IO

Answer: (c)

9. The dual of the Boolean theorem $A.(B+C)$ is – [Model Question]
(a) $A + (B.C) = A.B + A.C$ (b) $A.(B+C) = (A+B).(A+C)$
(c) $A + (B.C) = (A+B).(A+C)$ (d) none of these

Answer: (c)

10. The universal gate is- [Model Question]
(a) NAND gate (b) OR gate
(c) AND gate (d) None of the above

Answer: (a)

11. The inverter is - [Model Question]
(a) NOT gate (b) OR gate
(c) AND gate (d) None of the above

Answer: (a)

12. The inputs of a NAND gate are connected together. The resulting circuit is – [Model Question]
(a) OR gate (b) AND gate
(c) NOT gate (d) None of the above

Answer: (d)

13. The NOR gate is OR gate followed by – [Model Question]
(a) AND gate (b) NAND gate
(c) NOT gate (d) None of the above

Answer: (c)

14. The NAND gate is AND gate followed by [Model Question]
(a) NOT gate (b) OR gate
(c) AND gate (d) None of the above

Answer: (a)

15. When signed numbers are used in binary arithmetic, then which one of the following notations would have unique representation of zero –

- (a) magnitude (b) 1's complement (c) 2's complement (d) none

[Model Question]

Answer: (a)

16. A logic circuit that provides a HIGH output for both inputs HIGH or both inputs LOW is a(n) –

- (a) EX-NOR gate (b) OR gate (c) EX-OR gate (d) NAND gate

[Model Question]

Answer: (a)

17. A logic circuit that provides a HIGH output if one input or the other input, but not both, is a HIGH, is a(n)

- (a) EX-NOR gate (b) OR gate (c) EX-OR gate (d) NAND gate

[Model Question]

Answer: (c)

18. Identify the type of gate below from the equation $X = A \oplus B = \overline{AB} + A\overline{B}$ –

- (a) EX-NOR gate (b) OR gate (c) EX-OR gate (d) NAND gate

[Model Question]

Answer: (c)

19. Which type of gate can be used to add two bits?

- (a) Ex-OR (b) Ex-NOR (c) Ex-NAND (d) NOR

[Model Question]

Answer: (a)

20. Why is an exclusive-NOR gate also called an equality gate?

- (a) The output is false if the inputs are equal
(b) The output is true if the inputs are opposite
(c) The output is true if the inputs are equal

[Model Question]

Answer: (b)

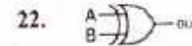
21. A NAND gate is called a universal gate because –

- (a) many computers use this gate
(b) any function can be realized by this gate
(c) all minimization techniques are applicable to this gate

[Model Question]

Answer: (b)

[Model Question]



This figure is the symbol of a –

- (a) OR gate, (b) NOR gate, (c) XOR gate.

Answer: (c)

23. The minimum number of NAND gates required to realize a XOR gate is –

- (a) 4 (b) 5 (c) 6

[Model Question]

Answer: (b)

24. The first contribution to logic was made by –

- (a) George Boole, (b) Copernicus, (c) Aristotle, (d) Shannon.

[Model Question]

Answer: (a)

25. Which logic gate is similar to the function of two series switches?

- (a) AND (b) OR (c) XOR (d) NAND

[Model Question]

Answer: (a)

26. The minimum number of NAND gates required to implement $A + AB' + AB'C$ is equal to –

- (a) 0 (b) 1 (c) 2 (d) 4

[Model Question]

Answer: (a)

27. The Boolean expression $AB + AB' + AC + AC'$ is unaffected by the value of the Boolean variable –

- (a) A (b) B (c) C (d) all of these

[Model Question]

Answer: (d)

28. $A+B$ can be implemented by using –

- (a) AND gates only (b) XOR gates only
(c) NAND gates only (d) none of the above

[Model Question]

Answer: (c)

29. The dual of $x.0=0$ is –

- (a) $x+1=1$ (b) $x+0=x$ (c) $x+x=x$ (d) $x.0=0$

[Model Question]

Answer: (a)

30. Which of the following gate can be used as a controlled inverter? [Model Question]
 (a) AND (b) OR (c) XOR (d) NAND

Answer: (c)

31. Any combinational circuit can be designed using - [Model Question]
 (a) only NAND (b) AND and OR (c) only AND

Answer: (a)

32. Exponent in floating point number can be represented by - [Model Question]
 (a) Sign-magnitude form, (b) 1's complemented from,
 (c) 2's complemented from, (d) biased form.

Answer: (a)

33. A subtractor is not usually present in a computer because - [Model Question]
 (a) It is expensive (b) It is not possible to design it
 (c) The adder will take care of subtraction (d) None of the above

Answer: (c)

34. Processors of all computers, whether micro, mini or mainframe must have - [Model Question]
 (a) ALU (b) Primary Storage (c) Control unit (d) All of these

Answer: (d)

35. Processors of all computers, whether micro, mini or mainframe must have [Model Question]
 (a) ALU (b) Primary storage
 (c) Control Unit (d) all of these

Answer: (d)

36. A group of 4 bits is known as - [Model Question]
 (a) a nibble (b) a byte (c) a bit (d) an octal number

Answer: (a)

37. The binary equivalent of the hexadecimal number AOB5 is - [Model Question]
 (a) 10100001010 (b) 1001111101001010
 (c) 1010000010110101 (d) 1011000011000101

Answer: (c)

38. Decimal number 13 is represented in natural BCD as - [Model Question]
 (a) 1101 (b) 00010011 (c) 00001101 (d) 00011101

Answer: (a)

39. The gate ideally suited for bit comparison is a - [Model Question]
 (a) two input X-NOR gate (b) two input X-OR gate
 (c) two input NOR gate (d) two input NAND gate

Answer: (a)

40. The binary equivalent of 576 is - [Model Question]
 (a) 110000000, (b) 1000000100, (c) 1001000000

Answer: (c)

41. Which of the following 4-bit combination is invalid BCD? - [Model Question]
 (a) 0101 (b) 1010 (c) 1001 (d) 0011

Answer: (b)

42. The expression $(\bar{A} + AB)$ can be written as - [Model Question]
 (i) $\bar{A}$ (b) B (c) $\bar{A} + B$ (d) A

Answer: (c)

43. The dual of $(AB + CD)$: [Model Question]
 (a) $(A + B)(C + D)$ (b) $(A + B + C + D)$

Answer: (a)

44. Laws of tautology states - [Model Question]
 (a) $A.A = A$ and $A = \bar{A}$ (b) $\bar{A}.A = A$ and $A + A = \bar{A}$
 (c) $A.A = A$ and $A + A = A$

Answer: (c)

45. Minimization of Boolean logical expressions helps to reduce - [Model Question]
 (a) Space (b) Cost (c) Number of gate (d) All of these

Answer: (d)

46. Which of the following does not represent Boolean algebra operation? [Model Question]
 (a) AND (b) OR (c) NOT (d) XOR

Answer: (d)

47. Tri-state buffer has - [Model Question]
 (a) 3 output states (b) 2 output states (c) 1 output states (d) no output states

Answer: (a)

48. Reflected code is -

- (a) gray code (b) binary code

[Model Question]

Answer: (a)

49. In octal system the value 2^6 is -

- (a) 20 (b) 40 (c) 31 (d) 100

[Model Question]

Answer: (d)

50. In binary code, shifting a register to left by one bit position is equivalent to -

- (a) subtraction by 2 (b) addition by 2 [Model Question]
(c) division by 2 (d) multiplication by 2

Answer: (c)

B. State whether the following statements are True or False:

51. $A + (B.C) = (A + B) (A + C)$.

[Model Question]

Answer: True

52. $A + AB = A$

[Model Question]

Answer: True

53. $\overline{A+B+C} = \overline{A}.\overline{B}.\overline{C}$.

[Model Question]

Answer: False

54. According to Boolean laws $A + A = 1$

[Model Question]

Answer: False

55. The parity of the binary number 11010110 is odd.

[Model Question]

Answer: True

56. The dual of $A + 1 = 1$ is $A' + 0 = 0$

[Model Question]

Answer: False

[dual is $A.0=0$]

57. In 2's complement representation, MSB is the sign bit.

[Model Question]

Answer: True

58. Decimal number 90 equals 01011010 in 2's complement

[Model Question]

Answer: True

59. A 2's complement adder can add and subtract.

[Model Question]

Answer: True

60. The binary equivalent of 576 is 1100000000

[Model Question]

Answer: False

61. The binary equivalent of 247 is 110000010.

[Model Question]

Answer: True

62. A 4-variable Karnaugh map for SOP form has 16 cells.

[Model Question]

Answer: True

63. In a Karnaugh map, adjacent pair eliminates single variable.

[Model Question]

Answer: True

64. Max-term expression directly yields OR-AND logic circuit.

[Model Question]

Answer: True

65. A NOR gate is equivalent to bubble input AND gate.

[Model Question]

Answer: True

66. A NAND gate is equivalent to bubble input OR gate.

[Model Question]

Answer: True

67. If any input is low in an AND gate, the output is high.

[Model Question]

Answer: False

68. In a NOT gate the output is equal to magnitude to input.

[Model Question]

Answer: False

69. A two input XNOR gate recognizes these words which has similar inputs.

[Model Question]

Answer: True

70. XOR gate is used as a controlled inverter.

[Model Question]

Answer: True

71. NAND and NOR gates are called universal gates.

[Model Question]

Answer: True

72. Universal gates consume more power than other gates.

[Model Question]

Answer: False

73. XOR gate is used both for parity generating and checking purposes

[Model Question]

Answer: True

74. In a XOR gate, if odd number of inputs are high, the output will be high.

[Model Question]

Answer: False

75. A controlled inverter transmits a binary digit or its complement.

[Model Question]

Answer: True

76. In an AND gate, if any input is low, the output is high.

[Model Question]

Answer: False

77. In a NOT gate, the output always follows the input.

[Model Question]

Answer: False

78. An OR operation in negative logic is equivalent to AND operation in positive logic

[Model Question]

Answer: True

C. Fill in the blanks in the following statements:

79. 1 nibble = 4 bits?

[Model Question]

80. $(E5)_{16} = (229)_{10}$

[Model Question]

Hints:

$$E5_{16} = (E \times 16) + (5 \times 1) = (14 \times 16) + (5 \times 1) = 224 + 5 = 229$$

D. Answer the following questions:

81. What is binary equivalent of $(24.625)_{10}$?

[Model Question]

Answer:

Conversion of $(24.625)_{10}$ to binary

$$24/2 = 12 + \text{remainder } 0 \quad 0 \text{ (Least Significant Bit)}$$

$$12/2 = 6 + \text{remainder } 0 \quad 0$$

$$6/2 = 3 + \text{remainder } 0 \quad 0$$

$$\begin{array}{rcl} 3/2 & = & 1 + \text{remainder } 1 \\ 1/2 & = & 0 + \text{remainder } 1 \quad 1 \text{ (Most Significant Bit)} \\ \text{Result } 24_{10} & = & 11000_2 \end{array}$$

Fraction Number	Product	Fractional Part	Integer Part
0.625	$0.625 \times 2 = 1.25$	0.25	1 MSB
0.25	$0.25 \times 2 = 0.5$	0.5	0
0.5	$0.5 \times 2 = 1$	0	1 LSB

$(24.625)_{10}$ is equal to $(11000.101)_2$

82. Simplify the following logic expression using De-Morgan's rule $Y = (A+B+AB)$?

[Model Question]

Answer:

$$Y = A+B+AB$$

Applying DeMorgan's rule, we get

$$Y = A+B+AB$$

$$\Rightarrow = \overline{\overline{A} \cdot \overline{B} \cdot \overline{AB}}$$

$$\Rightarrow = \overline{\overline{A} \cdot \overline{B} \cdot (\overline{A+B})}$$

$$\Rightarrow = \overline{(\overline{A} \cdot \overline{B} \cdot \overline{A}) + (\overline{A} \cdot \overline{B} \cdot \overline{B})}$$

$$\Rightarrow = \overline{(\overline{A} \cdot \overline{B}) + (\overline{A} \cdot \overline{B})}$$

$$\Rightarrow = \overline{(\overline{A} \cdot \overline{B}) + (\overline{A} \cdot \overline{B})}$$

$$\Rightarrow = \overline{(\overline{A} \cdot \overline{B})}$$

$$\Rightarrow = A+B$$

83. For a three or four variable Karnaugh map, what is the code used for column and row headings?

[Model Question]

Answer: Gray code

84. Which code is used in Karnaugh map simplification?

[Model Question]

Answer: Gray code

85. For 8-variable K-map, how many numbers of cells are created?

[Model Question]

Answer: $2^8 = 256$

86. What is an intranet? What is the Internet?

[Model Question]

Answer:

When two or more networks are connected, they become an internetwork, or internet. The most notable internet is called the Internet, a collaboration of more than hundreds of thousands of interconnected networks. Private individuals as well as various organizations such as government agencies, schools, research facilities, corporation, and libraries in more than 100 countries use the internet.

87. Describe networking.
[Model Question]
Answer:

Networking refers to the inter connection between computers and peripherals for data communication. Networking can be done using wired cabling or through wireless link.

88. Differentiate among Internet, Intranet and Extranet.
[Model Question]
Answer:

- Intranet is shared content accessed by members within a single organization.
- Extranet is shared content accessed by groups through cross-enterprise boundaries.
- Internet is global communication accessed through the Web.

89. Discuss binary number system.
[Model Question]
Answer:

In mathematics and digital electronics, a binary number is a number expressed in the binary numeral system, or base-2 numeral system, which represents numeric values using two different symbols: typically 0 (zero) and 1 (one). More specifically, the usual base-2 system is a positional notation with a radix of 2. Because of its straightforward implementation in digital electronic circuitry using logic gates, the binary system is used internally by almost all modern computers and computer-based devices such as mobile phones.

90. What is the unique characteristic of NAND & NOR gate?
[Model Question]
Answer:

NAND & NOR gates are the universal gates, i.e. any logic can be designed by any one type of those gates.

91. Which gate does perform controlled inverter operation?
[Model Question]
Answer:

XOR gates performs the operation of the controlled inverter.

92. Which type of gate can be used to add two bits?
[Model Question]
Answer:

OR gates can be used to add two bits.

93. Name the CPU registers.
[Model Question]
Answer:

Actually A Register in CPU terminology is small Named chunk of memory Available inside a microprocessor (CPU). Registers have specific names, sizes and functions varies from processor to processor, for example, if take 8085 microprocessor is an 8-bit processor which has 8-bit Registers(A:Accumulator, B,C,D,E, H, and L Registers and one flag register all are 8-bit). Two 16-bit registers PC and SP all are having special function and functions comes into the picture during assembly programming. Few registers control is beyond the programmer. If you take another processor Registers will vary let say 8086 is 16-bit processor and it has AX, BX, CX and DX all are 16-bit, PC, SP and Flag registers.

Long Answer Type Questions

➤ 1. (a) What are the meant by 1's and 2's complements of a binary number? Add the following numbers using the 2's complement method:

(i) -48 and +31

(ii) -64 and +46

(b) Name the gates that are used as universal gates. Explain with example.

[WBSCTE 2022]
Answer:

(a) **1's complement** of a binary number is another binary number obtained by toggling all bits in it, i.e., transforming the 0 bit to 1 and the 1 bit to 0. In the 1's complement format, the positive numbers remain unchanged. The negative numbers are obtained by taking the 1's complement of positive counterparts.

For example +9 will be represented as 00001001 in eight-bit notation and -9 will be represented as 11110110, which is the 1's complement of 00001001.

2's complement of a binary number is 1, added to the 1's complement of the binary number. In the 2's complement representation of binary numbers, the MSB represents the sign with a '0' used for plus sign and a '1' used for a minus sign. the remaining bits are used for representing magnitude. positive magnitudes are represented in the same way as in the case of sign-bit or 1's complement representation. Negative magnitudes are represented by the 2's complement of their positive counterparts.

Examples:

2's complement of "0111" is "1001"

2's complement of "1100" is "0100"

i)

$$(-48) = (001111)$$

$$(31) = (011111)$$

+

100010 $\rightarrow$ 2's complement

∴ Answer is $\rightarrow (100010)$

ii)

$$(-64) = (100001)$$

$$(45) = (101110)$$

10011 $\rightarrow$ 2's complement

∴ Answer is $\rightarrow (10011)$

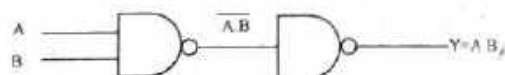
(b) **NAND Gate and NOR Gate are two universal gates.**

NAND Gate as a Universal Gate

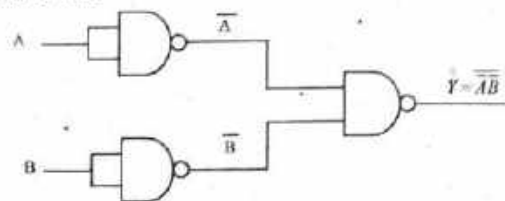
NOT Gate using NAND Gate



AND gate using NAND Gate



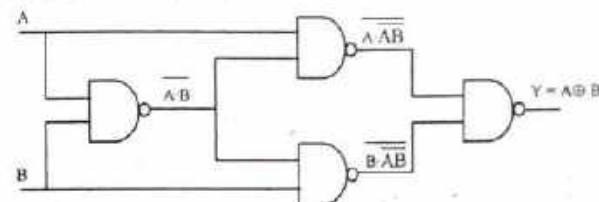
OR Gate using NAND Gate



Using Demorgan's theorem

$$Y = \overline{A \cdot B} = \overline{A} + \overline{B} = A + B$$

EX – OR Gate using NAND Gate



$$Y = \overline{\overline{A, AB, B, AB}}$$

Using Demorgan's 2nd theorem

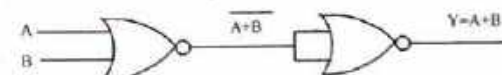
or, $Y = A.\overline{AB} + B.\overline{AB} = A(\overline{A} + \overline{B}) + B(\overline{A} + \overline{B}) = A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B} = \overline{AB} + \overline{AB} = A \oplus B$

NOR Gate as a Universal Gate

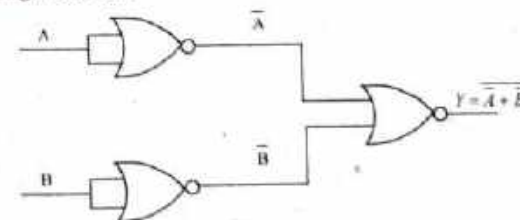
NOT Gate using NOR Gate



OR Gate using NOR Gate



AND Gate using NOR Gate



Using Demorgan's 1st theorem

$$Y = \overline{\overline{A} + \overline{B}}$$

$$Y = \overline{A} \cdot \overline{B}$$

$$Y = A \cdot B$$

2. Write short note on type of software.

[WBSCTE 2022]

Answer:

Among the various categories of software, the most common types include the following:

• **Application software:**

The most common type of software, application software is a computer software package that performs a specific function for a user, or in some cases, for another application. An application can be self-contained, or it can be a group of programs that run the application for the user. Examples of modern applications include office suites, graphics software, databases and database management programs, web browsers, word processors, software development tools, image editors and communication platforms.

• **System software:**

These software programs are designed to run a computer's application programs and hardware. System software coordinates the activities and functions of the hardware and software. In addition, it controls the operations of the computer hardware and provides an environment or platform for all the other types of software to work in. The OS is the best example of system software; it manages all the other computer programs. Other examples of system software include the firmware, computer language translators and system utilities.

• **Driver software:**

It is also known as device drivers, this software is often considered a type of system software. Device drivers control the devices and peripherals connected to a computer, enabling them to perform their specific tasks. Every device that is connected to a computer needs at least one device driver to function. Examples include software that comes with any nonstandard hardware, including special game controllers, as well as the software that enables standard hardware, such as USB storage devices, keyboards, headphones and printers.

• **Middleware:**

The term *middleware* describes software that mediates between application and system software or between two different kinds of application software. For example, middleware enables Microsoft Windows to talk to Excel and Word. It is also used to send a remote work request from an application in a computer that has one kind of OS, to an application in a computer with a different OS. It also enables newer applications to work with legacy ones.

• **Programming software:**

Computer programmers use programming software to write code. Programming software and programming tools enable developers to develop, write, test and debug other

software programs. Examples of programming software include assemblers, compilers, debuggers and interpreters.

3. (a) Draw the logic diagram of an Ex-OR gate and discuss its operation.

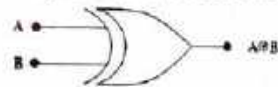
(b) Prove that:

[WBSCTE 2022]

$$\overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C + A\overline{B}\overline{C} = C$$

Answer:

(a) The exclusive-OR or XOR gate has two or more inputs. In general, an XOR gate gives an output value of 1 when there are an odd number of 1's on the inputs to the gate.

Symbol of the XOR Gate		Truth Table:		
		A	B	A ⊕ B
Logic Equation: $Y = A \oplus B = \overline{A}B + A\overline{B}$		0	0	0
		0	1	1
		1	0	1
		1	1	0

$$\begin{aligned}
 (b) \quad & \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C + A\overline{B}\overline{C} \quad (\text{L.H.S.}) \\
 &= \overline{A}C(\overline{B} + B) + A\overline{C}(\overline{B} + B) \\
 &= \overline{A}C \cdot 1 + A\overline{C} \cdot 1 \quad [\because A + \overline{A} = 1, B + \overline{B} = 1] \\
 &= \overline{A}C + A\overline{C} = C = \text{R.H.S.}
 \end{aligned}$$

$$\therefore \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C + A\overline{B}\overline{C} = C \quad [\text{Proved}]$$

4. (a) What is a gray code? Why is it important? Express the following number in Gray code from: i) 15 ii) 27 [WBSCTE 2022]

(b) Prove that:

$$AB + \overline{A}C + \overline{A}BC(AB + C) = 1$$

Answer:

(a) The **Gray Code** is a sequence of binary number systems, which is also known as reflected binary code. It is a non-weighted code. In this code a decimal number is represented in Binary form in such a way such that each gray code number differs from the preceding and succeeding number by a single bit. This code is a reflected code.

Presuming the question is about Gray Codes, aka reflected binary code, the primary benefit is that it requires only a single binary digit to change when incrementing or decrementing a binary number. That is, the Grey Code representation of X (base 10) has N binary digits in it. The Grey Code representation of either X-1 or X+1 requires only a single binary digit to change, out of the N digits, which make up the number. Normal

binary code can require up to all N digits to change when adding or subtracting a single increment. The advantage here is cleanliness, in particular for mechanical and electrical circuits.

Decimal Number	Binary Number	Gray code
15	1111	1000
27	11011	10110

(b) L.H.S., $AB + \overline{AC} + \overline{ABC}(AB + C)$

$$\Rightarrow AB + \overline{AC} + \overline{ABC} \cdot AB + \overline{ABC} \cdot C$$

$$\Rightarrow AB + \overline{A} + \overline{C} + \overline{ABC} \quad [(\overline{AB}) + \overline{A} + \overline{B} \text{ FROM De-Morgan's } B \cdot \overline{B} = C]$$

$$\overline{A} + B + \overline{C} + \overline{AB}$$

$$= \overline{A} + \overline{AB} + B + \overline{C} \quad \overline{A} + AB = \overline{A} + B \text{ (Redundant Literal rule) } 1 + A = 1]$$

$$= \overline{A} + (B + \overline{B}) + \overline{C} = \overline{A} + \overline{C} + 1 = 1 = \text{R.H.S.}$$

5. What are address, data and control bus?

[Model Question]

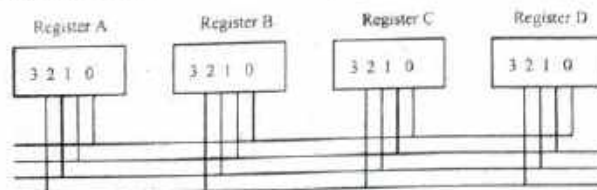
Answer:

Data Lines: These lines provide a path for moving data between registers. These are designed to transmit all bits of an n-bit word in parallel. So, they either consist of two sets of n-unidirectional lines or a single set of n-bi-directional lines.

Data lines are collectively called the **data bus**. This bus of size 'n' is usually a multiple of 8, with n = 8, 16 or 32 etc. separate lines. Since each line can carry only 1 bit at a time, the number of lines determines how many bits can be transferred at a time.

Example with Diagram

Let us now consider an example. In fig, there are four registers A, B, C and D with four bits (3, 2, 1, 0) each connected by a common (shared) bus system.



As each line of the data bus can carry one bit at a time, so the data bus must have **either** two sets of 4 unidirectional lines [i.e. $2 \times (n=4) = 8$] or 8 lines, out of which one set (i.e. 4 lines) will transmit data in the forward direction & the other set will transmit data in the

reverse direction. **Otherwise**, the data bus will have 4 (i.e. 1 set) bi-directional lines. Number of bi-directional lines in the data bus = number of bits in each register.

Address Lines: Address lines of the bus are used to transfer the addresses of the source register sending the data and the destination register receiving the data. Address lines are collectively called the **address bus**.

Example

Consider Fig. Suppose register A is sending data to register C. So via the address lines of the bus, both the sender's address (i.e. register A) and the receiver's address (i.e. register C) are transmitted to let all the registers sharing the bus identify the sender as well as the receiver of the data.

Control Lines: These lines, transmitting control signals, are used to control the access to the data & address lines i.e. control lines select which register, out of the multiple registers, will transmit its information through the bus.

Also control lines are used to transfer timing signals & status information about the registers in the system as well as to indicate the type of information present on the data lines.

Since all registers share the data & address lines, there must be means of controlling their use, which is done by the control lines.

Example

Fig. shows the control lines of the shared bus among the 4 registers.

6. Perform the operation (20) – (5) by using 2's complement.

[Model Question]

Answer:

Let us assume given number is represented by 8-bit representation.

20 is given by 00010100 while -5 in its two's complement representation is given as 11111011. On addition of these two numbers the result obtained is 11101110 or -18 (the two's complement form of the result is 00010010 which is binary equivalent of + 18. hence -18 is the result).

Converting decimal to binary,

$$(20)_{10} = (00010100)_2$$

$$(5)_{10} = (00000101)_2$$

$$\text{Again 2's complement of } (00000101)_2 = (11111011)_2$$

Adding both numbers,

$$\text{Ignoring the carry bit we get } (00001111)_2 = (15)_{10}$$

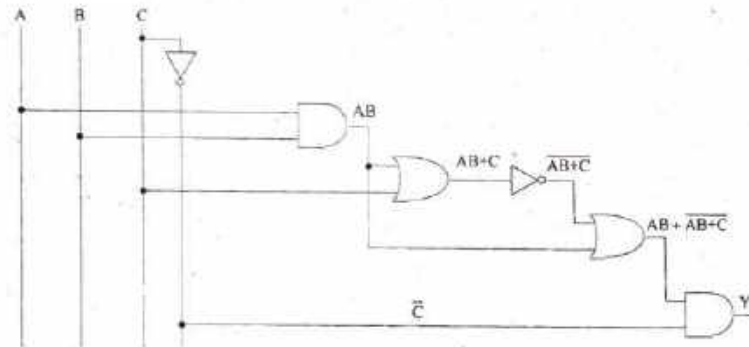
7. Realize the Boolean expression using basic gates.

[Model Question]

$$Y = \overline{C}(AB + \overline{AB} + C)$$

Answer:

$$Y = \overline{C}(AB + \overline{AB} + C)$$



8. Prove the following:

(i) $A + \overline{A}.B + A.\overline{B} = A + B$

(ii) $(A + B)(\overline{A} + C) = AC + \overline{A}B$

Answer:

(i) LHS : $(A + \overline{A})(A + B) + A\overline{B}$ [By distributive law]

$$\Rightarrow (A+B) + A\overline{B}, \text{ Since } A + \overline{A} = 1$$

$$\Rightarrow A + B + A\overline{B}$$

$$\Rightarrow A + A\overline{B} + B$$

$$\Rightarrow A(1 + \overline{B}) + B [\because 1 + \overline{B} = 1]$$

$$\Rightarrow A + B = RHS$$

(ii) LHS : $A\overline{A} + \overline{A}B + AC + BC$

$$\overline{A}B + AC + BC [\because A.\overline{A} = 0]$$

$$\overline{A}B + AC + BC(\overline{A} + A)$$

$$\overline{A}B + \overline{A}BC + AC + ABC$$

$$\overline{A}B + \overline{A}BC + AC + ABC$$

$$\overline{A}B(1 + C) + AC(B + 1)$$

$$\overline{A}B + AC = RHS$$

 9. Given the logic equation $F = ABC + \overline{BC}D + \overline{A}BC$. Make a truth table for the same.

[Model Question]

POLY-ITS

Answer:

$$F = ABC + \overline{BC}D + \overline{A}BC$$

Put the value of A,B,C from the truth table and get the output F

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

10. Prove the following:

[Model Question]

(i) $(AB + C + D)(\overline{C} + D)(\overline{C} + D + E) = ABC\overline{C} + D$

Answer:

$$(AB + C + D)(\overline{C} + D)(\overline{C} + D + E)$$

$$= (ABC\overline{C} + C\overline{C} + \overline{C}D + ABD + CD + D)(\overline{C} + D + E) \quad [\because C\overline{C} = 0 \text{ \& } ABD + CD + D = D]$$

$$= (ABC\overline{C} + \overline{C}D + D)(\overline{C} + D + E)$$

$$= \{ABC\overline{C} + D(\overline{C} + 1)\}(\overline{C} + D + E)$$

$$= (ABC\overline{C} + D)(\overline{C} + D + E)$$

$$= ABC\overline{C} + \overline{C}D + ABC\overline{C}D + D + ABC\overline{C}E + ED$$

$$= ABC\overline{C}(1 + D + E) + D(\overline{C} + 1 + E)$$

$$= ABC\overline{C} + D \quad [\because 1 + D + E + 1C + E + 1 = 1]$$

$$= RHS$$

(ii) $\overline{A}BC + \overline{A}BC + \overline{A}BC + \overline{A}BC = AB + BC + CA$

Answer:

$$\overline{A}BC + \overline{A}BC + \overline{A}BC + \overline{A}BC$$

$$\begin{aligned}
 &= \overline{A}BC + A\overline{B}C + ABC\overline{C} + ABC + ABC + ABC \\
 &\quad [\because ABC + ABC + ABC = ABC] \\
 &= ABC + A\overline{B}C + ABC + \overline{A}BC + ABC + A\overline{B}C \\
 &= AB(C + \overline{C}) + BC(A + \overline{A}) + AC(B + \overline{B}) [\because C + \overline{C} = 1, A + \overline{A} = 1 \& \\
 &= AB + BC + CA = R.H.S. \quad [B + \overline{B} = 1]
 \end{aligned}$$

11. (a) State Demorgans's theorem.

(b) Make a Karnaugh map of the function $F(A, B, C, D) = \sum m(5, 8, 9, 11, 13, 15) + d(7, 12)$ and

(c) Explain the applications of parity bit.

[Model Question]

Answer:

(a) De Morgan has suggested two theorems which are extremely useful in Boolean Algebra. The two theorems are discussed below.

Theorem 1

The left hand side (LHS) of this theorem represents a NAND gate with inputs A and B, whereas the right hand side (RHS) of the theorem represents an OR gate with inverted inputs.

Theorem 2

The LHS of this theorem represents a NOR gate with inputs A and B, whereas the RHS represents an AND gate with inverted inputs.

(b)

AB \ CD				
	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	2	3	15	14
10	8	9	11	10

$$\begin{aligned}
 Y &= A\overline{C} + AD + BD \\
 &= A(\overline{C} + D) + BD
 \end{aligned}$$

(c) A parity bit, or check bit, is a bit added to a string of binary code that indicates whether the number of 1-bits in the string is even or odd. Parity bits are used as the simplest form of error detecting code. There are two variants of parity bits: even parity bit and odd parity bit.

12. Implement the following function by NAND gates only.

[Model Question]

$$Y = A\overline{B}C + AB\overline{C} + A$$

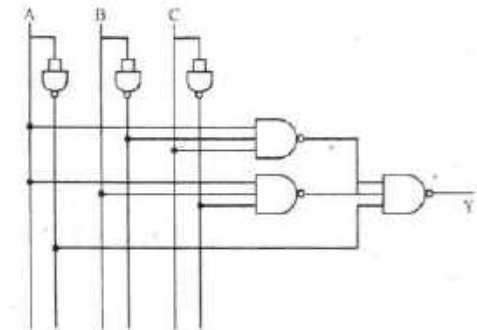
Answer:

We know that $\overline{x} \cdot \overline{y} = \overline{x + y} = X + Y$

So, $Y = A\overline{B}C + AB\overline{C} + A$

$$= A\overline{B}C + AB\overline{C} + A$$

$$= \overline{A\overline{B}C} \cdot \overline{AB\overline{C}} \cdot \overline{A}$$



13. Show that a positive logic NAND operator is equivalent to a negative logic NOR operation and vice versa.

[Model Question]

Answer:

In the positive logic low voltage signal is represented by 0 & high voltage signal is represented by 1 but in negative logic high voltage signal is represented by 0 & low voltage signal is represented by 1.

The truth table of positive logic NAND gate is: The truth table of negative logic NOR gate is:

A	B	$Y_{NAND} = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$Y = \frac{NOR}{A+B}$
1	1	0
1	0	1
0	1	1
0	0	1

So it can be seen that the both truth table are same, so positive logic NAND operator is equivalent to a negative logic NOR operator.

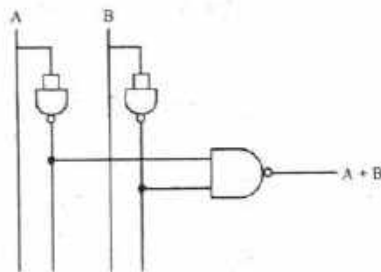
14. Construct the OR gate equivalent circuit using NAND gates.

[Model Question]

Answer:

Implementing OR Using only NAND Gates

An OR gate can be replaced by NAND gates as shown in the figure (The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters).



15. Implement the following function by NAND gate:

[Model Question]

$$Y = A\bar{B}C + AB(\bar{C} + A)$$

Answer:

$$Y = A\bar{B}C + AB(\bar{C} + A)$$

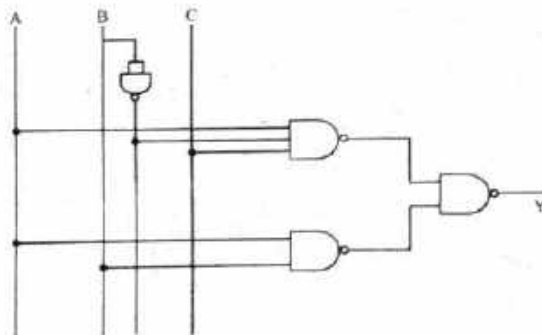
$$= A\bar{B}C + ABC + AB$$

$$= A\bar{B}C + AB(\bar{C} + 1)$$

$$= A\bar{B}C + AB$$

We know that, $\overline{X \cdot Y} = \overline{X} + \overline{Y} = X + Y$

Using this we can say that -

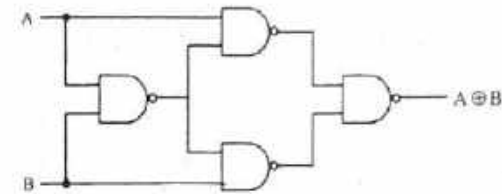


16. Construct a XOR gate using NAND gates only. Construct AND gate using NOR gate only.

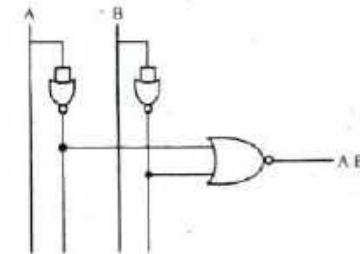
[Model Question]

Answer:

XOR gate:



AND gate:



17. What are the different components of a CPU? Explain with suitable diagram.

[Model Question]

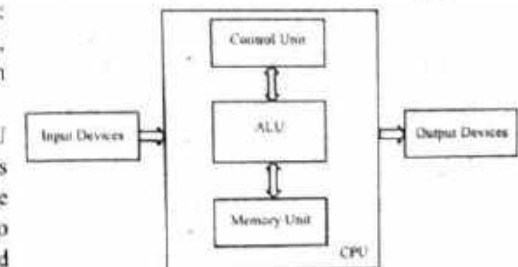
Answer:

The "heart" of a computer system is the central processing unit (or CPU) - the part where the actual processing of data takes place. It consists of three separate sub-parts, namely

- The main memory or store (where instructions and data are kept in coded form)
- The arithmetic/logic unit (ALU) contains the electronic circuitry that executes all arithmetic and logical operations like addition, subtraction, multiplication, and division and logical operation is usually a comparison. The unit can compare numbers, letters, or special characters. The computer can then take action based on the result of the comparison.

A computer can simultaneously test for more than one condition. In fact, a logic unit can perform six logical relationships: equal to, less than, greater than, less than or equal to, greater than or equal to, and not equal.

- The control unit of the CPU contains circuitry that uses electrical signals to direct the entire computer system to carry out, or execute, stored



program instructions. The control unit does not execute program instructions, rather, it directs other parts of the system to do so. The control unit must communicate with both the arithmetic/logic unit and memory.

Working:

- CPU consists of three basic units: control unit, Arithmetic Logical Unit (ALU) and memory unit.
- Input is given through the input devices to CPU.
- Control unit controls communication within ALU and memory unit.
- Decides which circuit is to be activated.
- For reading instruction it uses Fetch-execute mechanism.
- Control unit gets instruction from memory.
- Control unit decides what to do of that instruction and transfers it to the ALU.
- ALU performs various arithmetic operations like addition, subtraction, multiplication, division and logical operations like AND, OR, NOT, NAND etc. on that instruction.
- Results of ALU are stored in the memory or register for its further operations.
- After completing the instruction, stored results are passed to the output devices.
- To synchronize all these operations CPU uses its own system clock.

Functions:

- Executes stored instructions called as program.
- Tells rest of the computer system what to do.
- Executes arithmetic calculation and data manipulation.
- Holds data and instruction which are in the current use.
- Responsible for storing and retrieving information on disks and other media.

Chapter 2: Overview of Operating systems

Chapter at a glance:

2.1 Operating system, Evolution, Generations –1st, 2nd, 3rd, 4th

Operating systems have evolved through a number of distinct phases or generations which corresponds roughly to the decades.

The 1940's - First Generations

The earliest electronic digital computers had no operating systems. Machines of the time were so primitive that programs were often entered one bit at a time on rows of mechanical switches (plug boards). Programming languages were unknown (not even assembly languages). Operating systems were unheard of.

The 1950's - Second Generation

By the early 1950's, the routine had improved somewhat with the introduction of punch cards. The General Motors Research Laboratories implemented the first operating systems in early 1950's for their IBM 701. The system of the 50's generally ran one job at a time. These were called single-stream batch processing systems because programs and data were submitted in groups or batches.

The 1960's - Third Generation

The systems of the 1960's were also batch processing systems, but they were able to take better advantage of the computer's resources by running several jobs at once. So operating systems designers developed the concept of multiprogramming in which several jobs are in main memory at once; a processor is switched from job to job as needed to keep several jobs advancing while keeping the peripheral devices in use.

For example, on the system with no multiprogramming, when the current job paused to wait for other I/O operation to complete, the CPU simply sat idle until the I/O finished. The solution for this problem that evolved was to partition memory into several pieces, with a different job in each partition. While one job was waiting for I/O to complete, another job could be using the CPU.

Another major feature in third-generation operating system was the technique called spooling (simultaneous peripheral operations on line). In spooling, a high-speed device like a disk interposed between a running program and a low-speed device involved with the program in input/output.

Another feature present in this generation was time-sharing technique, a variant of multiprogramming technique, in which each user has an on-line (i.e., directly connected) terminal.

Fourth Generation

With the development of LSI (Large Scale Integration) circuits, chips, operating system entered in the system entered in the personal computer and the workstation age. Microprocessor technology evolved to the point that it become possible to build desktop computers as powerful as the mainframes of the 1970s. Two operating systems have dominated the personal computer scene: MS-DOS, written by Microsoft, Inc. for the IBM PC and other machines using the Intel 8088 CPU and its successors, and UNIX, which is dominant on the large personal computers using the Motorola 6899 CPU family.

2.2 Mainframe Systems

2.2.1 Simple Batch Systems

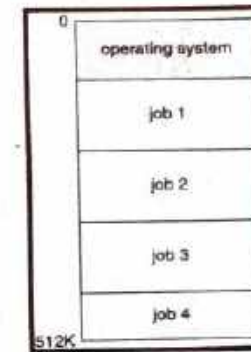
- Mainframe machines. Input devices were card readers. Output devices were line printer, tape drives, and card punch.
- A job (a single program+ associated data + control information) usually on the punch cards submitted to the operator.
- The output consisted of the results of the program or memory dump in case of error.
- The operator used to batch together similar programs and run as a group to reduce setup time.
- No user interaction while the job is executing.
- Current examples include .bat files under Dos – Windows and shell files under Unix/Linux.
- In batch systems:
 - Initial control is in monitor.
 - Load next program and transfer control to it.
 - When a job completes, the control transfers back to monitor.
 - Automatically transfer control from one job to another (Automatic job sequencing).

2.2.2 OS Features Needed for Multiprogramming

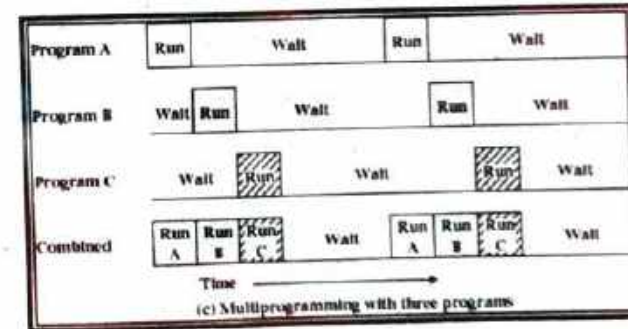
- I/O routine supplied by the system.
- Memory management – the system must allocate the memory to several jobs.
- CPU scheduling – the system must choose among several jobs ready to run.
- Allocation of devices.

2.2.3 Multiprogrammed Batch Systems

- Several jobs are kept in main memory at the same time, and the CPU is multiplexed among them.

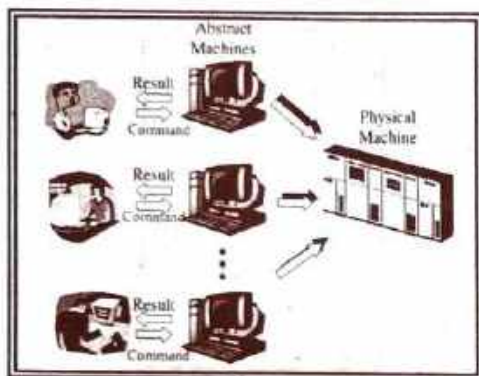


- If memory can hold several programs, then CPU can switch to another one whenever a program is waiting for an I/O to complete – This is multiprogramming.



2.2.4 Time Sharing Systems (Interactive Systems)

- TSS extends Batch multiprogramming to handle multiple interactive jobs – It's Interactive Multiprogramming.
- Multiple users simultaneously access the system through terminals.
- Processor's time is shared among multiple users, that is, the CPU is multiplexed among several jobs that are kept in memory and on disk (the CPU is allocated to a job only if the job is in memory).
- On-line communication between the user and the system is provided; when the operating system finishes the execution of one command, it seeks the next "control statement" from the user's keyboard.
- TS system provides each user with her/her own virtual machine.



2.2.5 Why Does Time Sharing Work?

- Because of slow human reaction time, a typical user needs 2 seconds of processing time per minute.
- Then many users should be able to share the same system without noticeable delay in the computer reaction time.

2.2.6 Batch Multiprogramming Vs. Time Sharing

	Batch Multi-Prog.	Time Sharing
Principle obj.	Max. Processor use	Min. response time
Source of inst. To OS	JCL provided with the job	Commands entered at the terminal

2.2.7 OS Features Needed for Time Sharing Systems

- On-line file system must be available for users to access data and code.
- Should do memory management
- Should do CPU scheduling
- Should do job synchronization and have communication facilities.
- Should ensure that dead lock and indefinite waiting does not occur.
- Should allow sharing of computer resources.

2.3 System components

Even though, not all systems have the same structure many modern operating systems share the same goal of supporting the following types of system components.

2.3.1 Process Management

The operating system manages many kinds of activities ranging from user programs to system programs like printer spooler, name servers, file server etc. Each of these activities is encapsulated in a process. A process includes the complete execution context (code, data, PC, registers, OS resources in use etc.).

It is important to note that a process is not a program. A process is only ONE instant of a program in execution. There are many processes can be running the same program. The five major activities of an operating system in regard to process management are

- Creation and deletion of user and system processes.
- Suspension and resumption of processes.
- A mechanism for process synchronization.
- A mechanism for process communication.
- A mechanism for deadlock handling.

2.3.2 Main-Memory Management

Primary-Memory or Main-Memory is a large array of words or bytes. Each word or byte has its own address. Main-memory provides storage that can be access directly by the CPU. That is to say for a program to be executed, it must in the main memory.

The major activities of an operating in regard to memory-management are:

- Keep track of which part of memory are currently being used and by whom.
- Decide which process are loaded into memory when memory space becomes available.
- Allocate and deallocate memory space as needed.

2.3.3 File Management

A file is a collected of related information defined by its creator. Computer can store files on the disk (secondary storage), which provide long term storage. Some examples of storage media are magnetic tape, magnetic disk and optical disk. Each of these media has its own properties like speed, capacity, data transfer rate and access methods.

A file systems normally organized into directories to ease their use. These directories may contain files and other directions.

The five main major activities of an operating system in regard to file management are

- The creation and deletion of files.
- The creation and deletion of directions.
- The support of primitives for manipulating files and directions.
- The mapping of files onto secondary storage.
- The back up of files on stable storage media.

2.3.4 I/O System Management

I/O subsystem hides the peculiarities of specific hardware devices from the user. Only the device driver knows the peculiarities of the specific device to whom it is assigned.

2.3.5 Secondary-Storage Management

Generally speaking, systems have several levels of storage, including primary storage, secondary storage and cache storage. Instructions and data must be placed in primary storage or cache to be referenced by a running program. Because main memory is too small to accommodate all data and programs, and its data are lost when power is lost, the computer system must provide secondary storage to back up main memory. Secondary storage consists of tapes, disks, and other media designed to hold information that will eventually be accessed in primary storage (primary, secondary, cache) is ordinarily divided into bytes or words consisting of a fixed number of bytes. Each location in storage has an address; the set of all addresses available to a program is called an address space.

The three major activities of an operating system in regard to secondary storage management are:

1. Managing the free space available on the secondary-storage device.
2. Allocation of storage space when new files have to be written.
3. Scheduling the requests for memory access.

2.4 Operating system services

An operating system provides basic services to the program for execution and the owner of the program. These are:

1. To load program into memory and execute it.
2. File system handling i.e., a program requires a file to read or to write, create or delete files.
3. I/O operations and resource allocation.
4. Communication between processes via shared memory or message passing.
5. Error handling i.e., error in the program, CPU, network, memory etc.
6. Security and protection.
7. Record keeping of users and system, i.e., which user and how many users are in queue and how much time they have allotted.

2.5 System calls

A system call is an instruction which generates an interrupt that causes the operating system to gain control of the processor. The operating system then determines what kind of system call it is and perform the appropriate service for the system caller. The essential difference between system call and conventional subroutine is that in the case of a conventional subroutine, the object code is part of the calling program while the system

call code is within the operating system. An operating system will only have a limited number of available system calls and in general these will be supplemented by standard subroutine libraries. Many of these standard subroutines which provide additional and/or higher level facilities for the programmer, will themselves use system calls.

2.6 Operating system structure, layered, monolithic, microkernel

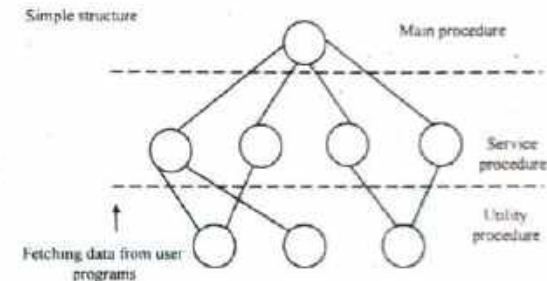


Fig. 1

Operating system structure is mainly based on hierarchy of layers. The bottom layer is the hardware and the highest layer is the user interface. In between the above two layers, there are so many layers like processor allocation layer, memory management layer, Input/Output management layer, device management layer etc. But since the operating system i.e., system software has improved rapidly from 1st generation to 5th generation computer system development, the structure will be somewhat different from the general concept. For example client-server model structure, parallel-processing structure, Distributed and real-time system structure is a variation from the traditional system structure. The fig. 1-4 shows the structure of different age.

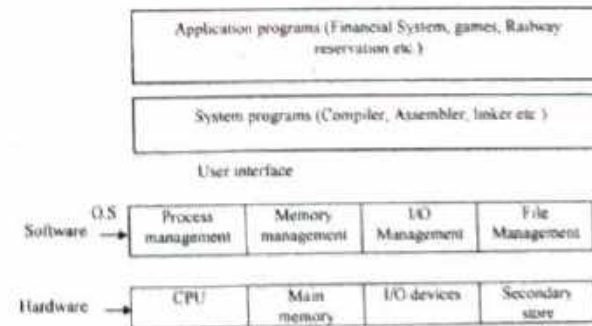


Fig. 2

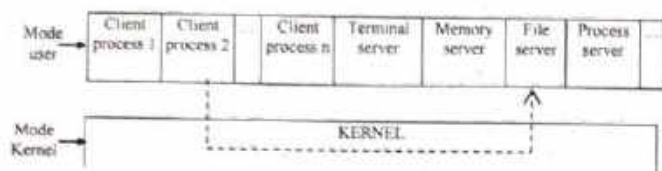


Fig: 3

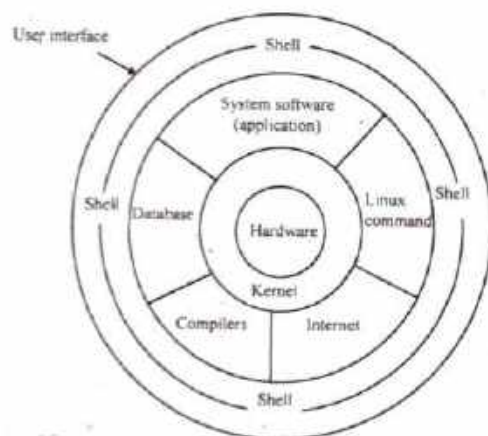


Fig: 4

Kernel and Shell:

In computer since, the **kernel** is the central component of most operating system. Its responsibilities include managing the system's resources. As a basic component of an operating system, a kernel provides the lowest level abstraction layer for the resources specially memory, processors and I/O devices that application software must control to perform its function. It typically makes these facilities available to application processes through inter-process communication mechanism and system calls. These tasks are done differently by different kernels, depending on their design and implementation. While **monolithic kernels** will try to achieve these goals by executing all the code in the same address space to increase the performance of the system, **micro kernels** run most of their services in user space, aiming to improve maintainability and modularity of the code base. A range of possibilities exists between these two extremes. The kernel's primary purpose is to manage the computer resources and allow other programs to run and use these resources.

Shell:

In computing, a shell is a piece of software that provides an interface for users. Typically, the term refers to an operating system shell, which provides access to the service of a kernel. However, the term is also applied very loosely to applications and may include any software that is built around a particular component, such as web browsers and e-mail clients that are "shells" for **HTML** rendering engines. The name "shell" originates from shells being an outer layer of interface between the user and the innards of the operating system (The Kernel). Operating system shells generally fall into one of two categories: Command line and graphical. Command line shells provide a **command line interface** to the operating system, while graphical shells provide a **graphical user interface** (GUI). In either category, the primary purpose of the shell is to invoke or launch another program; however shells frequently have additional capabilities such as viewing the contents of directories.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. Which of the following is an example of a Real Time Operating System

[WBSCTE 2022]

- (a) MAC (b) MS-DOS (c) Windows 10 (d) Process Control

Answer: (d)

2. Which of the following is not an operating system?

[WBSCTE 2022]

- (a) Windows (b) Linux (c) Oracle (d) DOS

Answer: (c)

3. Real time systems are _____

[Model Question]

- (a) Primarily used on mainframe computers
(b) Used for monitoring events as they occur
(c) Used for program development
(d) Used for real time interactive users

Answer: (b)

4. Super computers typically employ _____

[Model Question]

- (a) Real time Operating system (b) Multiprocessors OS
(c) desktop OS (d) None of these

Answer: (b)

5. The primary job of the operating system of a computer is to _____
 (a) Command Resources (b) Manage Resource [Model Question]
 (c) Provide Utilities (d) Be user friendly

Answer: (b)

6. In _____ OS, the response time is very critical. [Model Question]
 (a) Multitasking (b) Batch
 (c) Online (d) Real-time

Answer: (d)

7. Which of the following is not advantage of multiprogramming? [Model Question]
 (a) Increased throughput (b) Shorter response time
 (c) Decreased operating system overhead (d) Ability to assign priorities to jobs

Answer: (c)

8. Unix Operating System is an _____ [Model Question]
 (a) Time Sharing Operating System (b) Multi-User Operating System
 (c) Multi-tasking Operating System (d) All of these

Answer: (d)

9. Distributed OS works on the _____ principle. [Model Question]
 (a) File Foundation (b) Single system image
 (c) Multi system image (d) Networking image

Answer: (b)

10. Multiprogramming systems _____ [Model Question]
 (a) Are easier to develop than single programming systems
 (b) Execute each job faster
 (c) Execute more jobs in the same time
 (d) Are used only on large main frame computers

Answer: (c)

11. Multiprocessing _____ [Model Question]
 (a) Make the operating system simpler
 (b) Allows multiple processes to run simultaneously
 (c) Is completely understood by all major computer vendors
 (d) Allows the same computer to have the multiple processors

Answer: (d)

12. What is the first step in performing an operating system upgrade? [Model Question]
 (a) Partition the drive (b) Format the drive
 (c) Backup critical data (d) Backup old operating system

Answer: (c)

13. Which is not a layer of operating system? [Model Question]
 (a) Kernel (b) Shell
 (c) Application program (d) Critical section

Answer: (d)

14. In _____ mode, a machine executes operating system instructions. [Model Question]
 (a) user (b) system (c) safe (d) normal

Answer: (b)

15. Virtual memory involves [Model Question]
 (a) primary memory (b) secondary memory
 (c) both of these (d) none of these

Answer: (b)

16. In the layered approach of Operating Systems: (choose two) [Model Question]
 (a) Bottom Layer(0) is the User interface
 (b) Highest Layer(N) is the User interface
 (c) Bottom Layer(0) is the hardware
 (d) Highest Layer(N) is the hardware

Answer: (b) & (c)

17. The initial program that is run when the computer is powered up is called: [Model Question]
 (a) boot program (b) boot loader
 (c) initialize (d) bootstrap program

Answer: (d)

B. Answer the following questions:

18. What is the time sharing operating system? [WBSCTE 2022]
 Answer: Refer to Article No. 2.2.4 of Chapter at a Glance.

19. What are the advantages of multiprocessors system? [WBSCTE 2022]

Answer:

Some advantages of multiprocessor system are as follows:

- **Reduced Cost:** Multiple processors share the same resources. Separate power supply or mother board for each chip is not required. This reduces the cost.
- **Increased Reliability:** The reliability of system is also increased. The failure of one processor does not affect the other processors though it will slow down the machine. Several mechanisms are required to achieve increased reliability. If a processor fails, a job running on that processor also fails. The system must be able to reschedule the failed job or to alert the user that the job was not successfully completed.
- **Increased Throughput:** An increase in the number of processes completes the work in less time. It is important to note that doubling the number of processors does not halve the time to complete a job. It is due to the overhead in communication between processors and contention for shared resources etc.

20. What is operating system?

[Model Question]

Answer:

An Operating system (OS) is a software which acts as an interface between the end user and computer hardware. Every computer must have at least one OS to run other programs. An application like Chrome, MS Word, Games, etc needs some environment in which it will run and perform its task. The OS helps you to communicate with the computer without knowing how to speak the computer's language. It is not possible for the user to use any computer or mobile device without having an operating system.

21. State whether time sharing system can be used for multiprogramming or multi-user based OS design or not.

[Model Question]

Answer: Refer to Article No. 2.2.4 of Chapter at a Glance.

22. What is system call?

[Model Question]

Answer:

The invocation of an operating system routine. Operating systems contain sets of routines for performing various low-level operations. For example, all operating systems have a routine for creating a directory. If you want to execute an operating system routine from a program, you must make a system call.

23. What is a linker?

[Model Question]

Answer:

A linker is a program that combines object modules to form an executable program. Many programming languages allow us to write different pieces of code, called modules, separately. This simplifies the programming task because we can break a large program into small, more manageable pieces.

24. What is the purpose of command interpreter? Why is it usually separate from kernel?

[Model Question]

Answer:

It reads commands from the user or from a file of commands and executes them, usually by turning them into one or more system calls. It is usually not part of the kernel since the command interpreter is subject to changes.

25. State the importance of the system call in the context of operating system security.

[Model Question]

Answer:

System call in the context of operating system security gives us I/O protection which prevents users from performing illegal I.

26. What is booting?

[Model Question]

Answer:

Booting is the process of starting or restarting a computer.

Cold Boot: It is turning on a computer that has been turned off completely and properly

Warm boot: Process of using the operating system to restart a computer.

27. What operations are performed by loader?

[Model Question]

Answer:

For execution loading a program involving reading the contents of executable file contain program instruction into memory, and then carrying out other preparatory tasks to prepare the executable for running.

28. What is the difference of compiler and interpreter?

[Model Question]

Answer:

No.	Compiler	Interpreter
1.	Compiler Takes Entire program as input	Interpreter Takes Single instruction as input
2.	Intermediate Object Code is Generated	No Intermediate Object Code is Generated
3.	Conditional Control Statements are Executes faster	Conditional Control Statements are Executes slower
4.	Memory Requirement More (Since Object Code is Generated)	Memory Requirement is Less
5.	Program need not compiled every time	Every time higher level program is converted into lower level program
6.	Errors are displayed after entire program is checked	Errors are displayed for every instruction interpreted (if any)
7.	Example: C Compiler	Example: BASIC

29. What is Kernel?

[Model Question]

Answer:

In computer since, the kernel is the central component of most operating system. Its responsibilities include managing the system's resources. As a basic component of an operating system, a kernel provides the lowest level abstraction layer for the resources specially memory, processors and I/O devices that application software must control to perform its function. It typically makes these facilities available to application processes through inter-process communication mechanism and system calls. These tasks are done differently by different kernels, depending on their design and implementation. While monolithic kernels will try to achieve these goals by executing all the code in the same address space to increase the performance of the system, micro kernels run most of their services in user space, aiming to improve maintainability and modularity of the code base. A range of possibilities exists between these two extremes. The kernel's primary purpose is to manage the computer resources and allow other programs to run and use these resources.

30. What is the relationship between kernel and operating system? [Model Question]
Answer:

Kernel is the core of the operating system. It is the first program of operating system that is loaded into the main memory to start the working of the system. Kernel remains in the main memory till the system is shut down. Kernel basically translates the commands entered by the user in a way to make the computer understand that what has user requested.

Kernel acts as a bridge between application software and hardware of the system. Kernel directly communicates with the hardware and let it know what the application software has requested. An operating system is unable to run without the kernel as it is the important program for the working of the system.

Long Answer Type Questions

➤ 1. What is Operating system? What are the basic services provided by the operating system? [WBSCTE 2022]
Answer:
1st Part:

An Operating system (OS) is a software which acts as an interface between the end user and computer hardware. Every computer must have at least one OS to run other programs. An application like Chrome, MS Word, Games, etc needs some environment in which it will run and perform its task. The OS helps you to communicate with the

computer without knowing how to speak the computer's language. It is not possible for the user to use any computer or mobile device without having an operating system.

2nd Part:
(1) Program execution:

The system must be able to load a program into memory and run the program.

(2) I/O operations:

All the programs dealing with I/O operations relating to specific devices are to be dealt by the operating symbols.

(3) File system implementation/manipulation:

The file system is of particular interest. Programs need to read and write files. Files can also be labeled by their unique names.

(4) Communications:

The communication, which takes place between the concurrent processes, can be divided into two parts. The first one takes place between the processes that are running on the same computer and other type of processes are those that are being executed on different computer systems through a computer network. These two schemes can be implemented by shared memory or by message passing.

(5) Error detection:

There are various types of errors that occur when the process is running. Errors may be between CPU and memory or between CPU and I/O. OS handles these errors and gives information to the user.

(6) Resource allocation:

When the number of users are more than one, multi-programming (resource sharing) has to be implemented. There are various methods for handling these by special allocation coding, CPU scheduling etc.

(7) Protection:

In multi-user environment, protection of valuable resources plays an important role. This is implemented by the help of security. Each layer can be accessed by the help of a user name and password by the system administrator.

(8) Accounting:

The record keeping works as to which resource has been utilized by which process and for how long is being cared by OS.

➤ 2. What is CPU and I/O bound jobs? [WBSCTE 2022]
Answer:

The term **CPU-bound** describes a scenario where the execution of a task or program is highly dependent on the CPU. The CPU usually refers to the central processing unit of a computing device, such as a desktop computer. It's responsible for controlling the

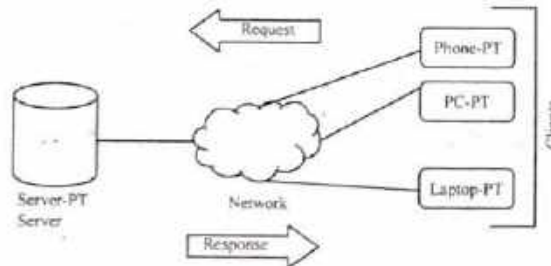
execution of tasks and programs in a computer system. As such, a computing device can't really function without it.

In contrast, a task or program is **I/O bound** if its execution is dependent on the input-output system and its resources, such as disk drives and peripheral devices. In **I/O bound environments**, we wait for resources to be retrieved through the input-output systems. Hence, it's safe to say that a program or task would execute faster if the input-output system worked faster.

➔ 3. What do you mean by client server architecture? Explain. [WBSCTE 2022]

Answer:

Client-server architecture, alternatively called a client-server model, is a network application that breaks down tasks and workloads between clients and servers that reside on the same system or are linked by a computer network. Client-server architecture typically features multiple users' workstations, PCs, or other devices, connected to a central server via an Internet connection or other network. The client sends a request for data, and the server accepts and accommodates the request, sending the data packets back to the user who needs them. **Examples of Client-Server Model** are Email, World Wide Web, etc.



There are few steps to follow to interact with the servers a client.

- User enters the **URL**(Uniform Resource Locator) of the website or file. The Browser then requests the **DNS**(DOMAIN NAME SYSTEM) Server.
- **DNS Server** lookup for the address of the **WEB Server**.
- **DNS Server** responds with the **IP address** of the **WEB Server**.
- Browser sends over an **HTTP/HTTPS** request to **WEB Server's IP** (provided by DNS server).
- Server sends over the necessary files of the website.
- Browser then renders the files and the website is displayed. This rendering is done with the help of **DOM** (Document Object Model) interpreter, **CSS** interpreter and **JS Engine** collectively known as the **JIT** or (Just in Time) Compilers.

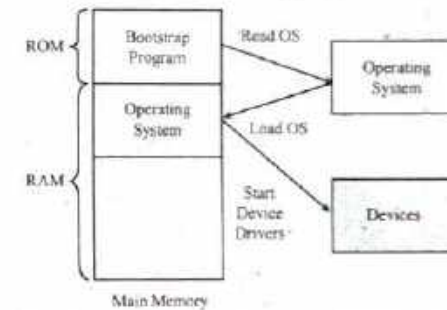
➔ 4. What is bootstrap program in OS? What is Batch operating System? [WBSCTE 2022]

Answer:

1st Part:

A bootstrap program is the first code that is executed when the computer system is started. The entire operating system depends on the bootstrap program to work correctly as it loads the operating system.

A figure that demonstrates the use of the bootstrap program is as follows –

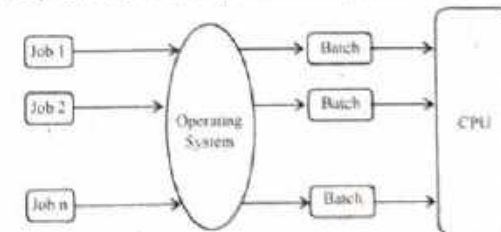


In the above image, the bootstrap program is a part of ROM which is the non-volatile memory. The operating system is loaded into the RAM by the bootstrap program after the start of the computer system. Then the operating system starts the device drivers.

2nd Part:

Batch Operating System:

This type of operating system does not interact with the computer directly. There is an operator which takes similar jobs having the same requirement and group them into batches. It is the responsibility of the operator to sort jobs with similar needs.



Advantages of Batch Operating System:

1. It is very difficult to know the time required for any job to complete. Processors of the batch systems know how long the job would be when it is in queue.
2. Multiple users can share the batch systems.

3. The idle time for the batch system is very less.
4. It is easy to manage large work repeatedly in batch systems.

➤ 5. Write short notes on Types of OS.

[WBSCTE 2022]

Answer:

Types of Operating Systems:

i) Batch Operating System:

This type of operating system does not interact with the computer directly. There is an operator which takes similar jobs having the same requirement and group them into batches.

Examples of Batch based Operating System: Payroll System, Bank Statements, etc.

ii) Time-Sharing Operating Systems:

Each task is given some time to execute so that all the tasks work smoothly. Each user gets the time of CPU as they use a single system. These systems are also known as Multitasking Systems. The task can be from a single user or different users also. The time that each task gets to execute is called quantum. After this time interval is over OS switches over to the next task.

Examples of Time-Sharing OSs are: Multics, Unix, etc.

iii) Distributed Operating System:

These types of the operating system is a recent advancement in the world of computer technology and are being widely accepted all over the world and, that too, with a great pace. Various autonomous interconnected computers communicate with each other using a shared communication network. Independent systems possess their own memory unit and CPU. These are referred to as **loosely coupled systems** or distributed systems.

Examples of Distributed Operating System are- LOCUS, etc.

iv) Real-Time Operating System:

These types of OSs serve real-time systems. The time interval required to process and respond to inputs is very small. This time interval is called **response time**.

Real-time systems are used when there are time requirements that are very strict like missile systems, air traffic control systems, robots, etc.

Examples of Real-Time Operating Systems are: Scientific experiments, medical imaging systems, industrial control systems, weapon systems, robots, air traffic control systems, etc.

➤ 6. What is the difference between a multiprogramming and multitasking operating system?

[Model Question]

POLY-ITS

Answer:

Multi-user - A multi-user Operating System allows for multiple users to use the same computer at the same time and/or different times. See our multi-user dictionary definition for a complete definition for a complete definition. Below are some examples of multi-user Operating Systems.

Linux

Unix

Windows 2000

Windows XP

Mac OS X

Multiprocessing - An Operating System capable of supporting and utilizing more than one computer processor. Below are some examples of multiprocessing Operating Systems.

Linux

Unix

Windows 2000

Windows XP

Mac OS X

Multitasking - An Operating system that is capable of allowing multiple software processes to run at the same time. Below are some examples of multitasking Operating Systems.

Unix

Windows 2000

Windows XP

Mac OS X

Multithreading - Operating systems that allow different parts of a software program to run concurrently. Operating systems that would fall into this category are:

Linux

Unix

Windows 2000

Windows XP

Mac OS X

➤ 7. Write the difference between batch processing and multiprogramming based OS system.

[Model Question]

Answer:

Batch processing environment it requires grouping of similar jobs which consist of programs, data and system commands.

The suitability of this type of processing is in programs with large computation time with no need of user interaction/involvement. Some examples of such programs include payroll, forecasting, statistical analysis and large scientific number crunching programs. Users are not required to wait while the job is being processed. They can submit their programs to operators and return later to collect them.

But it has two major disadvantages:

- i. Non-interactive environment
- ii. Off-line debugging

Multiprogramming operating systems compared to batch operating systems are fairly sophisticated. Multiprogramming has a significant potential for improving system throughput and resource utilisation with very minor differences. Different forms of multiprogramming operating system are multitasking, multiprocess and multiuser operating systems.

➤ **8. What is system call? Write down different system calls for performing different kinds of tasks.**
[Model Question]

Answer:

A system call is a request made by any program to the operating system for performing tasks — picked from a predefined set — which the said program does not have required permissions to execute in its own flow of execution. System calls provide the interface between a process and the operating system. Most operations interacting with the system require permissions not available to a user level process, e.g. I/O performed with a device present on the system or any form of communication with other processes requires the use of system calls.

The main types of system calls are as follows:

- **Process Control:** These types of system calls are used to control the processes. Some examples are end, abort, load, execute, create process, terminate process etc.
- **File Management:** These types of system calls are used to manage files. Some examples are Create file, delete file, open, close, read, write etc.
- **Device Management:** These types of system calls are used to manage devices. Some examples are Request device, release device, read, write, get device attributes etc.

➤ **9. (a) How to analyse a malware and protect it?**

(b) Discuss the methods for intrusion detection and virus protections.

[Model Question]

Answer:

a) The malware analysis knows how malware functions allow for better defenses to protect resources from any attack of malware. There are two types of malware analysis: static analysis or dynamic analysis. Static analysis is the actual viewing of code to get a better understanding of the malware and what it is. Dynamic analysis is how the malware behaves when executed.

b) Intrusion Detection methods

Host-based intrusion detection techniques revolve around individual hosts — usually servers — by monitoring the hard drive and both inbound and outbound packets, and constantly comparing the results against a pre-created image of the host and the host's expected packet flow. The idea is to look for malicious changes both in the logical contents of the host as well as the host's activity. It often relies on a local client or agent of the IDS system to be installed on the host.

Application-based intrusion detection techniques widen the scope to an application in an abstract sense — meaning, everything in the infrastructure that's involved in the way that application functions, but only that application. These solutions are used for applications that perform particularly crucial functions for the organization, because the potential consequences of a breach are high.

Network-based intrusion detection techniques expand the scope of coverage still further to all devices on a network or subnetwork (sometimes, multiple instances of solutions collaborate to accomplish this, due to the volume of traffic). Because they are the most general, they sometimes miss problems the other two might detect.

Virus Protection methods

Our main defense and protection is to detect and identify specific virus attacks to our computer. There are three methods in general use. Each has pros and cons. Often, a given anti-virus software program will use some combination of the three techniques for maximum possibility of detection.

- Scanning
- Integrity Checking
- Interception

Once a virus has been detected, it is possible to write scanning programs that look for telltale code (signature strings) characteristic of the virus. The writers of the scanner extract identifying strings from the virus. The scanner uses these signature strings to search memory, files, and system sectors. If the scanner finds a match, it announces that it has found a virus. This obviously detects only known, pre-existing, viruses. Many so-called "virus writers" create "new" viruses by modifying existing viruses. This takes only a few minutes but creates what appears to be a new virus. It happens all too often that these changes are simply to fool the scanners.

Integrity checking products work by reading our entire disk and recording integrity data that acts as a signature for the files and system sectors. An integrity check program with built-in intelligence is the only solution that can handle all the threats to our data as well as viruses. Integrity checkers also provide the only reliable way to discover what damage a virus has done.

Interceptors (also known as resident monitors) are particularly useful for deflecting logic bombs and Trojans. The interceptor monitors operating system requests that write to disk or do other things that the program considers threatening (such as installing itself as a resident program). If it finds such a request, the interceptor generally pops up and asks you if you want to allow the request to continue. There is, however, no reliable way to intercept direct branches into low level code or to intercept direct input and output instructions done by the virus itself. Some viruses even manage to disable the monitoring program itself. Indeed, for one widely-distributed anti-virus program several years back it only took eight bytes of code to turn its monitoring functions off.

- 10. Define the essential properties of the following types of operating systems: a. Batch, b. Time sharing c. Real time. [Model Question]

Answer:

Batch:

Jobs with similar needs are batched together and run through the computer as a group by an operator or automatic job sequencer. Performance is increased by attempting to keep CPU and I/O devices busy at all times through buffering, off-line operation, spooling, and multiprogramming. Batch is good for executing large jobs that need little interaction; it can be submitted and picked up later.

Time sharing:

This system uses CPU scheduling and multiprogramming to provide economical interactive use of a system.

The CPU switches rapidly from one user to another. Instead of having a job defined by spooled card images, each program reads its next control card from the terminal, and output is normally printed immediately to the screen.

Real time:

Often used in a dedicated application, this system reads information from sensors and must respond within a fixed amount of time to ensure correct performance.

- 11. What is the important aspect of a real-time system or Mission Critical Systems? [Model Question]

Answer:

A real time operating system has well defined fixed time constraints. Process must be done within the defined constraints or the system will fail. An example is the operating

system for a flight control computer or an advanced jet airplane. Often used as a control device in a dedicated application such as controlling scientific experiments, medical imaging systems, industrial control systems, and some display systems. Real-Time systems may be either hard or soft real-time. Hard real-time: Secondary storage limited or absent, data stored in short term memory, or read-only memory (ROM). Conflicts with time-sharing systems, not supported by general-purpose operating systems. Soft real-time: Limited utility in industrial control of robotics, Useful in applications (multimedia, virtual reality) requiring advanced operating-system features.

- 12. Explain the important differences between hard, firm and soft real-time systems. [Model Question]

Answer:

A hard real-time task is one that is constrained to produce its results within certain predefined time bounds. The system is considered to have failed whenever any of its hard real-time tasks does not produce its required results before the specified time bound. Unlike a hard real-time task, even when a firm real-time task does not complete within its deadline, the system does not fail. The late results are merely discarded. In other words, the utility of the results computed by a real-time task becomes zero after the deadline. Soft real-time tasks also have time bounds associated with them. However, unlike hard and firm real-time tasks, the timing constraints on soft real-time tasks are not expressed as absolute values. Instead, the constraints are expressed in terms of the average response times required.

- 13. With a suitable example explain the difference between the traditional notion of time and real-time. [Model Question]

Answer:

In a real-time application, the notion of time stands for the absolute time which is quantifiable. In contrast to real time, logical time, used in most general category applications, deals with a qualitative notion of time and are expressed using event ordering relations. For example, consider the following part of the behavior of library automation software used to automate the bookkeeping activities of a college library: "After a query book command is given by the user, the details of all the matching books are displayed by the software".

- 14. What are the characteristics of distributed system? [Model Question]

Answer:

- Distributed system distribute the computation among several physically separated processors.

- Loosely coupled system – each processor has its own local memory; processors communicate with one another through various communications lines, such as high-speed buses or telephone lines.
- Advantages of distributed systems.
 - Resources Sharing
 - Computation speed up – load sharing
 - Reliability and fault tolerance
 - Communications
- Requires networking infrastructure - Local area networks (LAN) or Wide area networks (WAN)
- May be either client-server or peer-to-peer systems.
- Less autonomy between computers
- Gives the impression there is a single operating system controlling the network.

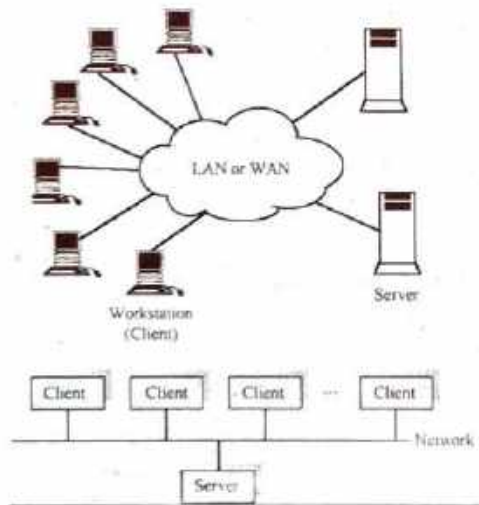


Fig: General structure of client-server

15. What is clustered System?

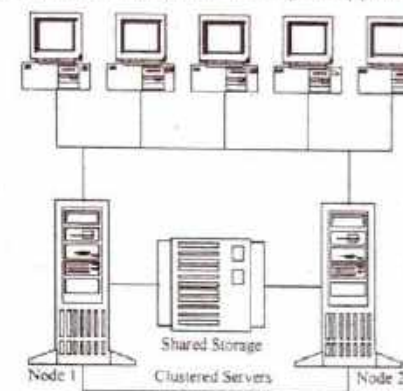
[Model Question]

Answer:

Clustered Systems

- Clustering allows two or more systems to share external storage and balance CPU load.
- Asymmetric clustering: one server runs the application while other servers standby.

Symmetric clustering: all N hosts are running the application.



16. Differences between Network operating system and distributed operating system. [Model Question]

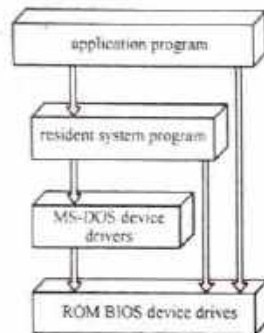
Answer:

Network operating system	Distributed operating system
1) Users view as a collection of distinct machine in a communication subsystem.	1) Users view as a single system image by hiding the existence of multiple users.
2) In network operating system, a different set of system calls have to be used for accessing local and remote resources.	2) In distributed operating system, same set of system calls is used for accessing both local and remote resources.
3) In case of a network operating system, each computer has its own local operating system and no co-ordination at all is required except special situation like interprocess communication.	3) In case of a distributed operating system there is a single system wide operating system and each computer runs a part of global operating system.
4) A network operating system provides a little or no fault tolerance capability i.e., in case of machine failures.	4) A distributed operating system are normally unaffected by the failed machines i.e., fault tolerance capability is usually very high.

17. What are the various components of operating – system structure explain the simple and layered approach of operating system in details. [Model Question]

Answer:

Simple approach



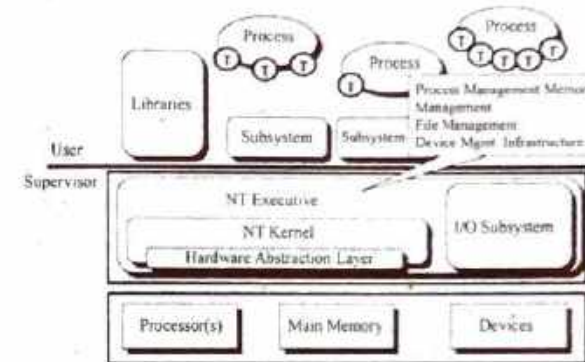
In MS-DOS, application may bypass the operating system

- Operating systems such as MS-DOS and the original UNIX did not have well-defined structures.
- There was no CPU Execution Mode (User and Kernel), and so errors in applications could cause the whole system to crash.

Layered approach

This approach breaks up the operating system into different layers.

- This allows implementers to change the inner workings, and increases modularity.
- As long as the external interface of the routines don't change, developers have more freedom to change the inner working of the routines.
- With the layered approach, the bottom layer is the hardware, while the highest layer is the user interface.
 - The main advantage is simplicity of construction and debugging.
 - The main difficulty is defining the various layers.
 - The main disadvantage is that the OS tends to be less efficient than other implementations.



The Microsoft Windows NT Operating System. The lowest level is a monolithic kernel, but many OS components are at a higher level, but still part of the OS.

18. What is the simple structure of operating system?

Answer:

There are so many systems that do not have a well defined structure. The reason is that they started with small, simple and limited systems and then grew beyond their original scope. The example is MS - DOS structure. Initially it has been started with a group of few people without the awareness that it would grow so popular. It has features of consuming least space, because of limited hardware on which it runs, so it was not carefully divided into modules. UNIX (original) old version is another example. MS-DOS structure is shown below.

[Model Question]

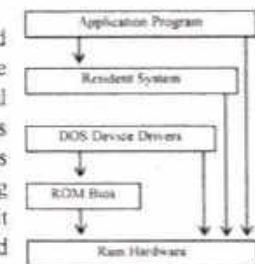


Fig. MS-DOS Structure

(The users)		
Shells and commands compilers and interpreters system libraries		
System call interface to the Kernel		
I/O system Terminal drivers	File system disk and tape Drivers	CPU scheduling page replacement demand paging Virtual memory
Kernel interface to the Kernel		
Terminal controllers	Device & memory controllers	
Terminals	Disk and tapes	Physical memory

UNIX System structure

19. Describe multiple OS boot process.

[Model Question]

Answer:

Whenever there are multiple OSes, be it multiple Windows or Windows with Linux, then Boot process will be slightly different.

Actually, there can be two different types of Boot Process in multiple OS environment, Microsoft way and Non-Microsoft way (or Third Party Boot Loader way!).

Microsoft Master Boot loaders doesn't recognize other types of OSes like Linux (in default, that is!), hence using Microsoft MBR in the presence of Linux is ruled out.

Consider the case that (*this is the general case*), there is one Primary partition and some Logical Partitions inside Extended Partition. Now if Win98 is installed in the Primary Partition, and afterwards WinXP is installed in a Logical Partition, then theoretically both OS should have their own Boot Records, i.e. PBR for Win98 and LBR for WinXP, which contain program to boot the respective OS, so that each individual OS can be booted up by the MBR by passing control to respective PBR of the OS as described in previous section.

But this does not happen in Microsoft Boot loader! It does a peculiar thing, that it always considers the current Active Partition as the default System/Boot Partition.

That is Primary Partition in which Win98 was installed is the Active Partition, then when WinXP is installed another partition, instead of writing the code for booting in it's partition, WinXP writes the code in current Active Partition (that is, where Win98 is installed!).

Program responsible for loading the WinXP is ntldr standing for NTLoader. Theoretically, this should be in it's partition, but this copied to that of Win98.

Then file responsible for Win98 booting is combined into a single file called bootsect.dos and this is also placed in the Win98 partition.

Then, WinXP creates another file called boot.ini which, contains the names of Microsoft OSes installed and path for System files of each OS.

After all this prelims, Windows multiboot can be represented as below:

1. When BIOS hands over control to Microsoft MBR, this program looks into Partition Table for Active Partition.
2. Then it hands over the control to the PBR of Active Partition. In this case, the Active Partition is where Win98 was installed.
3. But Win98 PBR has been altered by WinXP and it no longer contains Win98 boot program (like io.sys or msdos.sys). But it contains ntldr! Peculiarity is that one OS's Boot program is in another OS's Partition!
4. ntldr looks into boot.ini file and finds out the Microsoft OSes installed in the System and displays the option menu.
5. When user selects Win98, the file bootsect.dos (which is in same partition) is executed, and if WinXP is selected, ntoskrnl is executed (which is in another partition!).

The good thing about Microsoft way is that, it's very easy to configure (*you don't have to configure at all!*).

But the bad thing about Microsoft MBR is that, the two OSes are not independent of each other. It is because, that Microsoft MBR always boots into the Active Partition (that is it always boots into Win98 Partition, but executes WinXP program!) and from here other OSes are loaded. This does not provide flexibility of installing multiple Microsoft OSes in a random order, because here *older version* of OS should be installed first and then *newer versions* of OSes should be installed.

This boot process also has two limitations:

1. There can be only one Real Mode DOS based OSes like Win95/Win98 along with NT based OSes. If you want both Win95, Win98 with any NT based OS, then it's simply not possible.
2. Microsoft MBR looks for Active Status in Primary Partitions only and not in Logical Partitions. This means, Microsoft OSes should be installed in Primary Partitions only if it should be bootable (For this reason itself, WinXP boot file ntldr is placed in Primary Partition of Win98 instead of it's own Logical Partition).

20. What is the main advantage of the layered approach system design? What are the disadvantages of using layered approach? [Model Question]

Answer:

The main advantage of the layered approach is simplicity of construction and debugging.

The layers are selected so that each uses functions (operations) and services of only lower-level layers. This approach simplifies debugging and system verification. The major difficulty with the layered approach involves appropriately defining the various layers. Because a layer can use only lower-level layers, careful planning is necessary. Also, layered implementations tend to be less efficient than other types. For instance, when a user program executes an I/O operation, it executes a system call that is trapped to the I/O layer, which calls the memory management layer which in turn calls the CPU-scheduling layer, which is then passed to the hardware. At each layer, the parameters may be modified, data may need to be passed, and so on. Each layer adds overhead to the system call; the net result is a system call that takes longer than does one on a non-layered system.

➤ 21. List at least three of the major categories of system calls. [Model Question]

Answer:

The main types of system calls are as follows:

- **Process Control:** These types of system calls are used to control the processes. Some examples are end, abort, load, execute, create process, terminate process etc. Process control involves creating new processes to work concurrently on solving problems. There are four main system calls for process control:
 - **fork:** create a new process
 - **exit:** normal termination of process (exit is not actually a system call, but it calls _exit which is.)
 - **execve:** load and execute another program (there are five other variations in the exec family, but execve is the actual system call executed.)
 - **wait:** wait for another process to finish.
- **File Management:** These types of system calls are used to manage files. Some examples are Create file, delete file, open, close, read, write etc.
 - **open:** system call to open a file
 - **open** returns a file descriptor, an integer specifying the position of this open file in the table of open files for the current process
 - **create:** (no 'e') system call to create a file /* no longer necessary */
 - **close:** system call to close a file
 - **read:** read data from a file opened for reading
 - **write:** write data to a file opened for writing
 - **lseek:** seek to a specified position in a file (used for random access when reading or writing a file)
 - the original seek system call used a 16 bit address

- "lseek" is named because it uses a "long" integer as the address to seek to. In practice this is a 32 bit integer on most current machines
- the UNIX standards organizations have apparently not yet standardized a suitable seek system call for 64 bit addresses
- Currently (January, 2005), the llseek system call is available on Solaris (hercules), but there is no indication that this system call is portable
- Versions of Linux apparently support _llseek() or llseek() for raw devices. Compilers may support lseek() as accepting a 64-bit long or add lseek64(), according to a compile-time option.
- **Device Management:** These types of system calls are used to manage devices. Some examples are Request device, release device, read, write, get device attributes etc. Process usually require several resources to execute, if these resources are available, they will be granted and control returned to the user process. These resources are also thought of as devices. Some are physical, such as a video card, and others are abstract, such as a file. User programs *request* the device, and when finished they *release* the device. Similar to files, we can *read*, *write*, and *reposition* the device.

Chapter 3: Algorithm and Flowcharts

Chapter at a glance:

3.1 Algorithm

An algorithm is a finite sequential set of instructions which, if followed, accomplish a particular task or a set of tasks in finite time. Algorithms are used for calculation, data processing, and automated reasoning.

3.2 Complexity

There are two types of complexities of an algorithm, time complexity and space complexity.

- The **time complexity** of an algorithm is the amount of time the computer requires to execute the algorithm.
- The **space complexity** of an algorithm is the amount of memory space the computer requires, completing the execution of the algorithm.

3.3 Big OH Notation

The Big Oh (the "O" stands for "order of") notation is used to classify functions by their asymptotic growth function and hence finding the time complexity of an algorithm

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. A detailed flowchart is called _____ [WBSCTE 2022]
(a) Stack (b) Macro (c) Micro (d) Union

Answer: (c)

2. Flowcharts and algorithms are used for _____ [WBSCTE 2022]
(a) Better programming (b) Easy testing and debugging
(c) Efficient coding (d) All of the above

Answer: (d)

3. The operation represented by parallelogram _____ [WBSCTE 2022]
(a) Input/output (b) Assignment
(c) Comparison (d) Condition

Answer: (a)

Algorithm and Flowcharts

ITS.67

4. Which of the following is a pictorial representation of an algorithm? [WBSCTE 2022]
(a) Pseudocode (b) Program (c) Flowchart (d) None

Answer: (c)

5. Which data structure has fixed size? [Model Question]
(a) Arrays (b) Linked lists (c) Trees (d) Graphs

Answer: (a)

B. Fill in the blanks with the alternatives given in the bracket:

6. Integer is a primitive data type. [Model Question]

7. In 2s complement form, the highest integer that can be stored with n bits is 2^{n-1} [Model Question]

8. A data structure suitable for data remaining in secondary storage devices is structured. [Model Question]

9. A programming technique that never uses a GOTO statement is called structured programming. [Model Question]

C. Answer the following questions:

10. What are the main advantages of algorithm? [WBSCTE 2022]

Answer:

Advantages of Algorithm:

- It is easy to understand.
- An algorithm is a step-wise representation of a solution to a given problem.
- In Algorithm the problem is broken down into smaller pieces or steps hence, it is easier for the programmer to convert it into an actual program.

11. Differentiate between Storage structures and Data structures. [Model Question]

Answer:

The representation of a particular data structure in the memory of a computer is called a storage structure whereas a storage structure representation in auxiliary memory is often called a data structure.

12. What is data structure? [Model Question]

Answer:

A data structure is a mathematical or logical way of organizing data in the memory that consider not only the items stored but also the relationship to each other and also it is characterized by accessing functions.

13. Define an algorithm? [Model Question]

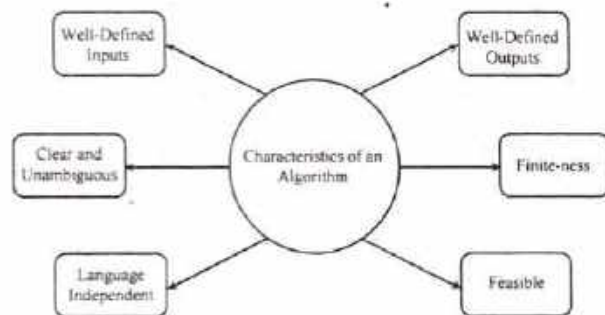
Answer: Refer to 3.1 of Chapter at a Glance.

14. What are the characteristics of algorithm?

[Model Question]

Answer:

- **Precision** – the steps are precisely stated (defined).
- **Uniqueness** – results of each step are uniquely defined and only depend on the input and the result of the preceding steps.
- **Finiteness** – the algorithm stops after a finite number of instructions are executed.
- **Input** – the algorithm receives input.
- **Output** – the algorithm produces output.
- **Generality** – the algorithm applies to a set of inputs.


15. Write down the properties of algorithm.

[Model Question]

Answer:

The properties of an algorithm are

- It should terminate after a finite time.
- It should produce at least one output.
- It should take zero or more input.
- It should be deterministic means giving the same output for the same input case.
- Every step in the algorithm must be effective i.e. every step should do some work.

16. What is flowchart? What are the guidelines of the flowchart? [Model Question]
Answer:

Flowchart is a diagrammatic representation of sequence of logical steps of a program. Flowcharts use simple geometric shapes to depict processes and arrows to show relationships and process/data flow.

Guidelines for Developing Flowcharts

These are some points to keep in mind while developing a flowchart –

- Flowchart can have only one start and one stop symbol
- On-page connectors are referenced using numbers
- Off-page connectors are referenced using alphabets

- General flow of processes is top to bottom or left to right
- Arrows should not cross each other

17. Explain the features of a good program.

[Model Question]

Answer:

A good computer program should have following characteristics:

- **Portability:** Portability refers to the ability of an application to run on different platforms (operating systems) with or without minimal changes. Due to rapid development in the hardware and the software, nowadays platform change is a common phenomenon. Hence, if a program is developed for a particular platform, then the life span of the program is severely affected.
- **Readability:** The program should be written in such a way that it makes other programmers or users to follow the logic of the program without much effort. If a program is written structurally, it helps the programmers to understand their own program in a better way. Even if some computational efficiency needs to be sacrificed for better readability, it is advisable to use a more user-friendly approach, unless the processing of an application is of almost importance.
- **Efficiency:** Every program requires certain processing time and memory to process the instructions and data. As the processing power and memory are the most precious resources of a computer, a program should be laid out in such a manner that it utilizes the least amount of memory and processing time.
- **Structural:** To develop a program, the task must be broken down into a number of subtasks. These subtasks are developed independently, and each subtask is able to perform the assigned job without the help of any other subtask. If a program is developed structurally, it becomes more readable, and the testing and documentation process also gets easier.
- **Flexibility:** A program should be flexible enough to handle most of the changes without having to rewrite the entire program. Most of the programs are developed for a certain period and they require modifications from time to time. For example, in case of payroll management, as the time progresses, some employees may leave the company while some others may join. Hence, the payroll application should be flexible enough to incorporate all the changes without having to reconstruct the entire application.
- **Generality:** Apart from flexibility, the program should also be general. Generality means that if a program is developed for a particular task, then it should also be used for all similar tasks of the same domain. For example, if a program is developed for a particular organization, then it should suit all the other similar organizations.
- **Documentation:** Documentation is one of the most important components of an application development. Even if a program is developed following the best programming practices, it will be rendered useless if the end user is not able to fully utilize the functionality of the application. A well-documented application is also useful for other programmers because even in the absence of the author, they can understand it.

Long Answer Type Questions

1. Write an algorithm that will read input temperature in Centigrade and convert to Fahrenheit. Draw a flowchart that will reads two values and determines the largest value and prints the largest value. [WBSCTE 2022]

Answer:

1st Part:

The algorithm for the conversion of temperature from Fahrenheit to Celsius is as follows:

Step 1: Read the temperature given in degree Centigrade.

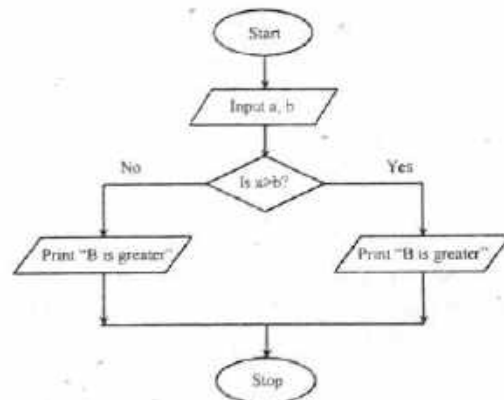
Step 2: Convert the temperature in Centigrade into Fahrenheit using the formula: $(30^{\circ}\text{C} \times 1.8) + 32$

Step 3: Print the Fahrenheit and Celsius value of temperature.

Step 4: End

2nd Part:

The flowchart that will reads two values and determines the largest value and prints that value is:



2. Write down the advantage and disadvantages of an algorithm.

Answer:

Advantages of Algorithms:

- It is easy to understand.
- An algorithm is a step-wise representation of a solution to a given problem.
- In Algorithm the problem is broken down into smaller pieces or steps hence, it is easier for the programmer to convert it into an actual program.

Disadvantages of Algorithms:

- Writing an algorithm takes a long time so it is time-consuming.
- Understanding complex logic through algorithms can be very difficult.
- Branching and Looping statements are difficult to show in Algorithms.

3. Define the symbols of flowchart.

[Model Question]

Answer:

Symbol	Name	Function
	Start/end	An oval represents a start or end point
	Arrows	A line is a connector that shows relationships between the representative shapes
	Input / Output	A parallelogram represents input or output
	Process	A rectangle represents a process
	Decision	A diamond indicates a decision

4. What is algorithm?

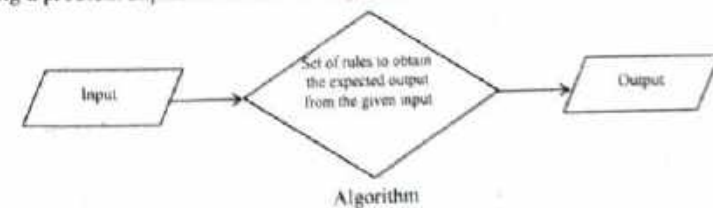
What do you mean by complexity of an algorithm? Show that the function $f(x)$ defined by: $f(1) = 1, f(n) = f(n-1) + 1$ has the complexity $O(n)$

[Model Question]

Answer:

1st Part:

Algorithm: An algorithm is a finite sequential set of instructions which, if followed, accomplish a particular task or a set of tasks in finite time, is an effective method for solving a problem expressed as a finite sequence of steps.



2nd Part:

There are two types of complexities of an algorithm, time complexity and space complexity.

Time Complexity: The time complexity of an algorithm quantifies the amount of time taken by an algorithm to run as a function of the size of the input to the problem. This commonly expressed using big O notation, which suppresses multiplicative constants and lower order terms. When expressed this way, the time complexity is said to be described *asymptotically*.

The **space complexity** of an algorithm is the amount of memory space the computer requires, completing the execution of the algorithm.

3rd part:

Applying the recursion up to $n-2$ times that is:

$$\begin{aligned} f(n) &= 1 + 1 + 1 + 1 + 1 + \dots + n - 2 \text{ times} + f(1) + 1 \\ &= (n-1) + f(1) \\ &= (n-1) + 1 \text{ where } f(1) = 1 \\ f(n) &= n \end{aligned}$$

$$\text{or, } |f(n)| \leq 2 * |g(n)| \text{ where } c = 2 \text{ \& } g(n) = n$$

$$\text{or, } f(n) = O(g(n))$$

$$\text{or, } f(n) = O(n)$$

➤ **5. Explain the terms: Best, Average and worst case complexities of an algorithm.**
[Model Question]

Answer:

There are different types of time complexities which can be analysed for an algorithm:

- Best-case time complexity, Average-case time complexity AND Worst-case time complexity.

The best-case time complexity of an algorithm is a measure of the minimum time that the algorithm will require for an input of size n . The running time of many algorithms varies not only for the inputs of different sizes but also for the different inputs of same size. The average-case time complexity of an algorithm is a measure of the average time that the algorithm will require for an input of size n .

The worst-case time complexity of an algorithm is a measure of the maximum time that the algorithm will require for an input of size n . Therefore, if various algorithms for sorting are taken into account and say, n input data items are supplied in reverse order for any sorting algorithm, then the algorithm will require n^2 operations to perform the sort which will correspond to the worst-case time complexity of the algorithm.

The work done by an algorithm for the execution of the input size n defines the time analysis as function $f(n)$ of the input data items.

➤ **6. How do you find the complexity of an algorithm? What is the relation between space complexity and time complexity of an algorithm? Justify your answer.**
[Model Question]

Answer:
1st Part:

Algorithmic complexity is concerned about how fast or slow particular algorithm performs. We define complexity as a numerical function $T(n)$ - time versus the input size n . We want to define time taken by an algorithm without depending on the implementation details. But we agree that $T(n)$ does depend on the implementation. A given algorithm will take different amounts of time on the same inputs depending on such factors as: processor speed; instruction set, disk speed, brand of compiler and etc. The way around is to estimate efficiency of each algorithm asymptotically. We measure time $T(n)$ as the number of elementary "steps" (defined in any way), provided each such step takes constant time.

Let us consider two classical examples: addition of two integers. We will add two integers digit by digit (or bit by bit), and this will define a "step" in our computational model. Therefore, we say that addition of two n -bit integers takes n steps. Consequently, the total computational time is $T(n) = c * n$, where c is time taken by addition of two bits. On different computers, addition of two bits might take different time, say c_1 and c_2 , thus the addition of two n -bit integers takes $T(n) = c_1 * n$ and $T(n) = c_2 * n$ respectively. This shows that different machines result in different slopes, but time $T(n)$ grows linearly as input size increases.

2nd Part:

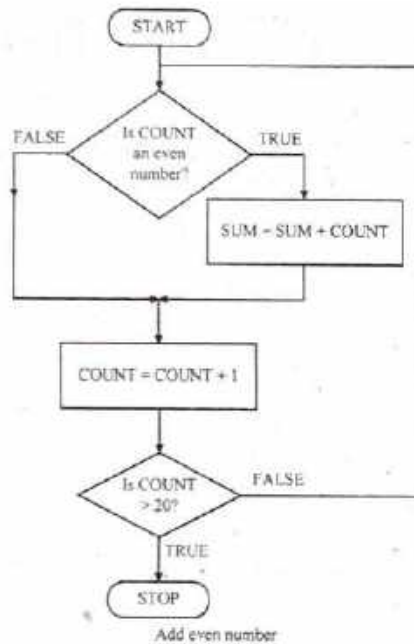
The time complexity of an algorithm is, the faster the algorithm will carry out its work in practice. Apart from time complexity, its space complexity is also important. This is essentially the number of memory cells which an algorithm needs. There is often a time-space-trade off involved in a problem, that is, it cannot be solved with few computing time and low memory consumption. One of them has to compromise and to exchange computing time for memory consumption or vice-versa. The space-complexity cannot be more than the time complexity because writing X units of space takes $\omega(X)$. Every algorithm that is $O(n)$ time is also $O(n^2)$ space.

➤ **7. By which features a program is not an algorithm? How can you describe an algorithm with the help of a flowchart?**
[Model Question]

Answer:

Actually, Algorithm + Data Structure = Program

The algorithm of addition of even numbers are shown below using a flow-chart.



Chapter 4: HTML, CSS & Javascript

Chapter at a glance:

4.1 HTML

HTML stands for HyperText Markup Language. It is used to design web pages using a markup language. HTML is the combination of Hypertext and Markup language. Hypertext defines the link between web pages. A markup language is used to define the text document within the tag which defines the structure of web pages. This language is used to annotate (make notes for the computer) text so that a machine can understand it and manipulate text accordingly. Most markup languages (e.g. HTML) are human-readable. The language uses tags to define what manipulation has to be done on the text.

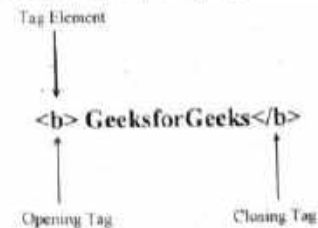
HTML is a markup language used by the browser to manipulate text, images, and other content, in order to display it in the required format. HTML was created by Tim Berners-Lee in 1991. The first-ever version of HTML was HTML 1.0, but the first standard version was HTML 2.0, published in 1995.

HTML Released Year



Elements and Tags:

HTML uses predefined tags and elements which tell the browser how to properly display the content. Remember to include closing tags. If omitted, the browser applies the effect of the opening tag until the end of the page.



HTML page structure:

The basic structure of an HTML page is laid out below. It contains the essential building-block elements (i.e. doctype declaration, HTML, head, title, and body elements) upon which all web pages are created.

HTML Page Structure

<!DOCTYPE html> ← Tells version of HTML
 <html> ← HTML Root Element
 <head> ← Used to contain page HTML metadata
 <title>Page Title</title> ← Title of HTML page
 </head>
 <body> ← Hold content of HTML
 <h2>Heading Content</h2> ← HTML heading tag
 <p>Paragraph Content</p> ← HTML paragraph tag
 </body>
 </html>

4.2 CSS

Cascading Style Sheets, fondly referred to as CSS, is a simply designed language intended to simplify the process of making web pages presentable. CSS allows you to apply styles to web pages. More importantly, CSS enables you to do this independent of the HTML that makes up each web page. It describes how a webpage should look: it prescribes colors, fonts, spacing, and much more. In short, you can make your website look however you want. CSS lets developers and designers define how it behaves, including how elements are positioned in the browser.

4.3 Javascript

JavaScript is a lightweight, cross-platform, and interpreted compiled programming language which is also known as the scripting language for web pages. It is well-known for the development of web pages, many non-browser environments also use it. JavaScript can be used for **Client-side** developments as well as **Server-side** developments. Javascript is both imperative and declarative type of language. JavaScript contains a standard library of objects, like **Array**, **Date**, and **Math**, and a core set of language elements like **operators**, **control structures**, and **statements**.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. tag makes the enclosed text bold. What is other tag to make text bold?

[WBSCTE 2022]

- (a) (b) <dar> (c) <black> (d) <cmp>

Answer: (a)

2. All html tags are must be enclosed in

[WBSCTE 2022]

- (a) ? And ! (b) <And > (c) { And } (d) # And #

Answer: (b)

3. CSS stands for

[WBSCTE 2022]

- (a) Cascade style sheets (b) Color and style sheets
 (c) Cascading style sheets (d) None of the above

Answer: (c)

4. If we want define style for a unique element, then which css selector will we use?

[WBSCTE 2022]

- (a) Id (b) text (c) class (d) name

Answer: (a)

5. Inside which HTML element do we put the JavaScript?

[WBSCTE 2022]

- (a) <js> (b) <script> (c) <scripting> (d) <javascript>

Answer: (b)

6. Tags and text that are not directly displayed on the page are written in _____ section.

[WBSCTE 2022]

- (a) <html> (b) <head> (c) <title> (d) <body>

Answer: (b)

7. What will the following code return:

[WBSCTE 2022]

Boolean(10>9)

- (a) NaN (b) true (c) false

Answer: (b)

8. Which of the following CSS property is used to set the blend mode for each background layer of an element? [WBSCTE 2022]
- (a) background-blend-mode property (b) background-collapse property
(c) background-transform property (d) background-origin property

Answer: (a)

9. Which HTML attribute is used to define inline styles? [Model Question]
- (a) style (b) font (c) styles (d) div

Answer: (a)

10. What does a Markup tag tell the web browser? [Model Question]
- (a) how to organize the page (b) how to display message box on page
(c) how to display the page (d) none of these

Answer: (b)

11. HTML documents are saved in [Model Question]
- (a) ASCII text (b) machine language codes
(c) special binary format (d) none of these

Answer: (a)

12. Which of the following is correct regarding meta tag in HTML? [Model Question]
- (a) <meta>...</meta> (b) <meta name = "" />
(c) <metadata>...</metadata> (d) <metadata name = "" />

Answer: (b)

13. Which of the following methods is used to get meta information of a resource? [Model Question]
- (a) TOP (b) HEAD (c) GET (d) GRAB

Answer: (b)

14. DHTML is for [Model Question]
- (a) model (b) view (c) controller (d) servlet

Answer: (b)

15. Which of the following technologies is used to display XML data? [Model Question]
- (a) CSS (b) XPath (c) XQuery (d) XHTML

Answer: (a)

16. Which of the following is an invalid CSS selector? [Model Question]
- (a) Title selectors (b) HEAD selectors
(c) TAG selectors (d) Id selectors

Answer: (b)

17. A declaration is terminated by a [Model Question]
- (a) } - end curly bracket (b) . - period
(c) ; - semi colon (d) ! - exclamation sign

Answer: (c)

18. The correct HTML tag for the biggest heading is [Model Question]
- (a) <h6> (b) <heading small> (c) <small heading> (d) <h1>

Answer: (d)

19. Which HTML attribute is used to define inline styles? [Model Question]
- (a) Class (b) Styles (c) Style (d) Font

Answer: (c)

20. Javascript's default object is [Model Question]
- (a) navigator (b) documents (c) window (d) browser

Answer: (c)

21. Choose the correct HTML tag for the smallest heading [Model Question]
- (a) <h6> (b) <heading small> (c) <small heading> (d) <h1>

Answer: (a)

22. Which of the following syntax is correct for commenting lines in XML? [Model Question]
- (a) /! (b) ! * comments * ! (c) :: (d) <!--comments-->

Answer: (d)

23. The in XML is equivalent to the schema in database. [Model Question]
- (a) element (b) attribute (c) parser (d) DTD

Answer: (d)

24. allow style sheets to differentiate between different element type. [Model Question]
- (a) Pseudo-classes (b) Pseudo-elements
(c) Contextual selectors (d) Classes

Answer: (b)

25. Which is the correct CSS syntax?

[Model Question]

- (a) {body;color = black(body)}
 (b) body {color : black}
 (c) body : color = black
 (d) {body;color:black}

Answer: (b)

26. In which method URL exposes the data?

[Model Question]

- (a) Post
 (b) Get
 (c) Both Post and Get
 (d) All of these

Answer: (b)

27. FORM tag is used in HTML inside _____ Tag.

[Model Question]

- (a) Head
 (b) Title
 (c) Body
 (d) All of these

Answer: (c)

28. Choose the correct HTML tag for largest heading.

[Model Question]

- (a) <h6>
 (b) Head
 (c) <h1>
 (d) <heading>

Answer: (c)

B. Fill in the blanks with the alternatives given in the bracket:

29. HTML stands for _____

[Model Question]

Answer: Hypertext Markup Language.

C. Answer the following questions:

30. What are the advantages of JavaScript over HTML?

[Model Question]

Answer:

The advantages of JavaScript over HTML is that, JavaScript enhances HTML by making it dynamic. HTML as we know is static and therefore once an HTML document has been created it cannot be changed.

JavaScript has two basic functionalities which are:

1. To build HTML dynamically as the web page is loaded and;
2. To monitor as well as react to User events.

31. What are the usage of: hover, active and: focus dynamic pseudo classes?

[Model Question]

Answer:

Hover pseudo-class: The hover pseudo-class applies while the user designates an element (with some pointing device), but does not activate it. For example, a visual user agent could apply this pseudo-class when the cursor (mouse pointer) hovers over a box generated by the element. User agents not supporting interactive media do not have to

support this pseudo-class. Some conforming user agents supporting interactive media may not be able to support this pseudo-class (e.g., a pen device).

Active pseudo-class: The active pseudo-class applies while an element is being activated by the user. For example, between the times the user presses the mouse button and releases it.

Focus pseudo-class: The focus pseudo-class applies while an element has the focus (accepts keyboard events or other forms of text input).

32. Design a HTML page on your friends. List your friends. Each friend's name is a link. Prepare separate HTML documents on each friend and call them in the appropriate link. [Model Question]

Answer:

```
<html>
<head><title> Display the links</title>
</head>
<body>
<h2>My Friends Web Pages?</h2>
<ol>
  <li><a href = "manju.html">Manju Biswas</a></li>
  <li><a href = "gopa.html">Gopa Mandal</a></li>
</ol>
</body>
</html>
```

33. What is Frame? Explain merits and demerits of frame.

[Model Question]

Answer:

HTML frames are used to divide your browser window into multiple sections where each section can load a separate HTML document. A collection of frames in the browser window is known as a frameset. The window is divided into frames in a similar way the tables are organized: into rows and columns.

Merits of Frame:

- The main advantage of frames is that it allows the user to view multiple documents within a single Web page.
- It is possible to load pages from different servers in a single frameset.
- The concern that older browsers do not support frames can be addressed using the <noframe> tag. This tag provides a section in an HTML document to include alternative content for browsers without support for frames. However, it requires that the Web designer provide two formats for one page.

Demerits of Frame:

There are few drawbacks with using frames, so it's never recommended to use frames in your webpages –

- Some smaller devices cannot cope with frames often because their screen is not big enough to be divided up.
- Sometimes your page will be displayed differently on different computers due to different screen resolution.
- The browser's back button might not work as the user hopes.
- There are still few browsers that do not support frame technology.

34. What is cookie?

[Model Question]

Answer:

Cookies are the name given to the small text files your browser stores on your computer, which contain information relevant to the sites you have visited in the past. Another definition is that, cookie is a variable that is stored on the visitor's computer. Each time the same computer requests a page with a browser, it will send the cookie too. With JavaScript, you can both create and retrieve cookie values.

35. Write a function to create a cookie in JavaScript.

[Model Question]

Answer:

```
function createCookie(name, value, days) {
    if (days) {
        var date = new Date();
        date.setTime(date.getTime() + (days * 24 * 60 * 60 * 1000));
        var expires = "; expires=" + date.toGMTString();
    }
    else var expires = "";
    document.cookie = name + "=" + value + expires + "; path=/";
}
```

Long Answer Type Questions

1. (a) What is the difference between static and dynamic web pages?

(b) What are the advantages of using Javascript?

[WBSCTE 2022]

Answer:

a) The difference between Static and dynamic web pages are-

Static Web page

Static web pages are also known as flat or stationary web page. They are loaded on the client's browser as exactly they are stored on the web server. Such web pages contain only static information. User can only read the information but can't do any modification or interact with the information.

Static web pages are created using only HTML. Static web pages are only used when the information is no more required to be modified.

Dynamic Web page

Dynamic web page shows different information at different point of time. It is possible to change a portion of a web page without loading the entire web page. It has been made possible using Ajax technology.

b) The advantages of using JavaScript are the following:

- **Less server interaction** – Validate user input before sending the page off to the server. This saves server traffic, which means less load on your server.
- **Immediate feedback to the visitors** – They don't have to wait for a page reload to see if they have forgotten to enter something.
- **Increased interactivity** – Create interfaces that react when the user hovers over them with a mouse or activates them via the keyboard.
- **Richer interfaces** – Use JavaScript to include such items as drag-and-drop components and sliders to give a Rich Interface to your site visitors.

2. (a) Write a Javascript program to find HCF or GCD.

(b) In how many ways a javascript code can be involved in an HTML file?

(c) What is the difference between the operators '==' & '==='?

[WBSCTE 2022]

Answer:

(a) Javascript program to find HCF or GCD:

```
// program to find the HCF or GCD of two integers
let hcf;
// take input
const number1 = prompt('Enter a first positive integer: ');
const number2 = prompt('Enter a second positive integer: ');
// looping from 1 to number1 and number2
for (let i = 1; i <= number1 && i <= number2; i++) {
    // check if is factor of both integers
    if (number1 % i == 0 && number2 % i == 0) {
        hcf = i;
    }
}
// display the hcf
console.log('HCF of $(number1) and $(number2) is $(hcf).');
```

(b) You can include JavaScript in your HTML in two ways:

- Writing the code in your HTML
- Including it as a link to an external file

Using the script tag to include an external JavaScript file:

To include an external JavaScript file, we can use the script tag with the attribute src and used the src attribute when we use images. The value for the src attribute should be the path to your JavaScript file. This script tag should be included between the <head> tags in your HTML document.

```
<script type="text/javascript" src="path-to-javascript-
file.js"></script>
```

(c) The '=' operator tests for abstract equality i.e. it does the necessary type conversions before doing the equality comparison.

But the '===' operator tests for strict equality i.e. it will not do the type conversion hence if the two values are not of the same type, when compared, it will return false.

Example:

```
<script>
// In R.H.S, string "9" is converted into
// number 9, hence returns true.
document.write(9 == "9");
// used for next line
document.write('<br>')
// Here no type conversion takes place,
// hence returns false
document.write(9 === "9");
</script>
```

Output:

true
false

3. Write a program to create HTML table with the following output:

[WBSCTE 2022]

Name	Department	Mobile Number
Rahul Das	DME	9123443518
Sanjib Paul	DCE	8513546728
Sibnath Dey	DSE	9175354578

Answer:

```
<!DOCTYPE html>
<html>
<body>
<table>
<tr>
<th>Name</th>
<th>Department</th>
<th>Mobile Number</th>
</tr>
<tr>
<td>Rahul Das </td>
<td>DME</td>
<td>9123443518</td>
</tr>
<tr>
<td>Sanjib Paul</td>
<td>DCE</td>
<td>8513546728</td>
</tr>
<tr>
<td>Sibnath Dey</td>
<td>DSE</td>
<td>9175354578</td>
</tr>
</table>
</body>
</html>
```

4. Illustrate the basic tags of a HTML page with an example. [Model Question]

Answer:

The basic structure for all HTML documents is simple and should include the following minimum elements or tags:

- <html> - The main container for HTML pages
- <head> - The container for page header information
- <title> - The title of the page
- <body> - The main body of the page

Remember that before an opening <html> tag, an XHTML document can contain the optional XML declaration, and it should always contain a DOCTYPE declaration indicating which version of XHTML it uses.

An HTML document starts and ends with `<html>` and `>/html>` tags. These tags tell the browser that the entire document is composed in HTML. Inside these two tags, the document is split into two sections:

- The `<head>...</head>` elements, which contain information about the document such as title of the document, author of the document etc. Information inside this tag does not display outside.
 - The `<title>...</title>` elements, which element is a child of the `<head>` element. It specifies a title for every page that you write inside the `<title>` element.
- The `<body>...</body>` elements, which contain the real content of the document that you see on your screen.

In spite of the above four tags there are lots tags available in HTML. We discuss some tags among them.

Example:

```
<html>
  <head>
    <title> My first page </title>
  </head>
  <body text="white" bgcolor="blue">
    This is a html page.
    <i>This is italic</i> and this is <b> bold</b>.
  </body>
</html>
```

Heading Tag

One of the earliest means of formatting text was the heading tag. It is available in six levels of importance from `<h1>` down to `<h6>`, as shown in the following code and in Figure. You might think of these headers as headlines for chunks of text.

```
<html>
  <head>
    <title>Header Example</title>
  </head>
  <body>
    <h1>This is an example of a level 1 header.</h1>
    <h2>This is an example of a level 2 header.</h2>
    <h3>This is an example of a level 3 header.</h3>
    <h4>This is an example of a level 4 header.</h4>
    <h5>This is an example of a level 5 header.</h5>
    <h6>This is an example of a level 6 header.</h6>
  </body>
</html>
```

Paragraph Tag

The `<p>` element offers a way to structure your text. The `p` tag signifies a paragraph break. The paragraph breaks cause the browser to skip a line. Also, the `p` tag is considered a *container* tag because its opening and closing tags should be used to contain paragraphs of content.

Each paragraph of text should go in between an opening `<p>` and closing `</p>` tag.

`</p>` tag as shown below in the example:

```
<p>Here is a paragraph of text.</p>
<p>Here is a second paragraph of text.</p>
<p>Here is a third paragraph of text.</p>
```

You can use `align` attribute to align your paragraphs.

```
<p align="left">This is left aligned.</p>
<p align="center">This is center aligned.</p>
<p align="right">This is right aligned.</p>
<p align="justify">This is justified. This works when you have multiple lines in
your paragraph and you want to justify all the lines so that they can look more
nice.</p>
```

Line break:

The `br` tag inserts a simple line break. Whenever you use the `<br />` element, anything following it starts on the next line. This tag is an example of an **empty** element, where you do not need opening and closing tags, as there is nothing to go in between them. It tells the browser to drop down to the next line before continuing. If you insert multiple `br` tags, the browser will drop down several lines before continuing.

Example:

```
<p>
  Jack and Jill went up a hill <br />
  To catch a pail of water <br />
  Jack fell down and broke his crown <br />
  And Jill came tumbling after <br />
</p>
```

5. (a) State different types of lists with examples in HTML.
(b) State the use of `<pre>` and `<del>` tags.

[Model Question]

Answer:

- (a) There are three types of lists in HTML:

- **Ordered Lists:** These are sometimes called numbered lists, and list items that have a specific numerical order or ranking. This list is created by `<ol>` tag. The numbering starts at one and is incremented by one for each successive ordered list element tagged with `<li>`. You can use `type` attribute to specify the type of numbers you like

(Attributes: type = I|A|a|I|i). By default it's a generic numbers. You can also use *start* attribute to specify the beginning of any index. For example:

```
<ol>
  <li>Entry 1</li>
  <li>Entry 2</li>
  <li>Entry 3</li>
</ol>
```

- **Unordered Lists:** These are sometimes called bulleted lists, because they have small bullet icons in front of the list items. They are for lists that don't have a required order. Use the `<ul>` tag (the ending `</ul>` tag is required) to create a list with bullets instead of numbers. Just like with the ordered list, the elements are created with the `<li>` tag. For example:

```
<ul>
  <li>Entry 1</li>
  <li>Entry 2</li>
  <li>Entry 3</li>
</ul>
```

- **Definition Lists:** These are lists of items that have two parts, a term to be defined and the definition. This creates lists similar to a dictionary or glossary. There are three tags associated with the definition list:

- i) `<dl>` to define the list
- ii) `<dt>` to define the definition term
- iii) `<dd>` to define the definition of the term

Here is how a definition list looks:

```
<dl>
  <dt>This is a definition term</dt>
  <dd>And this is the definition</dd>
  <dt>term 2</dt>
  <dd>definition 2</dd>
  <dt>term 3</dt>
  <dd>definition 3</dd>
</dl>
```

(b) Use of pre tag: Sometimes you want your text to follow the exact format of how it is written in the HTML document. In those cases, you can use the preformatted tag (`<pre>`). Any text between the opening `<pre>` tag and the closing `</pre>` tag will preserve the formatting of the source document.

Use of del tag: The *HTML Deleted Text Element* (`<del>`) represents a range of text that has been deleted from a document. This element is often (but need not be) rendered with strike-through text.

```
<html>
<head><title>Use of del tag in html</title>
```

```
</head>
<body>
<p><del>This text has been deleted</del>, here is the rest
of the paragraph.
</p>
</body>
</html>
```

6. (a) What is the difference between class selector and id selector in CSS.

(b) Give coding example for inline styles of CSS.

[Model Question]

Answer:

(a) i) The difference between class selector and id selector is that an id selector can be called only once in a document, but a class selector can be called multiple times in a document.

ii) The id selector uses the id attribute of the HTML element, and is defined with a "#". But, the class selector uses the HTML class attribute, and is defined with a ".".

iii) The second difference is that id can be called by Javascript's `getElementById` function.

(b) Coding example for inline styles of CSS:

[Model Question]

```
<html>
<body>
<h1 style="color:red;margin-left:30px;">This is a heading.</h1>
<p>This is an example of inline styles .</p>
</body>
</html>
```

7. Why do we need to have a dynamic webpage? What is the advantage of using style sheet in formatting the different elements of webpages in a website? How can we link a style sheet to a webpage?

[Model Question]

Answer:

A dynamic web page is a web page with web content that varies based on specific parameters provided by a user or a computer program. Dynamic webpages manage the content in the website effectively and thereby make the site more attractive which in turn will attract more customers to the site. Also dynamic web designs empower the owners with the ability to upload information on their own, through password protected administrative platforms.

Benefits/Advantages of CSS:

1) We can write CSS once and then reuse as many times in the same pages. Thus it saves time.

- 2) If using CSS the pages load faster. If we are using CSS we do not need to write HTML tag attributes every time. So less code and faster download times.
- 3) CSS allows easy maintenance as to make a global change simply change the style, and all the elements in all the web pages will be updated automatically.
- 4) CSS has a much wider array of attributes than HTML.
- 5) CSS allow content to be optimized for more than one type of device.

There are four different ways of inserting a CSS styles in webpage.

1) Embedded or Internal CSS: Using the <style> Element:

We can put CSS rules into an HTML document using the <style> element. This tag is placed inside <head>...</head> tags. A generic syntax:

```
<head> <style type="text/css" media="..."> Style Rules ..... </style> </head>
```

2) Inline CSS: Uses the style Attribute of any HTML element to define style rules. These rules will be applied to that element only.

Generic syntax:

```
<element style="...style rules...">
```

3) External CSS: Uses the <link> Element to include an external stylesheet file in the HTML document. An external style sheet is a separate text file with .css extension. Define all the Style rules within this text file and then you can include this file in any HTML document using <link> element.

Generic syntax of including external CSS file:

```
<head> <link type="text/css" href="..." media="..." /> </head>
```

4) Imported CSS: Uses @import Rule to import an external stylesheet in a manner similar to the <link> element.

Generic syntax of @import rule.

```
<head> <@import "URL"; </head>
```

8. (a) Using the FROM tag of HTML create a registration form containing the below mentioned fields:

- (i) First Name (Using text boxes)
- (ii) Last Name (Using text boxes)
- (iii) E-mail ID (Using text boxes)
- (iv) Gender (Using radio button)
- (v) Hobbies (Using check boxes)
- (vi) Submit Button and Reset Button.

(b) In your browser there will be a textbox to enter your name. There will be a SUBMIT button. If your name field is empty then on submit this will give an alert that your name field is empty. If your name is more than 10 characters

long then it will give an alert to enter your name properly. Else it will display your name in capital, bold and in red colour. [Model Question]

Answer:

```
(a) <html>
<head><title>Welcome to Registration Page</title></head>
<body>
<form>
<pre>
First Name <input type="text" size="10" maxlength="40"
name="name"><br />
Last Name <input type="text" size="10" maxlength="40"
name="name"><br />
Email ID <input type="text" size="10" maxlength="40"
name="name"><br />
Gender <input type="radio" name="Registration"
value="Male">Male<br />
<input type="radio" name="Registration"
value="Female">Female<br />
Hobbies
<input type="checkbox" name="Registration"
value="Drawing">Drawing<br />
<input type="checkbox" name="Registration"
value="Singing">Singing<br />
<input type="checkbox" name="Registration"
value="Dancing">Dancing<br />
</pre>
<button type="submit" name="Registration"
value="Submit">Submit</button>
<input type="reset" value="Reset" />
</form>
</body>
</html>
```

(b)

```
<html>
<head>
<script type="text/javascript">
function abc()
{
var name=document.getElementById("name").value;
if(name=="")
alert("Your name field is empty!");
else if(name.length > 10)
alert("Enter your name properly!");
}
```



```

else
{
    name=name.toUpperCase();
    name=name.bold();
    document.write(name.fontcolor("red"));
}
}
</script>
</head>
<body>
<input type='text' title='Name' id='name'>
<input type='submit' onclick='abc()' value='Submit'>
</body>
</html>

```

9. (a) Write down the features of scripting language. How markup language is different from scripting language?

(b) Create a HTML page with the following features.
Hyperlink, List and Form.

[Model Question]

Answer:

(a) A markup language is simply a set of tags that are used to "mark up" text documents so that sections of text can be logically arranged and labeled. These documents can be viewed as plain text, or, more commonly, are viewed through a browser. The browser parses the document, looking for markup tags, and it then arranges the text and/or formats it according to the values in the tags.

Scripting languages, on the other hand, are interpreted at run-time. This means that every time you want to run the program, a separate program needs to read the code, interpret it, and then follow the instructions in the code. Compiled code has already been interpreted into machine language, so it will typically execute faster because the conversion into machine language has already been done.

(b)

```

<html>
<body>
<h2>Introduction to Hyperlink</h2>
<p><a href="#C4">Jump to Chapter 4</a></p>
<h2>Chapter 1</h2>
<p>This chapter explains 1</p>
<h2>Chapter 2</h2>
<p>This chapter explains 2</p>
<h2>Chapter 3</h2>
<p>This chapter explains 3</p>
<h2 id="C4">Chapter 4</h2>

```

```

<p>This chapter explains 4</p>
<h2>Chapter 5</h2>
<p>This chapter explains 5</p>
<h2>Introduction to Unordered List without Bullets.....</h2>
<ul style="list-style-type:none">
    <li>Coffee</li>
    <li>Tea</li>
    <li>Milk</li>
</ul>
<h2> Introduction to Ordered List.....</h2>
<ol>
    <li>Coffee</li>
    <li>Tea</li>
    <li>Milk</li>
</ol>
<h2> Introduction to Forms....</h2>
<form>
    First name:<br>
    <input type="text" name="firstname">
    <br>
    Last name:<br>
    <input type="text" name="lastname">
</form>
</body>
</html>

```

10. Write a simple servlet program which will return the current date and time.
[Model Question]

Answer:

```

import java.io.*;
import java.text.*;
import java.util.Date;
import javax.servlet.*;
import javax.servlet.http.*;

public class CurrentDate extends HttpServlet {
    public void doGet(HttpServletRequest request, HttpServletResponse
    response) throws ServletException, IOException {
        // Set response content type
        response.setContentType("text/html");
        PrintWriter out = response.getWriter();
        String title = "Display Current Date & Time";
        Date dNow = new Date();
        SimpleDateFormat ft = new SimpleDateFormat ("E yyyy.MM.dd 'at'
        hh:mm:ss a zzz");

```



```
String docType = "<!doctype html public \"-//w3c//dtd html 4.0 \"
+
*transitional//en\">\n";
out.println(docType +
* <html>\n" +
* <head><title>* + title + "</title></head>\n" +
* <body bgcolor = \"#f0f0f0\">\n" +
* <h1 align = \"center\">* + title + "</h1>\n" +
* <h2 align = \"center\">* + ft.format(dNow) + "</h2>\n" +
* </body>
* </html>*
};
}
```

Output:

Display Current Date & Time
Mon 2010.06.21 at 10:06:44 PM GMT+04:00

11. Create a JSP page to find the factorial of a number.

[Model Question]

Answer:

First install Netbeans Software. Then choose new project (web application). Now select a index.jsp page within "Web pages". and paste the following code to create a simple html page(i.e. your index page)

CODE:

```
<%-
Document : index
Created on : 12 Oct, 2012, 7:35:11 PM
Author : nested code team
--%>
<%@page contentType="text/html" pageEncoding="UTF-8"%>
<!DOCTYPE HTML PUBLIC "-//W3C//DTD HTML 4.01
Transitional//EN"
"http://www.w3.org/TR/html4/loose.dtd">
<html>
<head>
<meta http-equiv="Content-Type" content="text/html;
charset=UTF-
8">
<title>JSP Page</title>
</head>
```

```
<body><br><br><center>
<form action="fact.jsp ">
<h1>Enter the no :: <input type="text" name="n"><br><br>
<input type="submit" value="Submit"></h1>
</form></center>
</body>
</html>
```

Now we have to create the "fact.jsp" page, so choose again a new jsp page and give the name as "fact.jsp". Now paste the following code.

```
<HTML>
<HEAD>
<TITLE>Recursion</TITLE>
</HEAD>
<BODY>
<H1>factorial number Using Recursion</H1>
<%!
int numberfactorial(int number)
{
if (number == 1) {
return number;
}
else {
return number * numberfactorial(number - 1);
}
}
%>
<%
out.println("The factorial of 5 is " + numberfactorial(5));
%>
</BODY>
</HTML>
```

12. What are the different ways of adding styles (CSS) to a webpage? Write the precedence rule (or priority) for the same with suitable example.

[Model Question]

Answer:

1st part:

There are four ways to associate styles in the HTML document.

1) Embedded CSS – Using the <style> Element:

We can put CSS rules into an HTML document using the <style> element. This tag is placed inside <head>...</head> tags. Rules defined using this syntax will be applied to all the elements available in the document.

A generic syntax:

```
<head> <style type="text/css" media="..."> Style Rules
..... </style> </head>
```

2) Inline CSS: Uses the style Attribute of any HTML element to define style rules. These rules will be applied to that element only.

Generic syntax:

```
<element style="...style rules....">
```

3) External CSS: Uses the <link> Element to include an external stylesheet file in the HTML document. An external style sheet is a separate text file with .css extension. Define all the Style rules within this text file and then you can include this file in any HTML document using <link> element.

Generic syntax of including external CSS file:

```
<head> <link type="text/css" href="..." media="..." /> </head>
```

4) Imported CSS: Uses @import Rule to import an external stylesheet in a manner similar to the <link> element.

Generic syntax of @import rule.

```
<head> <@import "URL"; </head>
```

Here URL is the URL of the style sheet file having style rules.

Another syntax can be used as well:

```
<head> <@import url("URL"); </head>
```

2nd part:

```
<!DOCTYPE html>
<html>
<head>
<style>
#main {
    width: 400px;
    height: 150px;
    border: 1px solid #c3c3c3;
    display: -webkit-flex; /* Safari */
    display: flex;
}
#main div {
    width: 70px;
    height: 70px;
}
/* Safari 6.1+ */
div#myRedDIV {-webkit-order: 2;}
```

```
div#myBlueDIV {-webkit-order: 4;}
div#myGreenDIV {-webkit-order: 3;}
div#myPinkDIV {-webkit-order: 1;}
/* Standard syntax */
div#myRedDIV {order: 2;}
div#myBlueDIV {order: 4;}
div#myGreenDIV {order: 3;}
div#myPinkDIV {order: 1;}
</style>
</head>
<body>
<h1>The order Property</h1>
<div id="main">
    <div style="background-color:coral;" id="myRedDIV"></div>
    <div style="background-color:lightblue;" id="myBlueDIV"></div>
    <div style="background-color:lightgreen;"
id="myGreenDIV"></div>
    <div style="background-color:pink;" id="myPinkDIV"></div>
</div>
<p><b>Note:</b> Internet Explorer 10 and earlier versions do not
support the order property.</p>
<p><b>Note:</b> Safari 6.1 (and newer) supports an alternative,
the -webkit-order property.</p>
</body>
</html>
```


Chapter 5: Introduction to Computer Security

Chapter at a glance:

5.1 Computer Security

Computer security is a field of computer science concerned with the control of risks related to computer use. The subject deals with the objective to create a secure computing platform, designed so that agents (users or programs) can only perform actions that have been allowed. This involves specifying and implementing a security policy. The actions in question can be reduced to operations of access, modification and deletion. In a secure system the legitimate users of that system are still able to do what they should be able to do.

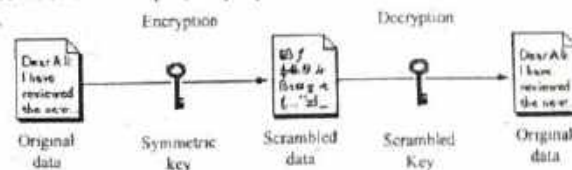
In the networked world, the concepts associated with security are described below.

5.2 Authentication, Data Encryption, Secure Transmission

Authentication is the process of verifying an identity claimed by or for a system entity. Authentication process consists of two steps --- Identification step, where an identifier is presented to the security system and a Verification step, where authentication information that corroborates the binding between the entity and the presented identifier, is generated. One common method of authentication is the through the familiar username-password verification. However, more complex schemes are also employed in situations that demand high security.

In Data Encryption, techniques of cryptography are used to "encrypt" a bunch of data sent over the network. The idea is that even if a snooper is able to acquire such a data, he may not be able to put it to use because of the inability to "decrypt" it. During encryption, the original data is encrypted based on a "cipher" --- an algorithm that transforms the original data into encrypted data based on a "key". The decryption can be carried out easily only if the key is known. Otherwise, it may be very difficult, if not impossible, to do so.

Till about 1978, the concept that was used is known as "private-key cryptography" --- the encrypting and the decrypting keys were the same. This method has an inherent weakness, namely, the key itself. The legitimate receiver must be made aware of the key and the process of "key exchange" may as well be vulnerable to snooping. If the key becomes known to the snooper, its purpose is lost. The figure below explains the concept.



Security was revolutionized with the invention of "public-key cryptography".

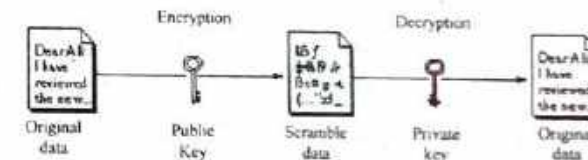
Public Key Cryptography is a form of cryptography, which generally allows users to communicate securely without having prior access to a shared secret key. This is done by using a pair of cryptographic keys, designated as **public key** and **private key**, which are related mathematically.

In public key cryptography, the *private key is generally kept secret*, while the *public key may be widely distributed*. In a sense, one key "locks" a lock; while the other is required to unlock it. It should not be possible to deduce the private key of a pair given the public key.

Any data encrypted with the private key can be decrypted with the corresponding public key.

Any data encrypted with the public key can be decrypted with the corresponding private key.

The figure below explains the above concepts.



Key Length and Encryption Strength: In general, the strength of encryption is related to the difficulty of discovering the key, which in turn depends on both the cipher used and the length of the key. For example, the difficulty of discovering the key for the RSA cipher most commonly used for public-key encryption depends on the difficulty of factoring large numbers, a well-known mathematical problem. Encryption strength is often described in terms of the size of the keys used to perform the encryption: in general, longer keys provide stronger encryption. Key length is measured in bits. For example, 128-bit keys for use with the RC4 symmetric-key cipher supported by SSL provide significantly better cryptographic protection than 40-bit keys for use with the same cipher. Roughly speaking, 128-bit RC4 encryption is 3×10^{28} times stronger than 40-bit RC4 encryption.

Secure Communication: Here, the sender encrypts the data being sent with the public key of the receiver. By definition, the public key of the sender is known to everybody, including the sender and any potential snooper. If a snooper gets the encrypted data, he can do nothing since he does not know the private key of the receiver. But the receiver can easily decrypt the data.

Authenticated Communication: Here, the purpose is different --- the data received from a sender should be verifiable to be sent by nobody but the sender. Snooping is not of particular concern. Here, the sender encrypts the data with his private key. This in effect puts his signature --- called **digital signature**, on the encrypted data. Anybody can

decrypt the data using the sender's public key but the data must have come from the sender --- anybody masquerading as the sender would not have the sender's private key and hence message sent and decrypted using the original sender's public key would be meaningless.

Secure Authenticated Communication: Here sender first signs the data by encrypting with his public key and then secures it by encrypting it further with the receiver's public key. Upon receiving the message, the receiver first decrypts it with his private key and then further decrypts it with the sender's public key.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

- In ethical hacking and cyber security, there are _____ types of scanning.
(a) 1 (b) 2 (c) 3 (d) 4 [WBSCTE 2022]
Answer: (c)
- Which of the following is a programs that copy themselves throughout a computer or network?
(a) Worms (b) Trojans (c) Viruses (d) Rootkits [WBSCTE 2022]
Answer: (c)
- Which of the following port and IP address scanner famous among the users?
(a) Cain and Abel (b) Angry IP Scanner (c) Snort (d) Ettercap [WBSCTE 2022]
Answer: (b)
- Which of the following transmission directions listed in not a legitimate channel?
(a) Simplex (b) Half Duplex (c) Full Duplex (d) Double Duplex [WBSCTE 2022]
Answer: (d)
- Programs that multiply like viruses but spread from computer to computer are called as:
(a) Worms (b) Virus (c) Boot (d) None of these [Model Question]
Answer: (a)

- Which of the following is a cybercrime?
(a) Hacking (b) Cyber bullying (c) Virus attack (d) All of these [Model Question]
Answer: (d)
- DOS attacks are caused by
(a) authentication (b) fabrication (c) alteration (d) replay attacks [Model Question]
Answer: (b)
- Which of the following is a cybercrime?
(a) Hacking (b) Cyber bullying (c) Virus attack (d) All of these [Model Question]
Answer: (d)
- Sniffing is a technique used for
(a) Attacks on computer hardware (b) Attacks on computer software (c) Attacks on operating system (d) Attacks on wireless network [Model Question]
Answer: (d)
- The use of the Internet or other electronic means to stalk or harass an individual, a group of individuals, or an organization is termed:
(a) Cyberspace (b) Cyber Stalking (c) Pornography (d) None of these [Model Question]
Answer: (b)
- In cyber law terminology 'DDoS' means
(a) Distributed Denial of Service (b) Disc Operating System (c) Distant Operator Service (d) None of these [Model Question]
Answer: (a)
- In cryptography, what is cipher?
(a) algorithm for performing encryption and decryption (b) encrypted message (c) both (a) and (b) (d) none of the mentioned [Model Question]
Answer: (a)
- In asymmetric key cryptography, the private key is kept by
(a) sender (b) receiver (c) sender and receiver (d) all the connected devices to the network [Model Question]
Answer: (b)

B. Answer the following questions:

- What is a Denial of Service attack? [WBSCTE 2022]

Answer:

A denial of service (DoS) attack is an incident in which a user or organization is deprived of the services of a resource they would normally expect to have. In a distributed denial-of-service, large numbers of compromised systems (sometimes called a botnet) attack a single target.

15. Define a worm.

[WBSCTE 2022]

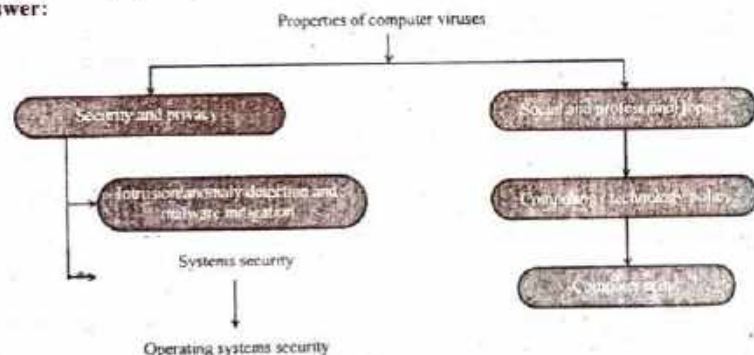
Answer:

Worms are very similar to viruses such that they are computer programs that replicate functional copies of themselves usually to other computer systems via network connections. Also worms often, but not always, contain some functionality that will interfere with the normal use of a computer or a program. Unlike viruses, however, worms exist as separate entities, they do not attach themselves to other files or programs. Due of their similarity to viruses, worms also are often referred to as viruses.

16. List some properties of virus.

[WBSCTE 2022]

Answer:



17. Explain the importance of authentication.

[Model Question]

Answer:

Authentication is the process of verifying a user's credentials before he can log into the network. It is normally performed using a username and password. This provides a secure means of limiting the access from unwanted intruders on the network.

18. What do you mean by the term encryption and decryption.

[Model Question]

Answer:

Encryption and decryption in asymmetric-key cryptography are mathematical functions applied over the numbers representing the plaintext and ciphertext. The ciphertext can be thought of as $C = f(K_{pub}, P)$; the plaintext can be thought of as $P = g(K_{priv}, C)$. The decryption function f is used only for encryption; the decryption function g is used only

for decryption. Next we show that the function f needs to be a trapdoor one-way function to allow Bob to decrypt the message but to prevent Eve from doing so.

19. What are the different types of Denial of Service attacks?

[Model Question]

Answer:

Different mechanisms are used by different types of denial service attacks.

Flood attacks

The flood attack is quite a simple concept where the attackers send a large number of requests which cannot be handled by a server. This is done in relentless way unless the server gets buckled and then gives up to the attacker. The server can again go back to normal operation when the attack ends. These attacks are quite common as they can be easily executed easily. You can easily get access to programs that can be used for such attacks. There are different forms of flood attacks. These include:

SYN flood- In this type of the flood attacks, SYN requests are sent repeatedly by the attackers, which the target ultimately accepts. The server can slow down or even get crashed due to such attacks. It involves the exchange of the ACK and the SYN messages.

Ping flooding- This is a process in which the target servers are flooded with ICMP echo requests. In this attack, the usage of bandwidth increases greatly. This eventually slows down or stops the operations of the server.

Smurf attack- The ping messages are pinged by the surf attacks to the broadcast IP addresses. If there is any response from the target machine and ICMP echo requests are broadcasted, then the attack spreads to a large number of machines. This problem has been almost fixed by the modern routers and this type of attack is now less common.

UDP attacks- In this kind of flood attack, high volume of UDP packet are sent for occupying the servers. This prevents the legitimate clients from getting access to the server. In this process, the attacker needs to find a free UDP port.

Logic attacks

The vulnerabilities of a network are exploited by this type of denial of service attacks. The intrusion method is quite different from the flood attacks. These attacks depend on the non standard traffic and the security holes in a system are exploited. These attacks locate the discoverable weakness of the network and then use them.

20. Name different virus, worms that can affect portable devices like laptop, mobile phones.

[Model Question]

Answer:

Virus: Duts, Cdripper, Fontal, 3gexplorer

Worms: lKee, Lasco

Trojans: Skulls, Gingermaster, DroidKungFu, MOSQUIT-A

Malwares: Cabir, Commwarrior

21. Classify attacks on mobile devices.

[Model Question]

Answer:

Mobile device attacks can be split into four main categories:

OS Attacks: Loopholes in operating systems create vulnerabilities that are open to attack. Vendors try to solve these with patches.

Malware or virus Attacks: There has been a constant rise in malware for mobile devices. The focus is on deleting files and rendering the system unusable.

Mobile App Attacks: Poor coding and improper development creates loopholes and compromises security.

Communication Network Attacks: Communications such as Bluetooth and Wi-Fi connections make devices vulnerable.

Long Answer Type Questions

1. What is malicious software? Give example of malicious program.

[WBSCTE 2022]

Answer:

These are Internet-based software or programs that are used to disrupt a network. The software is used to gain access to a system to steal sensitive information or data or causing damage to software present in the system.

Malicious software, commonly known as malware, is any software that brings harm to a computer system. Malware can be in the form of worms, viruses, trojans, spyware, adware and rootkits, etc., which steal protected data, delete documents or add software not approved by a user.

Malware is software designed to cause harm to a computer and user. Some forms of malware "spy" on user Internet traffic. Examples include spyware and adware. Spyware monitors a user's location and if enabled, it can capture sensitive information, e.g., credit card numbers, promoting identity theft. Adware also acquires user information, which is shared with advertisers and then integrated with unwanted, triggered pop-up ads.

2. Describe in brief the different types of Network Security Attacks.

[Model Question]

Answer:

Network security attacks are mainly classified as Passive and Active attacks. Passive attacks are generally of eavesdropping (monitoring) on transmissions, to obtain information that is being transmitted. Active attacks involve some data stream modification, or creation of a false stream. The different types of attacks are as given below:

- **Malicious code:** includes execution of viruses, worms, Trojan horses, and active Web scripts with intent to destroy or steal information.
- **Backdoor:** Gaining access to system or network using known or previously unknown/newly discovered access mechanism.
- **Password crack:** attempting to reverse calculate a password.
- **Brute force:** trying every possible combination of options of a password.
- **Dictionary:** selects specific accounts to attack and uses commonly used passwords (i.e., the dictionary) to guide guesses.
- **Denial-of-service (DoS):** attacker sends large number of connection or information requests to a Target. The target system cannot handle successfully along with other, legitimate service requests. May result a system crash or inability to perform ordinary functions.
- **Distributed denial-of-service (DDoS):** coordinated stream of requests is launched against target from many locations simultaneously.
- **Spoofing:** technique used to gain unauthorized access; intruder assumes a trusted IP address
- **Man-in-the-middle:** attacker monitors network packets, modifies them, and inserts them back into network.

3. Write down the difference between computer virus and worm.[Model Question]

Answer:

A computer virus attaches itself to a program or file enabling it to spread from one computer to another, leaving infections as it travels. Like a human virus, a computer virus can range in severity: some may cause only mildly annoying effects while others can damage your hardware, software or files. Almost all viruses are attached to an executable file, which means the virus may exist on your computer but it actually cannot infect your computer unless you run or open the malicious program.

A worm is similar to a virus by design and is considered to be a sub-class of a virus. Worms spread from computer to computer, but unlike a virus, it has the capability to travel without any human action. A worm takes advantage of file or information transport features on your system, which is what allows it to travel unaided.

4. Define Cybercrime? Discuss about various types of Cybercrime.

[Model Question]

Answer:

a) 1st Part:

Identity theft is a form of stealing someone's personal information and pretending to be that person in order to obtain financial resources or other benefits in that person's name without their consent. Identity theft is considered a cyber crime.

2nd Part:

When any crime is committed over the Internet it is referred to as a cyber crime. There are many types of cyber crimes and the most common ones are explained below:

Hacking: This is a type of crime wherein a person's computer is broken into so that his personal or sensitive information can be accessed. In the United States, hacking is classified as a felony and punishable as such. This is different from ethical hacking, which many organizations use to check their Internet security protection. In hacking, the criminal uses a variety of software to enter a person's computer and the person may not be aware that his computer is being accessed from a remote location.

Theft: This crime occurs when a person violates copyrights and downloads music, movies, games and software. There are even peer sharing websites which encourage software piracy and many of these websites are now being targeted by the FBI. Today, the justice system is addressing this cyber crime and there are laws that prevent people from illegal downloading.

Cyber Stalking: This is a kind of online harassment wherein the victim is subjected to a barrage of online messages and emails. Typically, these stalkers know their victims and instead of resorting to offline stalking, they use the Internet to stalk. However, if they notice that cyber stalking is not having the desired effect, they begin offline stalking along with cyber stalking to make the victims' lives more miserable.

Identity Theft: This has become a major problem with people using the Internet for cash transactions and banking services. In this cyber crime, a criminal accesses data about a person's bank account, credit cards, Social Security, debit card and other sensitive information to siphon money or to buy things online in the victim's name. It can result in major financial losses for the victim and even spoil the victim's credit history.

Malicious Software: These are Internet-based software or programs that are used to disrupt a network. The software is used to gain access to a system to steal sensitive information or data or causing damage to software present in the system.

Child soliciting and Abuse: This is also a type of cyber crime wherein criminals solicit minors via chat rooms for the purpose of child pornography. The FBI has been spending a lot of time monitoring chat rooms frequented by children with the hopes of reducing and preventing child abuse and soliciting.

➤ **5. What is software piracy? Discuss about the preventive measures against software piracy.** [Model Question]

Answer:

1st Part:

Software piracy is the stealing of legally protected software. Under copyright law, software piracy occurs when copyright protected software is copied, distributed, modified or sold. Software piracy is considered direct copyright infringement when it denies copyright holders due compensation for use of their creative works.

2nd Part:

Here are some of the tips to prevent or lessen software piracy:

Code Key - For the past 5 to 7 years, software developers have devised a plan to lessen or prevent software piracy. This is by using a code key. A code key comes with the software that we buy. Before fully installing the software on our computer, we must provide the specific code key that came with the software. After providing the code key, we can run the software on our computer. The code key also locks after it has been used. This is so that the software cannot be installed on other computers after it has been installed on one. We will have to call the manufacturer of the software to be able to use the code key again. This is not good news to people who pirate software.

Open Source - Open source software is described as a free software that anyone can download from the Internet. A lot of different software can be licensed as open source. Open source software are under the license free software license. There are also different upgrades for these open source software that can also be freely downloaded from the Internet. By using open source software, people don't need to worry about spending a lot of money for original copies of software. This will also lessen the number of people who are using pirated software as there are alternative programs that can be used and they do not have to pay fines when they are caught.

Hardware Key - A hardware key is a device that is used for anti-piracy. This tool prevents software vendors to distribute their products or use them without authorization from the copyright owner of the software. The hardware key works when it is attached to a computer. It monitors software licensing and enforces licensing of the protected software those are detected on the computer. This tool will lessen software vendors from illegally distributing the software that they have and is a good prevention measure for piracy.

Anti-Piracy Software - There are different types of anti-piracy software that are available for free. Anti-piracy software is used to prevent illegal duplication or illegal use of copyrighted software. There is also an anti-piracy software that prevents hackers from getting into the software and copying it without consent from the copyright owner. Some of them are also already integrated on the disks of the software that contains the program. This may also be for piracy music. The anti-piracy association is also looking for other ways to prevent software piracy. Reporting anti-piracy may prevent it from happening.

➤ **6. (a) What are the basic differences between Viruses and Trojan horses?**

(b) Explain the concept of tunneling in VPN.

[Model Question]

Answer:

a) Malicious code-viruses:

Malicious code, which can be also referred as viruses, is inserted onto a host to damage a system. It can corrupt a system, by replicating itself it can deny services or access to networks, systems or services. They can also allow sensitive information to be copied or echoed to other systems.

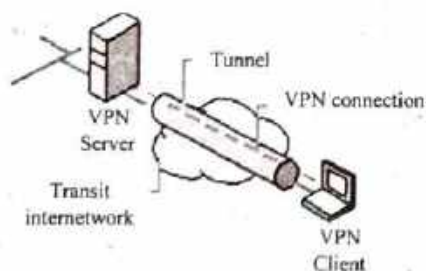
Trojan horses:

It is a program that performs a malicious action but cannot replicate itself. It may arrive as a harmless file or application with hidden, malicious code. When it runs, we may experience unwanted system problems and might sometimes lose information from our computer.

Example: Trojan horses can be used to ask the user to enter sensitive information in a commonly trusted utility like windows logon screen. Then when the user types his username and password, the Trojan would then send the information to the attacker and give error for bad password to the user. The logon screen then restarts again and now when the user carefully enters the username and password it is allowed to enter the system as if nothing has happened. But actually the username and password are stolen.

b) A virtual private network (VPN) is an extension of a private network that groups links across shared or public networks like the Internet. A VPN enables us to send data between two computers across a shared or public network such that it emulates the properties of a point-to-point private link.

Data is encapsulated or wrapped with a header that provides routing information allowing it to traverse the shared or public network to reach its destination. The data being sent is encrypted for confidentiality in the Private Network. The part of the connection in which the private data is encapsulated is known as the tunnelling. The part of the connection in which the private data is encrypted is known as the Virtual Private Network (VPN) connection as shown in figure, VPNs provide remote access to corporate resources over the public Internet, while maintaining privacy of information.



A VPN connection includes the following components:

- VPN server: A server is basically computer that accepts VPN connections from VPN clients.
- VPN client: It is also a computer that initiates a VPN connection to a VPN server. A VPN client can be an individual computer or a router.
- Tunnel: A tunnel is a portion of the connection in which your data is encapsulated.

- VPN connection: The portion of the connection in which your data is encrypted. For typical secure VPN connections, the data is encrypted and encapsulated along the same portion of the connection.
- Tunneling protocols: Tunneling protocols are protocols that are used to manage tunnels and encapsulate private data. Data that is tunneled must also be encrypted to be a VPN connection. The Windows Server 2003 family includes the PPTP and L2TP tunneling protocols.
- Tunneled data: Tunneled data are data that is usually sent across a private point-to-point link.
- Transit internetwork: The shared or public network crossed by the encapsulated data. For the Windows Server 2003 family, the transit internetwork is always an IP internetwork. The transit internetwork can be the Internet or a private IP-based intranet.

7. What is denial of service attacks?

[Model Question]

Answer:

A **denial-of-service attack (DoS attack)** is an attempt to make a computer or network resource unavailable to its intended users. Although the means to carry out, motives for, and targets of a DoS attack may vary, it generally consists of the concerted efforts of a person, or multiple people to prevent an Internet site or service from functioning efficiently or at all, temporarily or indefinitely.

A DoS attack does not usually result in the theft of information or other security loss but can cost the target person or company a great deal of time and money. A denial of service attack can also destroy programming and files in affected computer systems. Some common forms of denial of service attacks are:

Buffer Overflow Attacks: This is the most common kind of DoS attack that sends more traffic to a network address than the programmers who planned its data buffers anticipated someone might send.

SYN Attack which is initiated between a TCP client and server in a network in which an attacker sends a large number of connection requests very rapidly and then fail to respond to the reply. This may lead to several legitimate connection requests not to be accommodated.

Teardrop Attack: This type attack exploits IP packet fragmentation. The attacker's IP puts a confusing offset value in the second or later fragment causing an unprepared receiving system to crash.

Smurf Attack: In this attack, the perpetrator sends an IP ping request to a receiving site also indicating that the request is from another the targetted site using IP spoofing. As a result, a huge number of ping replies flooding back to the innocent spoofed host which can then be unable to receive or distinguish real traffic.

Both IP spoofing and DoS attacks can be prevented by the proper use of firewalls and configuration of modern switches and routers which provide techniques like Access Control Lists, delayed TCP binding, deep packet inspection, etc., for that purpose.

➤ 8. What is the difference between Worm and Trojan horse? [Model Question]

Answer:

Worms: Worms are very similar to viruses such that they are computer programs that replicate functional copies of themselves usually to other computer systems via network connections. Also worms often, but not always, contain some functionality that will interfere with the normal use of a computer or a program. Unlike viruses, however, worms exist as separate entities, they do not attach themselves to other files or programs. Due of their similarity to viruses, worms also are often referred to as viruses.

Trojan horses: It is a program that performs a malicious action but cannot replicate itself. It may arrive as a harmless file or application with hidden, malicious code. When it runs, we may experience unwanted system problems and might sometimes lose information from our computer.

Example: Trojan horses can be used to ask the user to enter sensitive information in a commonly trusted utility like windows logon screen. Then when the user types his username and password, the Trojan would then send the information to the attacker and give error for bad password to the user. The logon screen then restarts again and now when the user carefully enters the username and password it is allowed to enter the system as if nothing has happened. But actually the username and password are stolen.

➤ 9. (a) Define Cyberstalking. How stalking works?

(b) Explain the different attacks launched with attack vector.

(c) What is Cyberbullying?

[Model Question]

Answer:

(a) 1st part:

Cyberstalking is a crime in which the attacker harasses a victim using electronic communication, such as e-mail or instant messaging (IM), or messages posted to a Web site or a discussion group. A cyberstalker relies upon the anonymity afforded by the Internet to allow them to stalk their victim without being detected. Cyberstalking messages differ from ordinary spam in that a cyberstalker targets a specific victim with often threatening messages, while the spammer targets a multitude of recipients with simply annoying messages.

2nd part:

Cyber stalking works in many ways, depending on the technological capabilities of the cyber stalker.

- Multiple stalker utilize the services provided on the internet.

- Through Datafurnishing companies that provide information on a person's capability to function in a society.
- There are several other methods used by the cyber stalkers to stalk their victims
 - Spyware software
 - Phishing
 - Juice Jacking
 - Wi-Fi interface (Jacking)
 - Caller ID Spoofing

(b) An attack vector is a path or means by which a hacker (or cracker) can gain access to a computer or network server in order to deliver a payload or malicious outcome. Attack vectors enable hackers to exploit system vulnerabilities, including the human element. Just like we don't know whom we're up against, we don't always know what we're up against. There are different attack vectors, and we must have protection against any and all of them, before we can be secure. Here are some examples.

- **Social engineering** involves using social skills to obtain or compromise information. For instance, an attack can involve claiming to be a new employee, a friend stranded on vacation in need of money, a researcher or an acquaintance. Many times, they offer credentials and other information to make themselves appear more legitimate and support their identity. Phishing attacks are a form of social engineering attacks.
- **Brute force attacks** involve a trial-and-error method used to get information such as a PIN or password. In this type of attack, an automated software generates several consecutive guesses to try to isolate the correct solution. These attacks can be used by hackers to decrypt data, or by a security analysis team to test network security. These attacks are both time and resource consuming. When successful, they are usually based on computer power and the number of combinations tried, rather than an algorithm.
- **Distributed Denial of Service (DDoS)** involves using multiple compromised systems, typically infected with a Trojan to target a single system to cause a denial of service, or DDoS, attack. It floods the server with incoming traffic, which overwhelms it, shuts it down, and renders the website or online service useless for visitors. Since the flood of traffic comes from multiple sources, solving it is not as simple as blocking an IP address.
- **Cross-site scripting (XSS)** is a type of code injection attack. This happens by incorrectly validating user data, inserted on a page via an altered link or web form. The code can be injected can be any malicious client-side code, such as Flash, CSS, HTML, JavaScript, VBScript, and the like. These happen when developers don't take proper measures to secure their code. PHP developers must be diligent in knowing how attacks can be carried out to ensure they address potential vulnerabilities.

(c) Cyberbullying is bullying that takes place using electronic technology. Electronic technology includes devices and equipment such as cell phones, computers, and tablets as well as communication tools including social media sites, text messages, chat, and websites.

Examples of cyberbullying include mean text messages or emails, rumors sent by email or posted on social networking sites, and embarrassing pictures, videos, websites, or fake profiles.

FUNDAMENTALS OF ELECTRICAL & ELECTRONICS ENGINEERING

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Chapter 1: Overview of Electrical Components

Chapter at a glance:

1.1 Passive Element:

Elements, which are not energy sources, (which dissipate energy or consumed energy) are known as passive elements, e.g., Resistor, Inductor, Capacitor etc.

1.2 Charge

The basic quantity in an electric circuit is the electric charge. The unit of the charge is coulomb C and the symbol is Q or q . The magnitude of the charge is $1.602 \times 10^{-19} \text{C}$.

1.3 Current

Current is a flow of electrical charge carriers, usually electrons or electron-deficient atoms. The common symbol for current is the uppercase letter I . The standard unit is the ampere, symbolized by A.

$$I = \frac{dq}{dt} = \frac{Q}{t}$$

Electric current can be either direct or alternating. Direct current (DC) flows in the same direction at all points in time, although the instantaneous magnitude of the current might vary. In an alternating current (AC), the flow of charge carriers reverses direction periodically. Current per unit cross-sectional area is known as current density.

1.4 Voltage

Voltage, also called electromotive force, is a quantitative expression of the potential difference in charge between two points in an electrical field. Voltage is symbolized by an uppercase italic letter V or E . The standard unit is the volt, symbolized by a non-italic uppercase letter V .

Voltage can be direct or alternating. A direct voltage maintains the same polarity at all times. In an alternating voltage, the polarity reverses direction periodically.

1.5 Resistance

Electrical resistance is a measure of degree to which an object opposes the passage of an electric current.

The resistance is a ratio between voltage (V) across the object and current (I) passing

$$\text{through the object } R = \frac{V}{I}$$

1.6 Inductance

It is a measure of amount of magnetic flux produced for a given electric current. SI unit of inductance is Henry (H).

Mathematical representation of inductance is $L = \frac{\Phi}{i}$ where, Φ = Magnetic flux,

i = Current

1.7 Capacitance

It is measure of the amount of electric charge stored (or separated) for a given electric potential. The capacitance is usually defined as the total electric charge (Q) placed on the

object divided by potential (V) of the object. $C = \frac{Q}{V}$

1.8 Power

Electrical Power is defined as the rate at which electrical energy is transferred by an electric circuit. The SI unit of power is the Watt.

$$\text{Power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

where

P is the power (watt or W)

V is the potential difference (volt or V)

I is the current (ampere or A).

1.9 Energy

It defines as the work done over a time. It's unit is Kilo-Watt hour (KWh). It is measured by energy meter.

$$E = Pt \text{ (KWhr)}$$

Electrical energy is not available naturally and must be centrally generated and transmitted to the place to use and distribute to the loads.

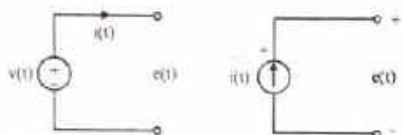
1.10 Energy Sources

• Active Element:

Basically active element is capable to generating the energy. Most important active elements are current and voltage source, OPAMP etc.

• Ideal sources:

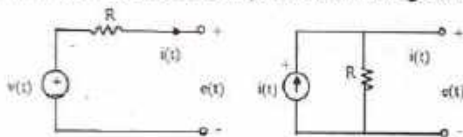
Ideal sources are those hypothetical devices, which maintain a constant rated value (voltage or current) at its terminals.



Above figures show ideal voltage and current source respectively, having no internal resistance.

• Practical sources:

Practical sources are those devices which do not maintain constant rated values (voltage or current) at its terminals due to internal drops of either voltage or current.



Above figures show practical voltage and current source respectively. For voltage source, internal resistance is connected in series whereas for current source internal resistance is shunt. Due to these internal resistance there must be some internal drops of either voltage / current.

• Independent voltage source:

Provides a specified voltage, it is independent of the current through the element.



• Independent current source:

Provides a specified current, it is independent of the voltage across the element.



• Dependent source or Controlled sources:

Dependent source / Controlled sources are voltage or current generator whose rated values are controlled by any other parameters in the circuit or the characteristics of which depends upon some other parameters of network (voltage or current), hence known as dependent sources also.



Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. Which component opposes any change in current? [WBSCTE 2022]
 (a) Resistor (b) Capacitor (c) Inductor (d) Diode

Answer: (a)

2. The duration of one cycle known as [WBSCTE 2022]
 (a) waveform (b) peak value (c) instantaneous value (d) period

Answer: (d)

3. The unit of resistivity of a material is - [Model Question]
 (a) mho (b) ohm (c) ohm-m² (d) ohm-m

Answer: (d)

4. Resistivity (ρ) of a metallic conductor - [Model Question]
 (a) decreases with the rise in temperature (b) increases with rise in temperature
 (c) remains unchanged (d) none of these

Answer: (a)

5. Elementary charge is - [Model Question]
 (a) 9.1×10^{-27} coulomb (b) 1.66×10^{-24} coulomb
 (c) 1.6×10^{-19} coulomb (d) none

Answer: (c)

6. An alternating voltage is given by $v = 30 \sin 2\pi t$. The frequency of the a.c. signal is- [Model Question]
 (a) π Hz (b) 2π Hz (c) 1 Hz (d) none

Answer: (c)

7. The capacitive impedance of a circuit is given by - [Model Question]
 (a) $\frac{1}{\omega C}$ (b) ωC (c) $\frac{\omega}{C}$ (d) none of these

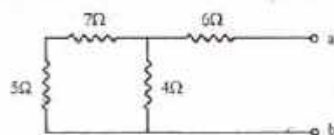
Answer: (a)

8. A 20Ω resistor is stretched to increase its length double. Its resistance will now be [Model Question]
 (a) 40Ω (b) 20Ω (c) 10Ω (d) 5Ω

Answer: 80Ω

9. The equivalent resistance across the terminal a-b will be [Model Question]

- (a) 9Ω
 (b) 3Ω
 (c) 6Ω
 (d) 2Ω



Answer: (a)

10. If a voltage source is to be neglected then the terminals across the source will be [Model Question]
 (a) open circuited (b) short circuited
 (c) both (A) and (B) (d) none of these

Answer: (b)

11. An open circuit may be considered as a resistor of value [Model Question]
 (a) zero (b) 100Ω (c) infinity (d) $10 M\Omega$

Answer: (c)

12. An ideal voltage source should have [Model Question]
 (a) large value of e.m.f. (b) small value of e.m.f.
 (c) infinite source resistance (d) zero source resistance

Answer: (d)

13. The time constant for an RC circuits is [Model Question]
 (a) RC (b) L/R (c) LR (d) R/C

Answer: (a)

14. The time constant for an RL circuits is [Model Question]
 (a) RC (b) L/R (c) LR (d) R/C
 Answer: (b)

B. State whether the following statements are True or False

15. Unit of reactance is same as that of impedance. [Model Question]
 Answer: True

16. The maximum value of power factor is 0.5. [Model Question]
 Answer: False

17. Kirchoff's laws are applied for a.c. circuit also. (True / False) [Model Question]
 Answer: False

C. Answer the following questions:

18. What is the unit of inductance? [Model Question]
 Answer: H(Henry)

19. What is the unit of capacitance? [Model Question]
 Answer: F(Farad)

20. State the unit of reactive power. [Model Question]
 Answer: VAR

21. Express the equivalent series impedance of $Z_1 = 3 + 4j$ and $Z_2 = 6 - 8j$ in polar form. [Model Question]

Answer:

$$Z_1 = 3 + 4j$$

$$Z_1 + Z_2 = 9 - 4j = \sqrt{9^2 + 4^2} \tan^{-1} \left(-\frac{4}{9} \right) = \sqrt{81 + 16} \angle -23.962^\circ = 9.84 \angle -23.962^\circ$$

22. Ampere hour is unit of which parameter? [Model Question]
 Answer:

Ampere hour is the unit of electric charge.

23. Define [Model Question]
 a) Linear circuit
 b) Non-linear circuit

- c) Bilateral circuit
- d) Unilateral circuit
- e) Network

Answer:

- a) **Linear Circuit:** A linear circuit is one which obeys the principle of superposition and homogeneity.
- b) **Non – linear circuit:** It is that circuit which does not obey the principle of superposition.
- c) **Bilateral Circuit:** A bilateral circuit is one whose properties or characteristics are same in either direction. It can work in both directions.
- d) **Unilateral Circuit:** It is that circuit whose properties and characteristics change with the direction of operation i.e. it can work in one direction only.
- e) **Network:** A network is a collection of interconnected components.

24. Give an example of passive element.

[Model Question]

Answer:

Resistance R , Inductance (L) and capacitance (C)

25. Determine whether the signal $x(t) = \sin^2 t$ is periodic. If periodic, find its fundamental period.

Answer:

$$x(t) = \sin^2 t = \frac{1}{2}(1 - \cos 2t) = \frac{1}{2} - \frac{1}{2}\cos 2t = x_1(t) + x_2(t)$$

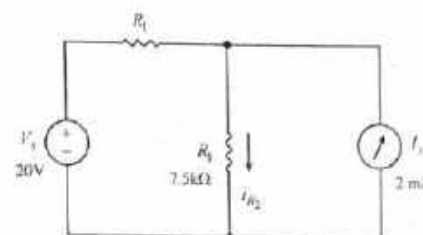
$$x_1(t) = \frac{1}{2} \text{ is a d.c. signal with arbitrary period.}$$

$$x_2(t) = -\frac{1}{2}\cos 2t = -\frac{1}{2}\cos \omega_2 t \text{ is periodic with period } T_2 = \frac{2\pi}{\omega_2} = \pi$$

Thus $x(t)$ is periodic with fundamental period $T_0 = \pi$.

Long Answer Type Questions

- 1. a) Define Periodic & non-periodic waveform with one example of each.
- b) Using Source transformation principle find out the current through resistance R_2 . [WBSCTE 2022]



Answer:

a) A signal $f(t)$ is called periodic if there exists a constant T such that $f(t) = f(t + T)$ for $-\infty < t < \infty$ where T is called the period of the signal.

A periodic signal exhibits the property of periodicity i.e. it exhibits a definite pattern that repeats over and over again with a repetition period of T .

A discrete-time signal $x(n)$ is periodic if $x(n) = x(n + N)$ for all n

Here N is the sampling period.

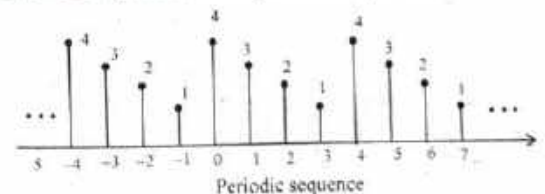
All signals other than periodic signals are called non-periodic or aperiodic signals.

A sinusoidal signal $x(n) = A \sin 2\pi f_0 n$ is periodic, when f_0 is a rational number i.e.

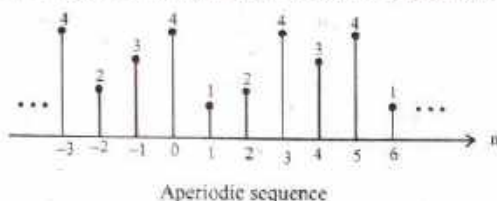
f_0 can be expressed as $f_0 = \frac{k}{N}$ where k and N are integers.

Remember the sum of two or more periodic continuous-time signals need not be periodic. They will be periodic only when the ratio of their fundamental periods is rational. In the case of discrete-time signals, the sum of a number of periodic signals is always periodic as the ratio of individual periods is always the ratio of integers, which is rational.

The figures below show periodic and aperiodic sequences.



This sequence is repeating for every 4 samples. Hence the period of the signal $N_0 = 4$



b) According to the circuit,

$$I_s \text{ (Source current)} = 2 \text{ mA} = 2 \times 10^{-3} \text{ A}$$

$$V_s \text{ (Source voltage)} = 20 \text{ V}$$

$$R_1 = 5 \text{ k}\Omega$$

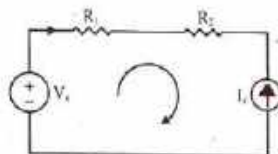
$$R_2 = 7.5 \text{ k}\Omega$$

Using source transformation principle,

The current through resistance (R_2) =

$$i = \frac{V_s}{R_1 + (1 + \beta)R_2} = \frac{20}{5 + (1 + 2) \times 7.5} = \frac{20}{27.5} = 0.7 \text{ mA}$$

$\therefore$ The current through resistance is 0.7 mA.



2. What is resistance?

[Model Question]

Answer: Refer to Article No. 1.5 of Chapter at a Glance.

3. Define with its units of (any two) – (i) inductance, (ii) capacitance, (iii) resistance.

[Model Question]

Answer: Refer to Chapter at a Glance.

4. (a) What is inductance? What is the unit of inductance?

(b) A resistor 'R' is connected across a potential difference of 110 V and dissipate energy at the rate of 220 W. Calculate the value of R. [Model Question]

Answer:

(a) Refer to Article No. 1.6 of Chapter at a Glance.

$$(b) V = 110 \text{ V}, P = 220 \text{ W}$$

$$P = V.I$$

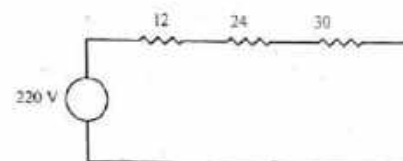
$$\text{or, } 220 = 110 \times I; \quad \text{or, } I = \frac{220}{110} = 2 \text{ A}$$

$$\therefore R = \frac{V}{I} = \frac{110}{2} = 55 \Omega \text{ (Ans.)}$$

5. Three Resistances of 12 ohms, 24 ohms, 30 ohms magnitude are connected in series and the series combination is connected across a 220 volt d.c source. Compute the power dissipated in each resistor and the total power drawn from the source.

[Model Question]

Answer:



The total resistance R_{tot} of the circuit is $R_{\text{tot}} = R_1 + R_2 + R_3 = 66 \text{ Ohms}$.

$$\text{We Know } I = \frac{V}{R_{\text{tot}}} = \frac{220}{66} = \frac{20}{6} \text{ Amps}$$

$$\text{Total power} = I^2 R_{\text{tot}} = \left(\frac{20}{6}\right)^2 \times 66 = \frac{400 \times 11}{6} = 733.3 \text{ W}$$

$$\text{Power in } R_1 = \left(\frac{20}{6}\right)^2 \times 12 = \frac{20}{6} \times \frac{20}{6} \times 12 = \frac{400}{3} \text{ W} = 133.33 \text{ W}$$

$$\text{Power in } R_2 = \left(\frac{20}{6}\right)^2 \times 24 = \frac{20 \times 20}{36} \times 24 = \frac{800}{3} \text{ W} = 266.67 \text{ W}$$

$$\text{Power in } R_3 = \left(\frac{20}{6}\right)^2 \times 30 = \frac{20 \times 20}{6 \times 6} \times 30 = \frac{2000}{6} \text{ W} = 333.33 \text{ W Answer.}$$

6. Explain why current through an inductance and voltage across capacitance cannot change instantaneously.

[Model Question]

Answer:

The instantaneous voltage developed across an inductance is expressed as $v_L = L \frac{di}{dt}$

In order to bring about an instantaneous change in the current i.e., to obtain a finite di for $dt \rightarrow 0$, the voltage should assume an infinite value.

It is impossible to realize such a function physically. It can only be realized mathematically by a dirac delta function $\delta(t)$. So it can be concluded that voltage through inductance cannot change instantaneously.

Current flowing through an capacitance can be expressed as $i_c = C \frac{dv}{dt}$

An instantaneous change in the capacitor voltage would require a current source of infinite magnitude. It can only be realized mathematically by a direct delta function. As it is not possible to construct an impulse function in physical reality, voltage across capacitance cannot change instantaneously.

Chapter 2: Electric & Magnetic Circuits

Chapter at a glance:

2.1 Definitions and units

Magnetizing Force (H):

Field intensity at any point within a magnetic field is numerically equal to the force experienced by a unit north (N) pole placed at that point.

To find field intensity at a point r meter distance away from a pole of m webers, let us imagine a pole of one Weber placed at that point. The force experienced by this pole is

$$F = \frac{m \times 1}{4\pi\mu_0 r^2}$$

Hence the magnetic field intensity $N = \frac{m}{4\pi\mu_0 r^2}$ N/wb (or AT/m)

Magnetic Flux:

Since a unit N-pole is supposed to radiate out a flux of one Weber, so, the flux coming out of a N-pole of m Weber is $\phi = m \text{ wb}$

Absolute Permeability (μ) and Relative Permeability (μ_r):

The phenomenon of magnetism and electromagnetism is dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities; (a) absolute permeability (μ) and (b) relative permeability (μ_r).

Permeance (P):

It is the reciprocal of reluctance. It is defined as the property of the magnetic material due to which it allows the establishment flux in the core. Its unit is wb/AT.

Magnetic Field Intensity (H):

It is the mmf per unit length along the path of magnetic flux. Thus magnetic field

intensity, $H = \frac{\text{mmf}}{l} = \frac{NI}{l} \text{ AT/m}$

Reluctance (R):

It is the resistance offered by the magnetic circuit to the flow of magnetic flux on it. Thus reluctance is the ratio of the drop in magneto-motive force to the flux produced in

that part of the circuit. Thus $R = \frac{\text{mmf}}{\phi}$ ampere turns / weber

The reluctance of a substance is:

- (i) Directly proportional to the length of flux path(l), and
- (ii) Inversely proportional to the cross-section(A).

Thus $R \propto \frac{l}{A}$, or, $R = \frac{l}{\mu_0 \mu_r A} \text{ ATWb}$

The constant of proportionality $\frac{1}{\mu_0 \mu_r}$ or $\frac{1}{\mu_0}$ show that reluctance depends on the relative permeability.

Magnetomotive force (m.m.f):

It is the force which drives or tends to drive magnetic flux through a magnetic circuit. When a coil of N turns has current I flowing through it, the magnetomotive force is given by $M = I.N$ ampere turns.

2.2 Force experienced by a current-carrying conductor placed in a magnetic field

If a current-carrying conductor is placed in a magnetic field, it experiences a force and starts moving in the direction of the force if it is free. The magnitude of the force is given by

$$F = BIl \sin \theta$$

where B = the magnetic flux density

I = the current in the conductor

l = the length of the conductor

θ = the angle between the conductor and the field

The direction of the force is given by Fleming's left-hand rule.

2.3 Definition of Hysteresis Loss

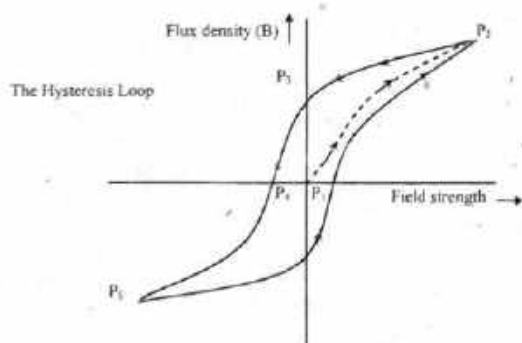


Fig: shows the effect of hysteresis in ferromagnetic materials.

We start with an unmagnetized sample at the origin (P_1) where both field strength and flux density are zero. The field strength is increased in the positive direction and the flux

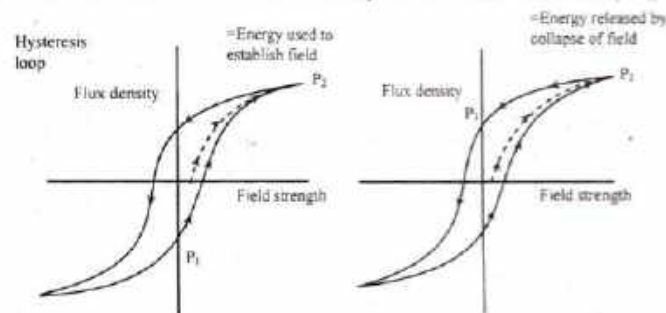
begins to grow along the dotted path until we reach P_2 . This is called the initial magnetization curve.

If the field strength is now relaxed then some curious behavior occurs. Instead of retracing the initial magnetization curve the flux falls more slowly. In fact, even when the applied field is returned to zero there will still be a remaining (**remnant or remanent**) flux density at P_3 . It is this phenomenon which makes permanent magnets possible.

To force the flux to go back to zero we have to reverse the applied field (P_4). The field strength here is called the **coercivity**. We can then continue reversing the field to get to P_5 , and so on round this type of magnetization curve called (by J. A. Ewing) a hysteresis loop.

Significance of this from the point of power losses

It is that we have had to spend energy in order to set up the remnant flux. To show this more clearly we'll look at separate figures. The area (shown shaded) between the B-H curve and the B axis represents the work done (per unit volume of material).



2.4 Analogy between Magnetic Circuit and Electric circuit

Magnetic circuits	Electrical circuits
1. Flux flows from N-pole to S-pole 2. Flux flow is due to mmf. 3. Resistance to flow of flux is offered by reluctance.	1. Current flows from +ve to -ve 2. Current flows due to e.m.f. 3. Resistance to flow of current is offered by resistance R .
$\text{Flux} = \frac{\text{mmf}}{\text{reluctance}}$ (i) mmf is expressed in ampere turns.	$\text{Current} = \frac{\text{emf}}{\text{resistance}}$ (i) emf is expressed in volts

Magnetic circuits	Electrical circuits
(ii) Flux is expressed in webers.	(ii) Current is expressed in amperes.
(iii) Flux density is expressed in Wb/m^2	(iii) Current density is expressed in (A/m^2) .
4. Reluctance $R = \frac{l}{\mu A}$ where μ is permeability.	4. Resistance $R = \frac{l}{\sigma A}$ where σ is the conductivity.
5. Permeance = $\frac{1}{\text{reluctance}}$	5. Conductance = $\frac{1}{\text{resistance}}$
6. Permeability = $\frac{1}{\text{reluctivity}}$	6. Conductivity = $\frac{1}{\text{resistivity}}$

2.5 Faraday's Law of Electromagnetic Induction

Faraday's first law states that "whenever magnetic flux linked with a close coil changes, a n induced emf is set up in the coil and the induced emf lasts as long as the change in magnetic flux continues."

Faraday's second law states that the magnitude of the induced emf is proportional to the rate of change of magnetic lines of force.

The phenomenon of electromagnetic induction plays a crucial role in the three most useful of all electrical devices: the electric generator, the electric motor, and the transformer.

2.6 Lenz's Law

Lenz's Law states that "the direction of the induced e m f. is such as to tend to oppose the change in the inducing flux." The mathematical expression of this law, in conjunction with Faraday's Law, introduces a negative sign, and gives

$$e = -N \frac{d\Phi}{dt}$$

2.7 Fleming's right hand rule

The direction of the induced emf (and hence the induced current) is given by Fleming's Right Hand Rule which states that "stretch the fore finger, middle finger and the thumb of right hand in such a way that all three are mutually perpendicular to each other. If fore finger points in the direction of field, thumb points in the direction of motion of conductor, then middle finger will point along the direction of induced conventional current as shown in figure.

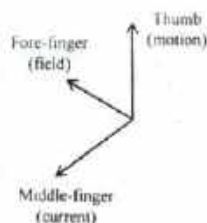
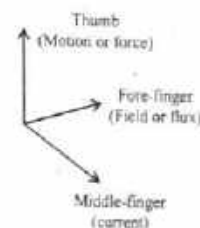


Fig: Fleming's Right hand rule

2.8 Fleming's left hand rule

This rule is used to determine the direction of force acting on a current carrying conductor placed in a magnetic field. According to this rule, if the middle finger, for finger and the thumb left hand are right angles to one another and if the middle finger and the fore finger represent the direction of current and magnetic field respectively, then the thumb will indicate the direction of force acting on the conductor. This rule is used to determine the direction of motion of a conductor (rotor) in the magnetic field produced by the stator for a motor.



2.9 Energy Stored in the Magnetic Field of an Inductor

The above inductor having inductance L is connected to the battery through Key K. In this case induced e m f is given by:

$$e = -L \frac{di}{dt}$$

To drive the current through the inductor against the induced emf 'e', the external voltage is applied from the battery which has emf 'E' and in this case $E = -e$.

Hence above equation becomes $E = L \frac{di}{dt}$

Let an infinitesimal charge dq be driven through the inductor. So the work done by the external voltage to do this work is given by:

$$dw = E dq$$

$$\text{or, } dw = L \frac{di}{dt} dq = L di \left(\frac{dq}{dt} \right)$$

$$\text{But } \frac{dq}{dt} = i$$

$$\therefore dw = L i di$$

Total work done to maintain the maximum value of current (I_m) through the inductor is given by

$$\int dw = \int_0^{I_m} L i di \quad W = L \left[\frac{i^2}{2} \right]_0^{I_m} = L \left[\frac{I_m^2}{2} \right]$$



The work done in increasing the current flowing through the inductor is stored as the potential energy (U) in its magnetic field. $U = \frac{1}{2} LI^2$

2.10 Concept of eddy current & eddy current loss

When a metallic body is moved in a magnetic field in such a way that the flux through it changes or is placed in a changing magnetic field, induced currents circulate throughout the volume of the body. These are called eddy currents. If the resistance of the said conductor is small, then the magnitudes of the eddy currents are large and the metal gets heated up. This heating effect is a source of power loss in iron-cored devices such as dynamos, motors and transformers.

The eddy current loss is given by

$$P_e = K_e \cdot f \cdot B_m^2 \cdot t^2 \cdot v \cdot W$$

$K_e \rightarrow$ constant

$B_m \rightarrow$ maximum flux density

$f \rightarrow$ frequency

$t \rightarrow$ thickness of the core material

$v \rightarrow$ total volume of the core material

Eddy current loss can also be reduced considerably by the use of silicon from alloy and employing conducting material of high resistivity.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. In case of ideal current sources, they have _____ [WBSCTE 2022]
 (a) zero internal resistance (b) low value of voltage
 (c) large value of current (d) infinite internal resistance

Answer: (d)

2. The unit of magnetomotive force is _____ [WBSCTE 2022]
 (a) ampere-turn (b) weber (c) mho (d) Maxwell

Answer: (a)

3. The property of a material which opposes the creation of magnetic flux is called _____ [WBSCTE 2022]
 (a) permeability (b) permittivity (c) reluctance (d) resistance

Answer: (c)

4. The energy stored in a magnetic field is given by (L = self inductance, I = current) [WBSCTE 2022]
 (a) $\frac{1}{2} LI^2$ (b) $\frac{1}{2} IL^2$ (c) IL^2 (d) LI^2

Answer: (a)

5. Eddy current loss is directly proportional to (f = frequency, B_m = maximum flux density) [WBSCTE 2022]
 (a) f^2 and B_m^2 (b) f and B_m
 (c) f and B_m^2 (d) f^2 and B_m

Answer: (c)

6. The direction of induced emf is such that it will oppose the cause to which it is created, is related to _____ law of electromagnetic induction. [WBSCTE 2022]
 (a) Faraday's (b) Lenz's (c) Ohm's (d) Ampere's

Answer: (b)

7. The reciprocal of permeability is known as- [Model Question]
 (a) conductivity (b) reluctance (c) m.m.f. (d) none of these

Answer: (b)

8. The strength of an electromagnet depends upon- [Model Question]
 (a) voltage & size of the wire (b) the current & size of the wire
 (c) the voltage & number of turns (d) the current and the number of turns

Answer: (b)

9. Magnetic lines of force- [Model Question]
 (a) never intersect (b) often intersect
 (c) intersect only in special circumstances (d) are not predictable

Answer: (b)

10. The ability of a material to set up a magnetic field is called its - [Model Question]
 (a) permeability (b) permittivity (c) retentivity (d) coactivity

Answer: (a)

11. The term 'reluctance' is associated with - [Model Question]
 (a) electrical circuit (b) equal to the iron part
 (c) higher than the iron part (d) all of these

Answer: (b)

12. MMF required for the air gap of an electro magnet is – [Model Question]
 (a) less than the iron part (b) equal to the iron part
 (c) higher than the iron part (d) all of these

Answer: (a)

13. Area of hysteresis loop measures – [Model Question]
 (a) copper loss (b) iron loss
 (c) both copper and iron losses (d) none of these losses

Answer: (b)

14. The shape of hysteresis loop depends on – [Model Question]
 (a) number of magnetic reversals per second
 (b) thickness of the laminations of the material
 (c) purity of the laminated material
 (d) laminated material

Answer: (a)

15. Reluctance of a magnetic material is – [Model Question]
 (a) directly proportional to absolute permeability of the magnetic material
 (b) inversely proportional to the area of cross section of the magnetic material
 (c) all of these

Answer: (a)

16. The value of permeability of free space μ_0 in rationalized M.K.S units – [Model Question]
 (a) 0.796×10^6 (b) 8.85×10^{-12} (c) $4\pi \times 10^{-7}$ (d) 2.998×10^6

Answer: (c)

17. Term 'Reluctance' is associated with – [Model Question]
 (a) electrical circuit (b) distribution line
 (c) magnetic circuit (d) generator

Answer: (c)

18. Value of permeability of free space μ_0 in MKS unit – [Model Question]
 (a) 0.796×10^6 (b) 8.85×10^{-12} (c) $4\pi \times 10^{-7}$ (d) 1

Answer: (c)

19. Strength of electromagnet depends upon – [Model Question]
 (a) voltage and size of wire (b) voltage and number of turns of wire
 (c) current and number of turns of wire (d) potential difference of the wire

Answer: (c)

20. Which of the following is not true of leakage flux? [Model Question]
 (a) It links both the winding through air
 (b) It links the primary winding through air
 (c) It links the secondary winding through air
 (d) It does not link both the windings

Answer: (d)

21. What is done to balance the *mmf* setup due to the secondary current? [Model Question]
 (a) The primary voltage is increased
 (b) The core flux is increased immediately
 (c) The current in the primary is increased
 (d) All of these

Answer: (c)

22. In a magnetic circuit, once a flux is set up [Model Question]
 (a) no further energy is required
 (b) energy is continuously required to maintain the flux
 (c) energy is released in the form of heat
 (d) none of these

Answer: (b)

23. Self inductance of a magnetic coil is proportional to [Model Question]
 (a) N (b) 1/N (c) N^2 (d) $1/N^2$
 [N = No. of turns]
 Answer: (c)

B. State the following statement as True or False

24. Permeance of magnetic circuit is similar to conductance of electric circuit. [Model Question]
 Answer: True

25. Lenz's law is associated with electrostatics. [Model Question]
 Answer: False

26. Fleming's right hand rule is also known as motor rule. [Model Question]
 Answer: False

27. Transformers are rated in KVA. [Model Question]
 Answer: True

C. Answer the following questions:

28. Define magnetic field intensity? [Model Question]

Answer:

Refer to Article No.2.1 of Chapter at a Glance.

29. What is the reciprocal of permeability?

[Model Question]

Answer: Reluctivity

30. What is the unit of magneto-motive force?

[Model Question]

Answer: AT (Ampere Turns)

31. What is the unit of reluctance in magnetic circuit?

[Model Question]

Answer: AT/Wb

32. Write down the relation between m.m.f. magnetic flux and reluctance in magnetic circuit.

[Model Question]

Answer:

Reluctance of a magnetic circuit is the ratio of m.m.f. and flux.

$$\text{Reluctance} = \frac{\text{m.m.f.}}{\text{flux}}$$

Long Answer Type Questions

1. (a) Define self & mutual inductance.

(b) State Faraday's laws of electromagnetic induction.

[WBSCTE 2022]

Answer:

a) Self Inductance:

When there is a change in the current or magnetic flux of the coil, an electromotive force is induced. This phenomenon is termed Self Inductance. When the current starts flowing through the coil at any instant, it is found that, the magnetic flux becomes directly proportional to the current passing through the circuit. The relation is given as:

$$\phi = L \times I$$

where L is termed as the self-inductance of the coil or the coefficient of self-inductance, the self-inductance depends on the cross-sectional area, the permeability of the material and the number of turns in the coil.

The rate of change of magnetic flux in the coil is given as,

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} = -L \frac{dI}{dt}$$

Self Inductance Formula:

$$L = N \frac{\phi}{I}$$

where, L is the self-inductance in Henries

N is the number of turns

ϕ is the magnetic flux

I is the current in amperes

Mutual inductance:

Consider two coils: P-coil (Primary coil) and S-coil (Secondary coil). A battery and a key are connected to the P-coil, whereas a galvanometer is connected across the S-coil. When there is a change in the current or magnetic flux linked with the two coils, an opposing electromotive force is produced across each coil and this phenomenon is termed Mutual Inductance.

This phenomenon is given by the relation:

$$\phi = MI$$

where M is termed as the mutual inductance of the two coils or the coefficient of the mutual inductance of the two coils.

The rate of change of magnetic flux in the coil is given as,

$$e = -\frac{d\phi}{dt} = -\frac{d(MI)}{dt} = -M \frac{dI}{dt}$$

Mutual Inductance Formula:

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

where, μ_0 is the permeability of free space

μ_r is the relative permeability of the soft iron core

N is the number of turns in coil

A is the cross-sectional area in m^2

l is the length of the coil in m

b) Faraday's Law:

1st law: when the flux linking with the coil or circuit changes (i.e. if a coil is subjected to an alternating flux), an emf is induced or if a conductor cuts magnetic flux an emf is induced in it.

2nd law: the magnitude of emf induced in a coil is proportional to the product of rate of change of flux and no of turns of the coil.

$$e \propto N \frac{d\phi}{dt}$$

2. (a) Define – (i) MMF and (ii) Reluctance of a magnetic circuit and state their units.

(b) A current of 20 amps flowing through a coil of 300 turns produces a flux of 100 mWb. Find the reluctance of the magnetic circuit. [Model Question]

Answer:

a) (i) **MMF:** Magneto-motive force is a force which establishes or tends to establish the magnetic flux in a magnetic circuit.

M.M.F is also defined as it is the difference of magnetic potential, a force which drives or tends to drive the magnetic flux through the magnetic circuit.

M.M.F. = Flux (ϕ) $\times$ reluctance (S)

= Ampere $\times$ turns

= Magnetic force (H) $\times$ length of the magnetic circuit (L)

(ii) Reluctance of the magnetic circuit is the property of the magnetic material by which it opposes or tries to oppose the production of the magnetic field in a material. Basically it is the property of a magnetic material. It is expressed by the symbol 'S'. The unit of the reluctance is AT/Weber.

b) Current flowing through the coil is 20 Amps. No of the turns = 300.

Magnetic flux (ϕ) = 100×10^{-3} Weber

We know that,

$$\text{Reluctance (S)} = \frac{L}{\mu_0 \mu_r N^2 A} = \frac{20 \times 300}{100 \times 10^{-3}} = 60000 \text{ AT/Weber}$$

3. Define – (i) magnetomotive force, (ii) permeance, (iii) magnetizing force. State the units of them. [Model Question]

Answer:

(i) **Magnetizing force:** It is also called magnetic field strength. It is the magnetomotive force per unit length at any given point in a magnetic circuit. Unit AT/m.

(ii) **Permeance (P):** It is the reciprocal of reluctance. It is defined as the property of the magnetic material due to which it allows the establishment flux in the core. Its unit is wb/AT.

(iii) **Magneto Motive Force:** It is expressed as a current, I flowing through a coil of N turns. The magnetic field strength of an electromagnet is therefore determined by the ampere turns of the coil with the more turns of wire in the coil the greater will be the strength of the magnetic field. Unit is AT.

4. What do you mean by magnetic hysteresis? Draw the hysteresis loop of a ferromagnetic material. [Model Question]

Answer:

Magnetic Hysteresis:

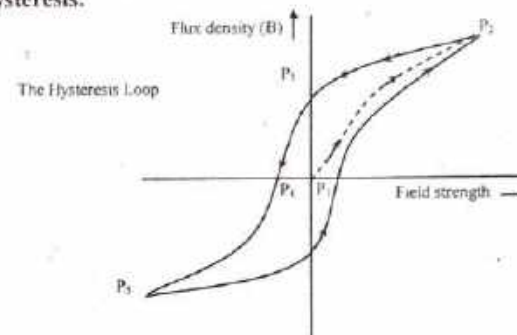


Fig. shows the effect of hysteresis in ferromagnetic materials

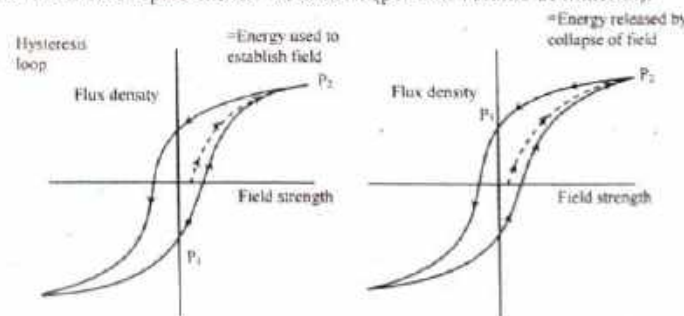
We start with an un-magnetized sample at the origin (P_1) where both field strength and flux density are zero. The field strength is increased in the positive direction and the flux begins to grow along the dotted path until we reach P_2 . This is called the initial magnetization curve.

If the field strength is now relaxed then some curious behavior occurs. Instead of retracing the initial magnetization curve the flux falls more slowly. In fact, even when the applied field is returned to zero there will still be a remaining (**remnant or remanent**) flux density at P_3 . It is this phenomenon which makes permanent magnets possible.

To force the flux to go back to zero we have to reverse the applied field (P_4). The field strength here is called the **coercivity**. We can then continue reversing the field to get to P_5 , and so on round this type of magnetization curve called (by J. A. Ewing) a hysteresis loop.

Significance of this from the point of power losses

It is that we have had to spend energy in order to set up the remnant flux. To show this more clearly we'll look at separate figures. The area (shown shaded) between the B-H curve and the B axis represents the work done (per unit volume of material).



5. State and explain Lenz's law.

[Model Question]

Answer:

Lenz's Law states that "the direction of the induced e.m.f. is such as to tend to oppose the change in the inducing flux." The mathematical expression of this law, in conjunction with Faraday's Law, introduces a negative sign, and gives

$$e = -N \frac{d\Phi}{dt}$$

6. (a) What is eddy current? How it can be minimized?

(b) What is 'Dot' current and 'Cross' current?

[Model Question]

Answer:

(a) When an alternating magnetic field is applied to a magnetic material an emf is induced in the material itself according to Faraday's Law of Electromagnetic induction. Since the magnetic material is a conducting material, these EMFs circulate currents within the body of the material. These circulating currents are called **Eddy Currents**. They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (I^2R loss) in the magnetic material known as an **Eddy Current Loss**.

 2nd part:

To reduce the eddy currents in the metal armature of motors the wire is wrapped around a number of thin metal sheets called laminations. This is done because the eddy currents would heat up the metal and so reduce (dampen) its efficiency. The laminations disrupt any induced eddy currents and consequently increase the efficiency of the motor. (The back emf in the iron core slows the motor but is not the back emf that appears in the windings.)

(b) A cross indicates current flowing in the direction perpendicular to the plane of the paper downwards.

A dot indicates that current is coming towards the observer looking into the paper.

7. (a) Write down the Fleming's Left Hand Rule.

(b) What do you understand by mutually induced e.m.f.?

[Model Question]

Answer:

(a) Refer to Article No. 2.8 of Chapter at a Glance.

(b) **Mutually induced e.m.f.** is the phenomenon in which a change of current in one coil causes an **induced emf** in another coil placed near to the first coil. The coil in which current is changed is called primary coil and the coil in which **emf is induced** is called secondary coil.

8. (a) State and explain Faraday's law of electromagnetic induction.

(b) A coil of 100 turns is wrapped around an iron core. If the flux in the core is given by the equation $\phi = 0.05 \sin(314t)$ Wb. What is the self-induced EMF in the coil at time $t = 0.02$ sec?

[Model Question]

Answer:

(a) Refer to Article No.2.5 of Chapter at a Glance.

(b) Emf induced $= N \cdot \frac{d\phi}{dt}$

$$e = 100 \times \frac{d}{dt}(0.05 \sin(314t))$$

$$e = 100 \times 0.05 \times 314 \times \cos(314t)$$

$$e \text{ at } t = 0.02 \text{ sec} = 1560.57 \text{ V}$$

 9. An iron ring of mean circumference 80 cm is uniformly wound with 500 turns of wire. When a current of 1 Amp is passed through the coil a flux density of 1.1 Wb/m² is produced in the iron. Find the permeability of the iron under these circumstances.

[Model Question]

Answer:

Mean circumference $= \pi D = 80 \text{ cm} = 0.8 \text{ m}$

$$\therefore D = \frac{0.8}{\pi} = 0.255 \text{ m}$$

Turn, $T = 500$

Current, $I = 1 \text{ Amp}$

$\therefore$ m.m.f. $= 500 \text{ AT}$

$$\therefore \text{Cross sectional area } a = \frac{\pi D^2}{4} = 0.05 \text{ m}^2$$

$$\therefore \text{Flux density} = \frac{\text{Flux}}{a}$$

or, Flux $= \text{Flux density} \times a$

$$= 1.1 \times 0.05 \text{ Wb} = 0.056 \text{ Wb}$$

$$\text{flux} = \frac{\text{m.m.f.}}{\text{Reluctance}}$$

$$\therefore \text{Reluctance, } S = \frac{\text{m.m.f.}}{\text{flux}} = \frac{500}{0.056} = 8928.6 \text{ AT/Wb}$$

$$S = \frac{L}{\mu_0 \times \mu_r \times a}$$

$$\therefore \mu_r = \frac{L}{S \times \mu_0 \times a} = 1426.79$$

10. A coil of 500 turns and resistance 20Ω is uniformly wound on an iron ring of mean circumference 50 cms and cross-sectional area 4 cm^2 . It is connected to a 24V D.C. supply. Under these conditions, the relative permeability of iron is 800. Calculate the total flux in the iron. [Model Question]

Answer:

$$I = \frac{V}{R} = \frac{24}{20} \text{ Amp} = 1.2 \text{ Amp}$$

$$\text{Mean circumference} = \pi D = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Turn, } T = 500$$

$$\text{Current, } I = 1.2 \text{ Amp}$$

$$\therefore \text{m.m.f.} = 500 \times 1.2 \text{ AT} = 600 \text{ AT}$$

$$\therefore \text{Cross sectional area } a = 4 \text{ cm}^2 = 0.0004 \text{ m}^2$$

$$\text{Reluctance, } S = \frac{L}{\mu_0 \times \mu_r \times a}$$

$$= \frac{0.5}{4\pi \times 10^{-7} \times 800 \times 0.0004} \text{ AT/Wb} = 1244090.6 \text{ AT/Wb}$$

$$\therefore \text{flux} = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{600}{1244090.6} \text{ Wb} = 4.82 \times 10^{-4} \text{ Wb}$$

11. (a) Define – (i) leakage flux, (ii) fringing of magnetic flux, (iii) useful flux.
(b) Compare between permanent magnet and electromagnet. Which one of these find wide applications. [Model Question]

Answer:

(a) **Leakage flux:** The amount of flux lines that do not follow the core and are lost to the surrounding air.

Fringing of magnetic flux: The effect of the fringing field is to increase the effective cross sectional area A_g of the air gap. Due to the fringing effect, the magnetic flux in the air gap of electrical machines is reduced. This leads to the larger effective air gap length.

Useful flux: Basically useful flux is the flux that flows within the magnetic circuit of an armature. That is why when there is an air gap the flux enters the armature which makes it useful flux

(b) A permanent magnet is an object made from a material that is magnetized and creates its own persistent magnetic field. As the name suggests, a permanent magnet is

'permanent'. This means that it always has a magnetic field and will display a magnetic behavior at all times.

An electromagnet is made from a coil of wire which acts as a magnet when an electric current passes through it. Often an electromagnet is wrapped around a core of ferromagnetic material like steel, which enhances the magnetic field produced by the coil.

Permanent Magnet v. Electromagnet: Magnetic Properties: A permanent magnet's magnetic properties exist when the magnet is (magnetized). An electromagnetic magnet only displays magnetic properties when an electric current is applied to it. That is how you can differentiate between the two. The magnets that you have affixed to your refrigerator are permanent magnets, while electromagnets are the principle behind AC motors.

Permanent Magnet v. Electromagnet: Magnetic Strength: Permanent magnet strength depends upon the material used in its creation. The strength of an electromagnet can be adjusted by the amount of electric current allowed to flow into it. As a result, the same electromagnet can be adjusted for different strength levels.

Permanent Magnet v. Electromagnet: Loss of Magnetic Properties: If a permanent magnet loses its magnetic properties, as it does by heating to a (maximum) temperature, it will be rendered useless and its magnetic properties can be only recovered by re-magnetizing. Contrarily, an electromagnet loses its magnetic power every time an electric current is removed and becomes magnetic once again when the electric field is introduced.

Permanent Magnet v. Electromagnet: Advantages: The main advantage of a permanent magnet over an electromagnet is that a permanent magnet does not require a continuous supply of electrical energy to maintain its magnetic field. However, an electromagnet's magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current supplied to the electromagnet.

12. A radial saw cut of 2 mm cut in an iron ring of mean diameter of 21 cms, radial thickness of 4 cms axial length of 2 cm. If it is wound with 1000 turns of wire, calculate the current required to produce a flux of 96 m Wb in the gap. Relative permeability is 3000. [Model Question]

Answer:

$$\text{Mean circumference} = \pi D = \pi \times \frac{21}{100} \text{ m} = 0.659 \text{ m}$$

$$\therefore \text{Cross sectional area, } a = 4 \times 2 \times 10^{-4} \text{ m}^2 = 0.0008 \text{ m}^2$$

$$\text{Turn, } T = 1000$$

Let Current 'I' is required to produce the said flux

$$\therefore \text{m.m.f.} = I \times 1000 \text{ AT}$$

Reluctance in air gap, $S_a = \frac{L_a}{\mu_0 \times a} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.0008} \text{ AT/Wb} = 1989436.78 \text{ AT/Wb}$

$$\therefore \text{Reluctance} = \frac{\text{m.m.f.}}{\text{Flux}}$$

$$\text{or, } 1989436.78 = \frac{I \times 1000}{36 \times 10^{-3}};$$

$$\text{or, } I \times 1000 = 1989436.78 \times 36 \times 10^{-3}$$

$$\text{or, } I = \frac{1989436.78 \times 36 \times 10^{-3}}{1000};$$

$$\text{or, } I = 71.6197 \text{ Amp}$$

$\therefore$ Required current is 71.6179 Amp.

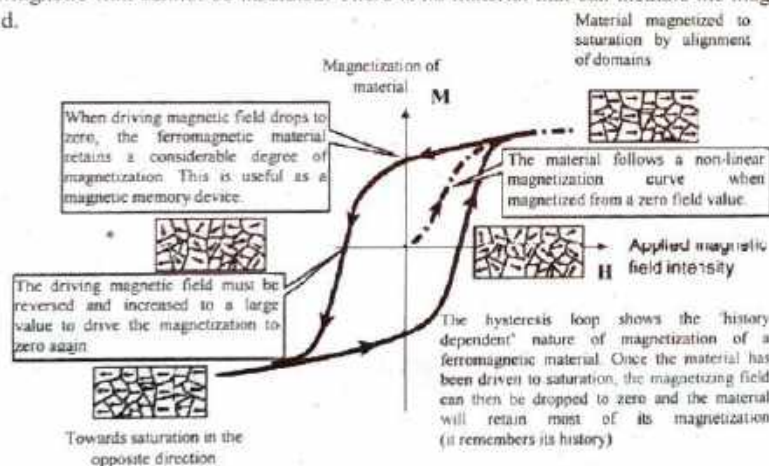
13. (a) Can the magnetic flux be insulated? Give the sketch of a magnetic hysteresis loop. Indicate in the loop the positions of remanent flux density.

(b) Distinguish between magnetizing force and magneto-motive force.

[Model Question]

Answer:

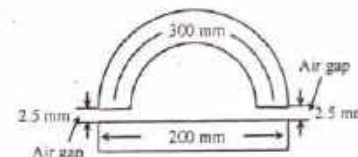
a) Magnetic flux cannot be insulated. There is no material that can insulate the magnetic field.



b) **Magnetizing force:** It is also called magnetic field strength. It is the magnetomotive force per unit length at any given point in a magnetic circuit.

Magneto Motive Force: It is expressed as a current, I flowing through a coil of N turns. The magnetic field strength of an electromagnet is therefore determined by the ampere turns of the coil with the more turns of wire in the coil the greater will be the strength of the magnetic field.

14. An electromagnet is of the form and dimensions shown. It is made of iron of square cross-section, 50 mm in side. A flux density of 1.2 m Wb is required in the air gaps. Neglecting leakage and fringing, find the number of AT required. Take μ_r to be 2000 at this flux density. [Model Question]



Answer:

Length of the air gap $L_a = 2.5 \text{ mm} = 0.0025 \text{ m}$

Cross sectional area $a = 50^2 \text{ mm}^2 = 0.0025 \text{ m}^2$

Reluctance in air gap,

$$S_a = \frac{L_a}{\mu_0 \times a} = \frac{0.0025}{4\pi \times 10^{-7} \times 0.0025} \text{ AT/Wb} = 795774.715 \text{ AT/Wb}$$

$$\therefore \text{Required flux} = 1.2 \text{ mWb} = 1.2 \times 10^{-3} \text{ Wb}$$

$$\text{flux} = \frac{\text{m.m.f.}}{\text{Reluctance}}$$

$$\therefore \text{m.m.f.} = \text{flux} \times \text{Reluctance} = 1.2 \times 10^{-3} \times 795774.715 \text{ AT} = 954.9296 \text{ AT}$$

$\therefore$ 957.9296 AT is required to produce said amount of flux in air.

15. What is relative permeability? Compare between magnetic and electric circuit. [Model Question]

Answer: Refer to Chapter at a Glance.

16. (a) Draw neatly the hysteresis loop and indicate hysteresis loss on it.

(b) A coil of 500 turns and resistance 20Ω is uniformly wound on an iron ring of mean circumference of 50 centimeters and cross sectional area of 4 cm^2 . It is connected to 24 V D.C. supplies. Under these conditions, the relative permeability of iron is 800. Calculate total flux in iron. [Model Question]

Answer:

a) Refer to Article No.2.3 of Chapter at a Glance.

$$\begin{aligned} \text{b) } \frac{I}{\mu_0 \mu_r A} &= \frac{.5}{4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4}} = \frac{.5}{4\pi \times 800 \times 4 \times 10^{-11}} \\ &= \frac{.5}{4\pi \times 32 \times 10^{-9}} = \frac{.5 \times 10^9}{4\pi \times 32} = \frac{500 \times 10^6}{4\pi \times 32} = 1.24 \times 10^6 \text{ henry}^{-1} \end{aligned}$$

$$\phi = \frac{NI}{S} = \frac{500 \times \frac{24}{20}}{1.24 \times 10^{-6}} = \frac{500 \times 1.2}{1.24 \times 10^{-6}} = \frac{600}{1.24 \times 10^{-6}}$$

$$= 483.87 \times 10^{-6} \text{ Wb} = 48387 \times 10^{-7} \text{ Wb} = 483 \text{ mWb}$$

17. (a) Define with its units of -

- (i) absolute permeability,
- (ii) reluctance,
- (iii) magnetomotive force,
- (iv) permeance.

(b) A current of 5A is passed through a 1000-turn coil wound on a circular magnetic circuit of radius 120 mm. Calculate (i) the magnetomotive force, and (ii) the magnetic field strength. [Model Question]

Answer:

a) Refer to Article No. 2.1 of Chapter at a Glance.

b) Electrical Technological

$$I = 5 \text{ A} \quad N = 1000 \text{ turns}$$

$$D = 120 \times 2 \text{ mm} \quad l = \pi d$$

$$= 24 \times 10^{-2} \text{ m} \quad = \pi \times 12 \times 2 \times 10^{-2} \text{ m}$$

$$= \pi \times 25 \times 10^{-2}$$

(i) magnetomotive force = $NI = 1000 \times 5 = 5000 \text{ Amp}$

(ii) Magnetic field strength

$$H = \frac{NI}{l} = \frac{1000 \times 5 \times 10^2}{\pi \times 12 \times 2} \text{ A/m} = \frac{13.269}{2} = 6,634.8 \text{ A/m}$$

18. (a) A coil of resistance 10 ohm and inductance 0.1 H is connected in series with a condenser of capacitance 150 μF across a 200 V, 50 Hz supply. Determine -

(i) total impedance of the circuit, (ii) current, (iii) power factor, (iv) voltage across the coil, and (v) voltage across the condenser.

(b) An electromagnet has an air gap of 4 mm. and flux density in the gap is 1.3 Wb/m². Determine the ampere-turns for the gap. Given $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$. [Model Question]

Answer:

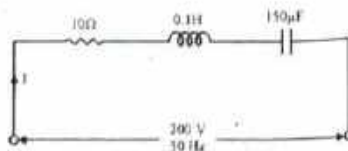
a)

$$R = 10 \Omega$$

$$L = 0.1 \text{ H}$$

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.1 = 31.4 \Omega$$

$$C = 150 \mu\text{F}$$



$$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 150} = \frac{10^3 \times 10^1}{100\pi \times 150} = 21.23 \Omega$$

$$i) Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (31.4 - 21.23)^2}$$

$$= \sqrt{10^2 + 103.4289} = \sqrt{203.4289} = 14.26 \Omega$$

$$ii) I = \frac{E}{Z} = \frac{200}{14.26} = 14.025 \text{ A}$$

$$iii) \text{ Power factor} = \frac{R}{Z} = \frac{10}{14.26} = 0.701$$

$$iv) \text{ Voltage across the coil } F_L = IX_L = 14.025 \times 31.4 = 440.385 \text{ V}$$

$$v) \text{ Voltage across the condenser } E_C = IX_C = 14.025 \times 21.23 = 297.750 \text{ V}$$

b) Here, $l_g = 4 \text{ mm} = 0.4 \text{ cm} = 4 \times 10^{-3} \text{ m}$; $B_g = 1.3 \text{ Wb/m}^2$

Ampere turns required for the gap

$$= H_g \times l_g = \frac{B_g}{\mu_0} \times l_g = \frac{1.3}{4\pi \times 10^{-7}} \times 4\pi \times 10^{-3} = 4,136.83 \text{ AT}$$

Chapter 3: A.C Circuits

Chapter at a glance:

3.1 Principle of generation of sinusoidal voltage and its waveform representation

The elementary AC generator (Fig below) consists of a conductor or loop of wire in a magnetic field that is produced by an electromagnet. The two ends of the loop are connected to slip rings, and they are in contact with two brushes. When the loop rotates it cuts magnetic lines of force, first in one direction and then the other.

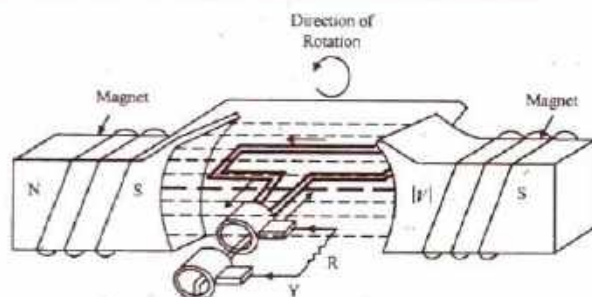


Fig: Simple AC Generator

3.2 Definition

i) Instantaneous Value

The value of alternating voltage or current at any instant is called its instantaneous value and is represented by small letter that is, e for emf, v for voltage and i for current.

ii) Cycle

Figure (i) shows a sine wave representing one complete cycle. The wave periodically passes through a definite cycle of changes, each complete cycle consisting of two half cycles (one positive and one negative) exactly identical, during one of which the voltage acts in one direction around a circuit and during the other, in the opposite direction. Hence, a complete set of positive and negative values of an alternating voltage or current can be termed as a cycle of ac wave. A cycle of ac wave is normally specified in terms of angular measure spread over 360° or 2π radians.

One half cycle is called alternation. One alternation corresponds to 180° or π radians.

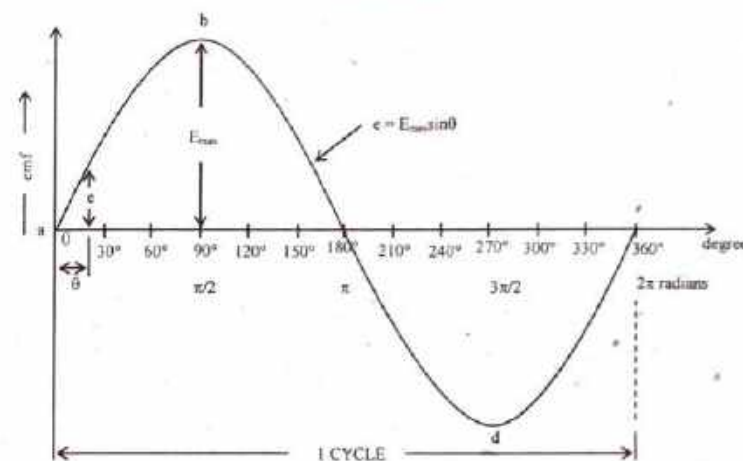


Fig: (i) Sine wave of emf

iii) Frequency

The number of cycles passed through in one second is called the frequency of the wave. It is denoted by f and is expressed in hertz (Hz) or cycles per second (c/s).

iv) Time Period

The time taken by an alternating voltage or current to complete one cycle is termed its time period. It is denoted by T and expressed in seconds. The relationship between frequency and periodic time is given by,

$$\text{Time period, } T = \frac{1}{f}$$

Angular Velocity

The angular distance covered per second is defined as angular velocity. It is represented by ω and expressed in rad/sec. The relationship between angular velocity and frequency is given by

$$\omega = 2\pi f$$

v) Amplitude

The alternating wave shape shown in fig. (i) has equal magnitude for the maximum value of positive and negative half of the wave shape. This maximum value of the positive or negative half of the ac wave is called its amplitude. The amplitude of an alternating emf, voltage and current is expressed by $E_{\max}$, $V_{\max}$ and $I_{\max}$ respectively.

vi) Average Value

The average value of the current or voltage of an alternating wave shape is the arithmetic mean of the ordinates at equal intervals over a half cycle of that wave. However, if the arithmetic mean is found out over the complete cycle, it will be zero for sinusoidal as well as for non-sinusoidal wave, provided the wave shape is symmetrical. In such a case, as much current flows in one direction during one half cycle, as flows in the opposite direction during the next half cycle.

Root Mean Square Value

The alternating voltage or current wave shape completes a certain number of cycles in one second, each cycle consisting of an identical positive and negative half cycle. The value of the quantity in such a wave shape changes from instant to instant. It becomes maximum at a certain instant only. In specifying such a varying quantity (current or voltage), its maximum or peak value is rarely used. The root mean square (rms) value of the alternating current or voltage is more extensively used in practice to express the quantity. It is also termed the effective or virtual value of the alternating quantity.

vii) Form Factor

The form factor of an alternating current wave shape is defined as the ratio of its rms value to the average value of current over a half cycle.

$$\text{Thus, form factor} = \frac{\text{rms value of current}}{\text{Average value of current}}$$

For sine wave,

$$\text{rms value of current, } I = 0.707 I_{\max}$$

$$\text{Average value of current, } I_{av} = 0.637 I_{\max}$$

$$\text{Thus, form factor for sine wave} = \frac{0.707 I_{\max}}{0.637 I_{\max}} = 1.11$$

Knowing the form factor of an alternating wave, its rms value can be ascertained from the average value and vice versa.

Peak Factor

The peak factor of an alternating current wave shape is defined as the ratio of its maximum value to the rms value.

$$\text{Peak factor} = \frac{\text{Maximum value of current}}{\text{rms value of current}}$$

This is also called crest factor or amplitude factor.

$$\text{For a sine wave, peak factor} = \frac{I_{\max}}{I_{\max}/\sqrt{2}} = \sqrt{2} = 1.414$$

Normally, the rms value of current or voltage of an alternating wave is specified. If peak factor of a wave is known, its maximum value can be determined, which is necessary to calculate the iron losses because these depend upon the value of maximum flux.

3.3 Representation of sinusoidal quantities

Before discussing the reactive components, we would like to recapitulate our knowledge of vector algebra.

In alternating current circuits generally, and especially in networks, the symbolic notation is of great use in simplifying the calculations of the various quantities involved. For this reason it will be considered here before proceeding to work in which such calculations are necessary.

Fig shows a vector representing (say) a voltage, which is expressed in the usual trigonometric notation, is given by

$$V = V_{\max} \sin(\omega t + \alpha)$$

Thus, symbolically, the vector is expressed as

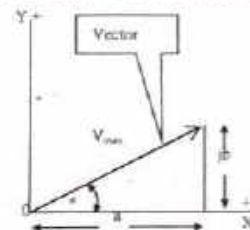
$$[V] = V_{\max} \cos \alpha + j V_{\max} \sin \alpha = V_{\max} [\cos \alpha + j \sin \alpha]$$

$$\text{or, } [V] = a + jb$$

The actual magnitude of V is $\sqrt{a^2 + b^2}$.

And its phase angle $\tan^{-1} \left(\frac{b}{a} \right)$

$$V = \sqrt{a^2 + b^2} \left[\tan^{-1} \left(\frac{b}{a} \right) \right] \text{ is the representation in the polar form.}$$



3.4 Expressions of voltage and current for sinusoidal sources through

Pure Inductance

Any device relying on magnetism or magnetic fields for operation is a form of inductor. The use of an inductor in a circuit can cause current and voltage to become out-of-phase and inefficient unless corrected.

Inductive Reactance (R-L Circuit)

In an inductive AC circuit, the current is continually changing and is continuously inducing an EMF.

$$\text{Inductive reactance: } l = \frac{E}{X_L}$$

Where I = effective current

X_L = inductive reactance (Ω)

E = effective voltage across the reactance (V)

$$X_L = 2\pi fL$$

Where $\pi = 3.14$

f = frequency (Hertz)

L = inductance (Henries)

In a purely inductive circuit, the resistance is negligible in comparison to the inductive reactance. The voltage applied to the circuit must always be equal and opposite to the EMF of self-induction.

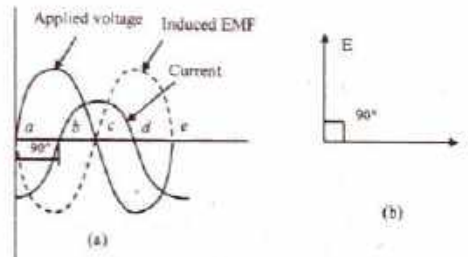
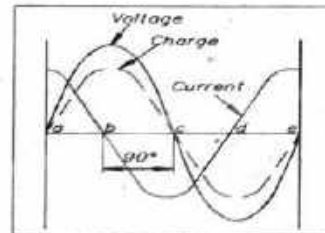


Fig: Current, Self-Induced EMF and Applied Voltage in an Inductive Circuit

Capacitance

The variation of an alternating voltage applied to a capacitor, the charge on the capacitor, and the current flowing through the capacitor is shown in figure below.

The current flow in a circuit containing capacitance depends on the rate at which the voltage changes.



In any purely capacitive circuit, current leads applied voltage by 90° .

Capacitive Reactance(R-C Circuit)

Capacitive reactance is the opposition by a capacitor or a capacitive circuit to the flow of current.

The current flowing in a capacitive circuit is directly proportional to the capacitance and to the rate at which the applied voltage is changing.

Capacitive reactance X_C is measured in ohms, as is inductive reactance.

Equation below is a mathematical representation for capacitive reactance.

$$X_C = \frac{1}{2\pi fC}$$

where

f = Frequency (Hz)

$\pi = 3.14$

C = capacitance (farads)

Equation below is the mathematical representation of capacitive reactance when capacitance is expressed in microfarads (F).

$$X_C = \frac{1,000,000}{2\pi fC}$$

If the applied voltage (E) is chosen as the reference phasor, then the current can be expressed as a phasor that leads the applied voltage by 90° .

Since the voltage induced in an capacitive circuit always lags the current by 90° , the

capacitive reactance can be expressed in vector notation as $X_C = \frac{1}{j2\pi fC}$

Equation below is the mathematical representation for the current that flows in a circuit

with only capacitive reactance $|I| = \frac{|E|}{X_C}$

Where

I = effective current (A)

E = effective voltage across the capacitive reactance (V)

X_C = capacitive reactance (Ω)

Impedance(R-L-C Circuit)

Equation below is the mathematical representation for the magnitude of impedance in an AC circuit.

$$Z = R + j2\pi fL + \frac{1}{j2\pi fC} = R + jX_L - jX_C = R + jX$$

Where $X = X_L - X_C$, the total reactance of the circuit.

$$|Z| = \sqrt{R^2 + X^2}$$

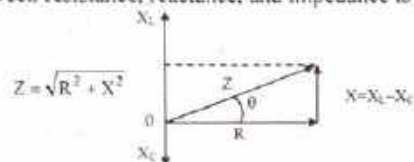
Where

Z = impedance (Ω)

R = resistance (Ω)

X = net reactance (Ω)

The relationship between resistance, reactance, and impedance is shown in Figure below.



This triangle is known as impedance triangle.

Noting the following points

- The current through a resistance is always in phase with the applied voltage. Resistance is shown on the zero axis.
- The current through an inductor lags applied voltage by 90° ; inductive reactance is shown along the 90° axis.
- Current through a capacitor leads applied voltage by 90° ; capacitive reactance is shown along the -90° axis.
- Net reactance in an AC circuit is the difference between inductive and capacitive reactance.
- Equation below is the mathematical representation for the calculation of net reactance when X_l is greater than X_c .

$$X = X_l - X_c$$

Where

X = net reactance (Ω)

X_l = inductive reactance (Ω)

X_c = capacitive reactance (Ω)

Mathematical representation

- Equation below is the mathematical representation for the calculation of net reactance when X_c is greater than X_l .

$$X = X_c - X_l$$

Impedance is the vector sum of the resistance and net reactance (X) in a circuit, as shown in Figure above. The angle θ is the phase angle and gives the phase relationship between the applied voltage and the current.

- Impedance in an AC circuit corresponds to the resistance of a DC circuit.

- The voltage drop across an AC circuit element equals the current times the impedance. Equation below is the mathematical representation of the voltage drop across an AC circuit.

$$V = IZ$$

Where

V = voltage drop (V)

I = current (A)

Z = impedance (Ω)

3.5 Expression of power factor, active, reactive and apparent power

Basic AC Power

Power Triangle

The power triangle represents comparable values that can be used directly to find the efficiency level of generated power to usable power, which is expressed as the power factor (discussed later). Apparent power, reactive power, and true power can be calculated by using the DC equivalent (RMS value) of the AC voltage and current components along with the power factor.

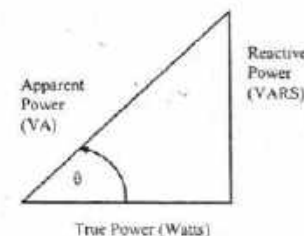


Fig: Power Triangle

Apparent Power

Apparent power (S) is the power delivered to an electrical circuit. The measurement of apparent power is in volt amperes (VA).

$$S = I^2 Z = IE$$

Where, S = apparent power (VA)

I = RMS current (A)

E = RMS voltage (V)

Z = impedance (Ω)

True Power

True power (P) is the power consumed by the resistive loads in an electrical circuit. Equation below is a mathematical representation of true power. The measurement of true power is in watts.

$$P = I^2 R = EI \cos \theta$$

Where, P = true power (watts)

I = RMS current (A)

E = RMS voltage (V)

R = resistance (Ω)

θ = angle between E and I sine waves

Reactive Power

Reactive power (Q) is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields. Reactive power is expressed in volt-amperes-reactive (VAR). Equation below is a mathematical representation for reactive power.

$$Q = I^2 X = EI \sin \theta$$

where, Q = reactive power (VAR)

I = RMS current (A)

X = net reactance (Ω)

E = RMS voltage (V)

θ = angle between E and I sine waves

Power Factor

Power factor (pf) is the ratio between true power and apparent power. True power is the power consumed by an AC circuit, and reactive power is the power that is stored in an AC circuit. $\cos \theta$ is called the power factor (pf) of an AC circuit.

$$\cos \theta = \frac{P}{S}$$

where $\cos \theta$ = power factor (pf)

P = true power (watts)

S = apparent power (VA)

Power factor also determines what part of the apparent power is real power. It can vary from 1, (when the phase angle is 0°), to 0, (when the phase angle is 90°). In an inductive circuit, the current lags the voltage and is said to have a lagging power factor, as shown in Figure below.

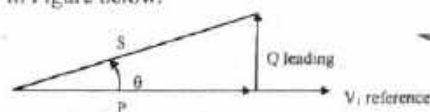


Fig: Leading Power Factor



Fig: Lagging Power Factor

In a capacitive circuit, the current leads the voltage and is said to have a leading power factor.

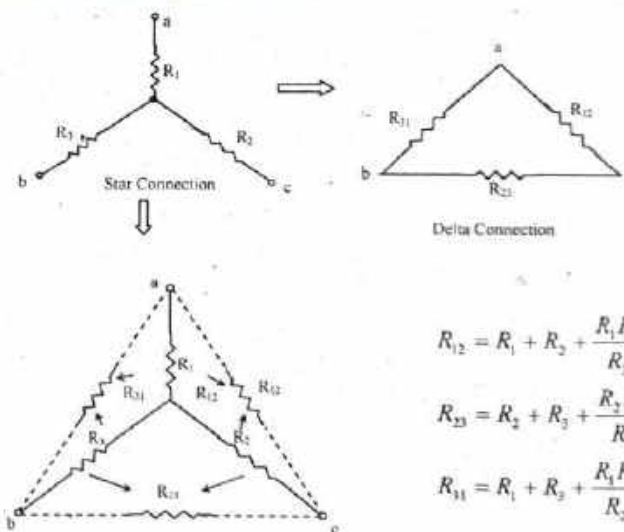
AC power relationships are summarized below:

- Observe the equations for apparent, true, and reactive power, and power factor:
 - Apparent power (S) = $I^2 Z = IE$
 - True power (P) = $I^2 R = EI \cos \theta$
 - Reactive power (Q) = $I^2 X = EI \sin \theta$
 - Power factor (pf) = $\frac{P}{S} = \cos \theta$

From observation, you can see that three power equations have the angle θ in common. θ is the angle between voltage and current. From this relationship, a power triangle, as shown in Figure above, is formed.

- Current (I) lags voltage (E) in an inductive circuit (L)
- Current (I) leads voltage (E) in a capacitive circuit (C)

3.6. Star to Delta Transformation



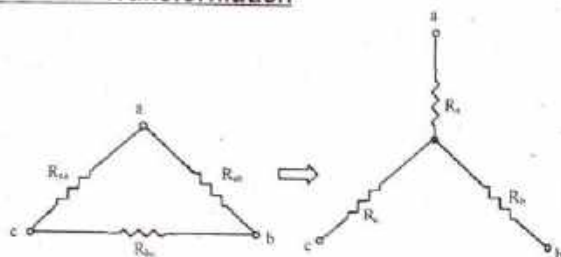
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

Here R_1, R_2, R_3 are the resistances in star connection; and R_{12}, R_{31}, R_{23} are the equivalent resistances in delta connection.

3.7 Delta to Star Transformation



$$R_a = \frac{R_{ab} \cdot R_{ac}}{R_{ab} + R_{bc} + R_{ca}}, \quad R_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab} + R_{bc} + R_{ca}}, \quad R_c = \frac{R_{bc} \cdot R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The unit of inductive reactance is [WBSCTE 2022]
 (a) Henry (b) Farad (c) Ohm (d) Joule

Answer: (c)

2. In an ac circuit ' $VI \sin \theta$ ' represents [WBSCTE 2022]
 (a) Active power (b) Reactive power
 (c) Apparent power (d) True power

Answer: (b)

3. When sinusoidal voltage is applied across a pure capacity the current in the capacitor [WBSCTE 2022]
 (a) lags applied voltage by 90°
 (b) leads applied voltage by 90°
 (c) is in phase with applied voltage
 (d) is in phase opposition with applied voltage

Answer: (b)

4. Peak factor for a sine wave is [WBSCTE 2022]
 (a) 1 (b) 1.414 (c) 1.11 (d) 2

Answer: (b)

A.C Circuits

FE.45

5. The maximum value of sine wave current is 100 A, the rms value of current in the circuit will be [WBSCTE 2022]
 (a) 70.7 A (b) 100 A (c) 50 A (d) 70.7 mA

Answer: (d)

6. The form factor for a sinusoidal a.c. voltage is— [Model Question]
 (a) 1.41 (b) 1.11 (c) 1.00 (d) 0.707

Answer: (b)

7. In an a.c system the ratio of the maximum current and average current is called — [Model Question]
 (a) form factor (b) peak factor (c) load factor (d) power factor

Answer: (b)

8. In a series R-L circuit, the applied voltage — [Model Question]
 (a) lags the current (b) leads the current
 (c) is in phase with the current (d) 90° ahead of the current

Answer: (b)

9. A sinusoidal voltage is passed through a series combination of an inductor (L) and a capacitor (C), the current flowing will — [Model Question]
 (a) be lagging (b) be leading
 (c) be in phase (d) lagging or leading will depend on $X_L - X_C$

Answer: (d)

10. In a R-L circuit, the phase difference ϕ between applied voltage and circuit current will increase if — [Model Question]
 (a) X_L is increased (b) R is increased
 (c) X_L is decreased (d) supply frequency is decreased

Answer: (a)

11. An alternating current is given by $i = 10 \sin 314t$ A. The time taken to generate two complete cycles of current is — [Model Question]
 (a) 0.02 second (b) 0.1 second
 (c) 0.04 second (d) 0.05 second

Answer: (c)

12. An alternating voltage is given by $v = 30 \sin 214t$. The time taken by the voltage to reach — 30V for the first time is — [Model Question]
 (a) 0.02 second (b) 0.01 second (c) 0.03 second (d) 0.05 second

Answer: (d)

13. An inductive circuit draws a line current of 10A. If the reactive component of line current is 6A, then power factor of the current is – [Model Question]
 (a) 0.6 lagging (b) 0.8 lagging (c) 0.5 lagging (d) none of these

Answer: (b)

14. In 3 phase AC system the magnitude of the voltage between two phases are – [Model Question]
 (a) 230V (b) 415V (c) 300V (d) 150V

Answer: (a)

15. Term 'Reluctance' is associated with – [Model Question]
 (a) electrical circuit (b) distribution line
 (c) magnetic circuit (d) generator

Answer: (c)

B. State the following statement as True or False

16. In an alternating current circuit the reactive power is the measure of actual power consumption. [Model Question]

Answer: False

17. An alternating voltage is represented by $20e^{-j2\pi/3}$, its polar form is $20+120j$. [Model Question]

Answer: False

18. Minimum value of power factor is 1. [Model Question]

Answer: False

C. Answer the following questions:

19. What is the value of form factor of a sine wave? [Model Question]

Answer: 1.41

20. What is the average value of an a.c. voltage over a cycle? [Model Question]

Answer: Zero

21. What is the power factor of purely capacitive circuit? [Model Question]

Answer: Leading

22. What is the value of power factor in purely capacitive circuit? [Model Question]

Answer: Leading

23. Can it possible to start an a.c. motor with the help of four point starter? [Model Question]

Answer:

The drawback of the 4 point starter is that it cannot limit or control the high current speed of the motor, so an A.C. Motor should not be started with the help of a four point starter.

24. Write down the expression of active power for a 3-phase a.c circuit in terms of phase quantities. [Model Question]

Answer: In a balanced three phase system magnitude of each line-to-neutral voltage (also line-line) is the same, as is the magnitude of each phase current. Therefore, the expression for total power in terms of phase current and phase voltage is given by:

$$P = 3 V I \cos \theta \text{ Watt, } \theta = \angle(\vec{I} \& \vec{V})$$

In-terms of line voltage & line current the above expression becomes:

$$P = \sqrt{3} V_L I_L \cos \theta \text{ Watt, } \theta = \angle(\vec{I} \& \vec{V})$$

In the above expression V and I are the r.m.s values of the phase voltage and phase current respectively. We can also calculate reactive power (Q) and complex power (S). For balanced load, the expressions for Q and S are given by:

$$Q = \sqrt{3} V_L I_L \sin \theta \text{ VAR, } \theta = \angle(\vec{I} \& \vec{V}), S = \sqrt{3} V_L I_L V_A$$

Long Answer Type Questions

1. (a) Define True & reactive power.
 (b) Show that in a pure capacitive circuit the current leads the applied voltage by 90 degree. Also show the phasor diagram for the same. [WBSCTE 2022]

Answer:

a) True Power

True power (P) is the power consumed by the resistive loads in an electrical circuit. Equation below is a mathematical representation of true power. The measurement of true power is in watts.

$$P = I^2 R = EI \cos \theta$$

Where, P = true power (watts)

I = RMS current (A)

E = RMS voltage (V)

R = resistance (Ω)

θ = angle between E and I sine waves

Reactive Power

Reactive power (Q) is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields. Reactive power is expressed in volt-amperes-reactive (VAR). Equation below is a mathematical representation for reactive power.

$$Q = I^2 X = EI \sin \theta$$

where, Q = reactive power (VAR)

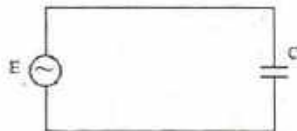
I = RMS current (A)

X = net reactance (Ω)

E = RMS voltage (V)

θ = angle between E and I sine waves

b) Let us consider a capacitor C connected to an AC source as shown below:



Let the AC voltage applied be

$$v = v_m \sin \omega t$$

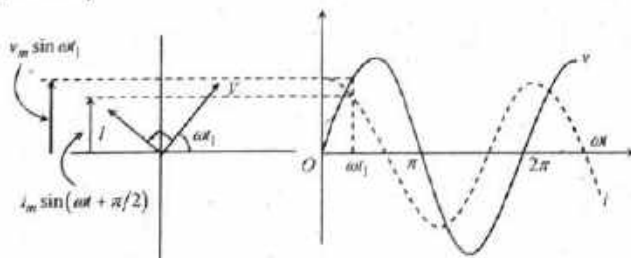
$$\therefore v = \frac{q}{C}$$

Applying Kirchhoff's loop rule, we have:

$$v_m \sin \omega t = \frac{q}{C}$$

$$i = \frac{dq}{dt}$$

$$\therefore i = \frac{d}{dt} (v_m C \sin \omega t)$$



$$\omega C V_m \cos(\omega t)$$

$$\cos \omega t = \sin \omega t + \frac{\pi}{2}$$

$$\therefore i = i_m \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$i_m = \omega C V_m$$

$$i = \frac{v_m}{\left(\frac{1}{\omega C} \right)}$$

Hence, the current leads the voltage in phase by $\frac{\pi}{2}$.

2. (a) Calculate the RMS and average values of purely sinusoidal current having peak value 15A.

(b) Draw STAR & DELTA connections. Also write voltage and current relations for both types. [WBSCTE 2022]

Answer:

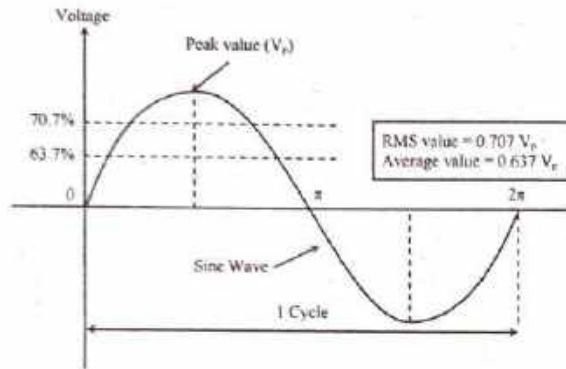
a) RMS Value:

RMS (Root mean square) value:

- RMS value is based on the heating effect of wave-forms.
- The value at which the heat dissipated in the AC circuit is the same as the heat dissipated in the DC circuit is called the RMS value provided, both the AC and DC circuit have equal value of resistance and are operated at the same time.
- RMS value 'or' the effective value of an alternating quantity is calculated as:

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

T = Time period



$$\text{RMS value} = V_m \sin \theta = V_m \sin 45^\circ = 0.707 V_m$$

Average Value:

The average value of sinusoidal waveform over one complete cycle is zero as two halves cancel each other, so the average value is taken over half a cycle.

$$V_{avg} = \frac{1}{\pi} \int_0^\pi V_p \sin \theta d\theta$$

$$V_{avg} = \frac{V_p}{\pi} (-\cos \theta)_0^\pi$$

$$V_{avg} = \frac{2V_p}{\pi} = 0.637 \times V_p$$

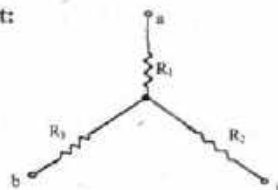
Therefore, the average value of a sinusoidal wave is 0.637 times the peak value.

Calculations:

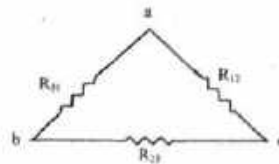
$$I_p = 15 \text{ A}$$

$$I_{avg} = \frac{2I_p}{\pi} = \frac{2 \times 15}{\pi} = 9.55 \text{ Amp.}$$

b) 1st Part:



Star Connection



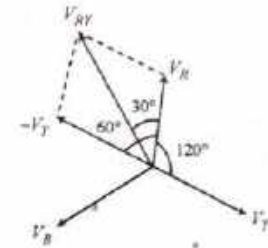
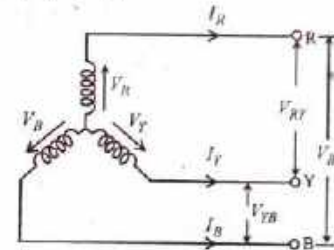
Delta Connection

2nd Part:

For Star connected System:

Let V_R , V_Y , V_B represents the three phase voltages while V_{RY} , V_{YB} and V_{BR} represents the line voltages. Assume that the system is balanced, so

$$|V_R| = |V_Y| = |V_B| = |V_{ph}|$$



From the circuit and phasor diagram of star connected load, it can be observed that the line voltage V_{RY} is a vector difference of V_R and V_Y or the vector sum of V_R and $-V_Y$, i.e.

$$V_{RY} = V_R + (-V_Y) = V_R - V_Y$$

Applying parallelogram law to obtain the magnitude of this, we get,

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^\circ}$$

$$\Rightarrow V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \cos 60^\circ} = \sqrt{3} V_{ph}$$

Similarly,

$$V_{YB} = V_Y - V_B = \sqrt{3} V_{ph}$$

$$V_{BR} = V_B - V_R = \sqrt{3} V_{ph}$$

$$\therefore V_{RY} = V_{YB} = V_{BR} = V_L = \text{Line Voltage}$$

$$\therefore V_L = \sqrt{3} V_{ph}$$

Therefore, in a star connected system, Line Voltage = $\sqrt{3} \times$ Phase Voltage.

Let I_R , I_Y and I_B being the currents in R, Y and B lines respectively. Since, the load is balanced, therefore,

$$I_R = I_Y = I_B = I_{ph} \text{ (say)}$$

Then,

$$I_L = I_{ph}$$

$$\Rightarrow \text{Line Current} = \text{Phase Current}$$

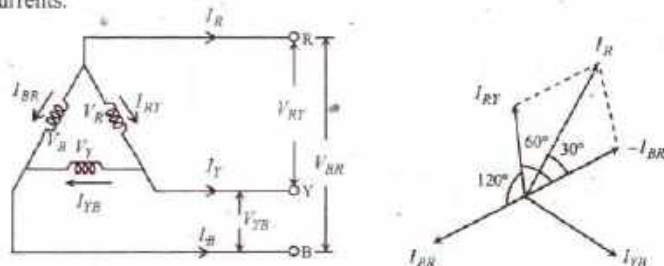
Note – for a balanced star connected system, the vector sum of line current is equal to zero,

$$\text{i.e. } I_R + I_Y + I_B = I_0 = 0$$

Where, I_n is the natural current.

For Delta connected System:

Let I_{RY} , I_{YB} and I_{BR} are the phase current in delta connected system while I_R , I_Y and I_B being the line currents.



By referring the circuit and phasor diagram, it can be seen that current in each line is the vector difference of corresponding phase currents and are given as,

$$I_R = I_{BR} - I_{RY}$$

$$I_Y = I_{RY} - I_{YB}$$

$$I_B = I_{YB} - I_{BR}$$

Now, the magnitude of current I_R can be obtained by parallelogram law of vector addition, as follows,

$$I_R = \sqrt{I_{BR}^2 + I_{RY}^2 + 2I_{BR}I_{RY}\cos 60^\circ}$$

Assumed the system is balanced, therefore,

$$|I_{RY}| = |I_{BR}| = |I_{YB}| = I_{ph}$$

$$\Rightarrow I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2\cos 60^\circ} = \sqrt{3}I_{ph}$$

Similarly,

$$I_Y = \sqrt{3}I_{ph} \text{ and } I_B = \sqrt{3}I_{ph}$$

$$\Rightarrow \text{Line Current} = \sqrt{3} \times \text{Phase Current}$$

Since, neutral does not exist in a delta connected system, thus the phase voltage and line voltage are same. Refer the circuit diagram,

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$\Rightarrow V_L = V_{ph}$$

3. (a) Define power factor of an a.c. circuit

(b) The alternating current flowing in a circuit is 5 Amps when the applied voltage is 250 volts. The power consumed is 1000 watts, calculate the resistance, reactance and power factor of the circuit. [Model Question]

Answer:

a) Power factor is the ratio of active power to apparent power.

$P.F. = \frac{VI \cos \phi}{VI} = \cos \phi$, in this term the ϕ is the angle between the voltage applied and current. If the circuit is resistive the value of ϕ is zero so, $\cos \phi$ is equal to 1. If the circuit is inductive the value of $\cos \phi$ is less than 1 so, the total apparent power is used as active power, the efficiency of system decreases.

$$\text{b) We know that } \cos \phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1000}{250 \times 5} = 0.8$$

$$\therefore \text{Phase angle } \phi = \cos^{-1} 0.8 \\ = 36.87^\circ$$

$$\text{Impedance } Z = \frac{V}{I} = \frac{250}{5} = 50 \Omega$$

Again,

$$\cos \phi = \frac{R}{Z} \quad \therefore R = Z \times \cos \phi = 50 \times 0.8 = 40 \Omega$$

$$Z = \sqrt{R^2 + X^2}$$

$$\therefore X = \sqrt{Z^2 - R^2} = \sqrt{50^2 - 40^2} = 30 \Omega$$

$$\therefore \text{Reactance} = 30 \Omega$$

4. (a) Define – (i) Form factor and (ii) crest factor.

(b) The r.m.s. value of a 50 Hz sinusoidal alternating current is 50 amps. Write down the equation for its instantaneous value. Also find the value of the current when $t = 0.002$ sec. [Model Question]

Answer:

a) (i) Form factor:

Form factor is defined by the ratio of R.M.S. value and Average value.

Form factor = $\frac{\text{R.M.S. value of any A.C. quantity}}{\text{Average value of any A.C. quantity}}$

for a sinusoidal A.C. quantity

$$\text{Form factor} = \frac{0.707 \times \text{peak value}}{0.637 \times \text{peak value}} = 1.11$$

The form factor of a half wave rectified A.C. = $\frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$

(ii) Crest factor:

It is defined as the ratio of the maximum value and R.M.S. value of any alternating quantity.

Crest factor = $\frac{\text{Maximum value of any A.C. quantity}}{\text{R.M.S. value of any A.C. quantity}}$

for a sinusoidal A.C. quantity

$$\begin{aligned} \text{Form factor} &= \frac{\text{Maximum Value}}{0.707 \times \text{Maximum value}} \\ &= \frac{1}{0.707} = 1.4142 \end{aligned}$$

b) The equation of instantaneous value of current

$$i = I_m \sin \omega t \text{ (with usual notation)}$$

$$\begin{aligned} \therefore i &= 50\sqrt{2} \times \sin 2\pi ft \\ &= 70.71 \sin 314t \end{aligned}$$

The value of the current when $t=0.002$ sec is

$$i = 70.71 \sin 314t$$

put, $t=0.002$ sec

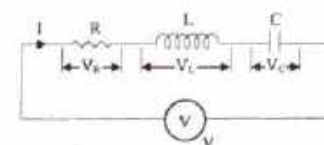
$$\begin{aligned} \therefore i &= 70.71 \sin (314 \times 0.002) \\ &= 41.54 \text{ Amp} \end{aligned}$$

5. A coil of resistance 8 ohms and inductance 0.1 Henry is connected in series with a capacitor of $75\mu\text{F}$ across a 240 volts, 50 Hz supply. Calculate –

- the impedance,
- the current and
- the angle of phase difference.

[Model Question]

Answer:



$$R = 8\Omega, L = 0.1 \text{ H}, C = 75\mu\text{F}$$

$$\therefore X_L = 2\pi fL = 31.4\Omega$$

$$\therefore X_C = \frac{1}{2\pi fL} = 46.46\Omega$$

$$\begin{aligned} \therefore \text{Impedance, } Z &= R + j(X_L - X_C) \\ &= 8 - j11.06 = 13.65 \angle 54.12^\circ (\text{leading}) \end{aligned}$$

$$\therefore \text{Phase difference} = 54.12^\circ$$

$$\therefore \text{Current } I = \frac{V}{Z} = \frac{240}{13.65} \text{ Amp} = 17.58 \text{ Amp}$$

6. (a) Define – (i) cycle, (ii) amplitude of a sinusoidal quantity.

(b) A vector is represented by $20e^{-j2\pi/3}$. Write down the – (i) rectangular form, (ii) polar form. [Model Question]

Answer:

a) Cycle is defined as a complete waveform, i.e. starting from a particular point again return back to that point.

Amplitude: Refer to Chapter at a Glance.

b) $20e^{-j2\pi/3}$ is the complex form

$$\text{Rectangular form} = 20 \times \cos \frac{\pi}{3} - j20 \times \sin \frac{\pi}{3} = 10 - j17.33$$

$$\text{Polar form} = \sqrt{10^2 + 17.33^2} \angle \tan^{-1} \frac{17.33}{10} = 20.01 \angle 60.01^\circ$$

7. A voltage represented by $230 \sin 314t$ is applied to a series circuit, the resulting current is represented by the equation $i = 10 \sin (314t + \pi/6)$. What is the frequency, resistance and reactance of the circuit. [Model Question]

Answer:

Maximum value of voltage 230 Volt & Maximum value of Current 10 Amp.

$$\text{Reactance } Z = \frac{230}{10} \Omega = 23 \Omega \quad \text{Phase difference} = \frac{\pi}{6}$$

$$\therefore \text{Resistance } R = Z \cos \frac{\pi}{6} = 23 \times 0.866 = 19.9 \Omega$$

$$\omega = 2\pi f = 314$$

$$\therefore \text{Frequency, } f = 50 \text{ Hz}$$

8. A square coil of 10cm side and with 100 turns is rotated at the uniform speed of 500 rpm about an axis at right angles to a uniform magnetic field of 0.5 Wb/m^2 . Calculate the instantaneous value of the induced emf when the plane of the coil is at 30° to the plane of the field. Find also the maximum value of the induced emf. [Model Question]

Answer:

$$\text{Length of the coil} = 4 \times 10 \text{ cm} = 40 \text{ cm}$$

We know that,

$$e = Blv \times T \sin \theta = 0.5 \times 10 \times \frac{500}{60} \times 40 \times \sin 30^\circ \text{ Volt} = 833.33 \text{ Volt}$$

Maximum value will be at $\theta = 90^\circ$

$$\therefore E = 0.5 \times 10 \times \frac{500}{60} \times 40 \times \sin 90^\circ \text{ Volt} = 1666.67 \text{ Volt}$$

9. (a) Prepare a comparative statement between resistance 'R' of a d.c. circuit and impedance 'Z' of an a.c. circuit having R-L-C connected in series having a frequency f Hz.
(b) The field coils of a generator have a resistance of 150 ohms at 16°C and after 6 hours run the resistance has increased to 162 ohms. What is the final average temperature of the coils? Resistance temperature coefficient 0.0043 per $^\circ\text{C}$ at 0°C . [Model Question]

Answer:

$$\text{a) } I = \frac{V}{R} \text{ (in case of DC)}$$

$$I = \frac{V}{Z} \text{ (in case of AC)}$$

$$\text{Where } Z = \sqrt{R^2 + X^2} \text{ (R is resistance, } X = X_L - X_C)$$

Resistance in DC circuit plays same role as in the AC. But reactance will cause in phase difference between voltage and current.

$$\text{b) } R_{16} = 150 \Omega \quad R_t = 162 \Omega$$

$$R_{16} = R_0(1 + \alpha \cdot 16) \quad \dots (1)$$

$$R_t = R_0(1 + \alpha \cdot t) \quad \dots (2)$$

Dividing (2) by (1),

$$\frac{R_t}{R_{16}} = \frac{R_0(1 + \alpha \cdot t)}{R_0(1 + \alpha \cdot 16)} \quad \therefore \frac{162}{150} = \frac{(1 + 0.0043 \cdot t)}{(1 + 0.0043 \cdot 16)}$$

Solving the equation we get, $t = 35.89^\circ$

10. (a) Define – (i) cycles, and (ii) phase difference of a sinusoidally induced e.m.f.
(b) A resistance of 10Ω is connected in series with an inductance of 0.05 H and a capacitance 'C' to a 100V , 50 Hz supply. Find at what value of 'C' the series circuit be a purely resistive one. [Model Question]

Answer:

a) i) **Cycle:** It is defined as a complete waveform, i.e. starting from a particular point again return back to that point.

ii) **Phase difference:** It is defined as the time difference between two waveform. e.g. Phase difference of 30° means the time difference between two waveforms.

$$\text{b) } X_L = 2\pi fL = 15.7 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$Z = R + j(X_L - X_C)$$

$$\text{To make the circuit pure resistive } (X_L - X_C) = 0$$

$$\therefore X_L = X_C$$

$$\text{or, } 15.7 = \frac{1}{2\pi fC}$$

$$\text{or, } C = \frac{1}{2\pi f \times 15.7} = 2.02 \times 10^{-4} \text{ F} = 202 \times 10^{-6} \text{ F} = 202 \mu\text{F}$$

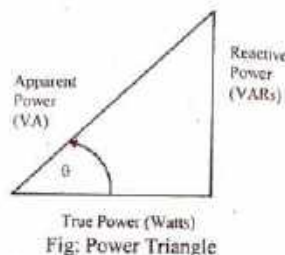
11. (a) Draw a power triangle of a series circuit have a R-L-C. This combination is connected across a V (r.m.s.) volt supply, frequency 'f' Hz. Hence state the expression for true power, apparent power and reactive power.

(b) An iron cored choke coil takes 5A when connected to 20V d.c. supply and takes 5A at 100V a.c and consume 250W . Determine – (i) power factor, and (ii) the iron loss. [Model Question]

Answer:

a) Power Triangle

The power triangle represents comparable values that can be used directly to find the efficiency level of generated power to usable power, which is expressed as the power factor (discussed later). Apparent power, reactive power, and true power can be calculated by using the DC equivalent (RMS value) of the AC voltage and current components along with the power factor.



Apparent Power

$$S = I^2 Z = IE$$

Where, S = apparent power (VA)

I = RMS current (A)

E = RMS voltage (V)

Z = impedance (Ω)

True Power

$$P = I^2 R = EI \cos \theta$$

where, P = true power (watts)

I = RMS current (A)

E = RMS voltage (V)

R = resistance (Ω)

θ = angle between E and I sine waves

Reactive Power

$$Q = I^2 X = EI \sin \theta$$

where, Q = reactive power (VAR)

I = RMS current (A)

X = net reactance (Ω)

E = RMS voltage (V)

θ = angle between E and I sine waves

$$\text{Power factor (pf)} = \frac{P}{S} = \cos \theta$$

b) A.C. Circuit $V = 100V$, $I = 5A$

$$Z = \frac{V}{I} = \frac{100}{5} = 20\Omega$$

$$i) P = VI \cos \phi$$

$$\text{or, } 250 = 100 \times 5 \times \cos \phi$$

$$\text{or, } \cos \phi = \frac{250}{500} = 0.5$$

Power factor is 0.5

ii) Total loss = loss in resistance + iron loss

$$\therefore 250 = 20 \times 5 + P_i$$

$$\text{or, } P_i = 150 \text{ W}$$

12. (a) An 8Ω resistor, 9Ω inductive reactance and unknown capacitor are connected in series across $100V$, 50 Hz supply. The current drawn by the circuit is found to be 10 Amp .

Determine the value of capacitance.

(b) Draw the impedance triangle of above example in actual scale (approx).

[Model Question]

Answer:

$$a) R^2 + (X_C - X_L)^2 = Z^2$$

$$\Rightarrow 8^2 + (X_C - 9)^2 = 10^2; \quad \Rightarrow (X_C - 9)^2 = 10^2 - 8^2 = 36; \quad \Rightarrow X_C - 9 = 6$$

$$\Rightarrow X_C = 15$$

$$X_C = 15$$

$$\Rightarrow \frac{1}{2\pi fC} = 15; \quad \Rightarrow \frac{1}{2\pi \times 50C} = 15; \quad \Rightarrow \frac{1}{2\pi \times 50C} = 15$$

$$\Rightarrow \frac{1}{100\pi C} = 15; \quad \Rightarrow \frac{1}{314C} = 15; \quad \Rightarrow C = \frac{1}{314 \times 15} = 2.12 \times 10^{-4} \text{ F}$$

b) **Impedance Triangle:** The resistance is a DC element; the reactance, as suggested, can be of inductive or capacitive thus displaced 90° from the resistance; the impedance Z , as is the vector sum of these two.

Based on this justification, a triangle formed by the three sides namely – the resistance, the reactance and the impedance – is called the impedance triangle.

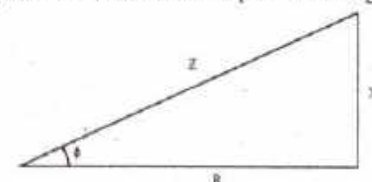
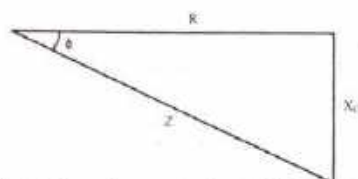
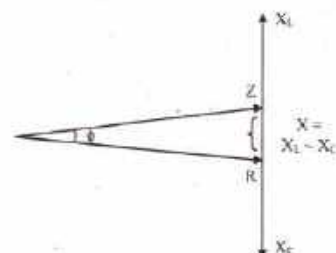


Fig: (a) Impedance triangle with R , X_L and Z


 Fig: (b) Impedance triangle with R , X_C and Z

 Fig: (c) Impedance triangle with R , X_L , X_C and Z

The impedance triangle with R , X_L , and Z is in Fig: (a).

The impedance triangle with R , X_C , and Z is in Fig: (b).

The impedance triangle with X_L and X_C will have the net X on the inductive side or capacitive side, depending on which is greater. Fig: (c) depicts the impedance triangle with X_L and X_C where $X_L > X_C$.

Note: In all these figures that the angle ϕ is the phase angle. From these three figures, the p.f., $\cos \phi$ can be found out to be, the ratio between R and Z , that is

$$\cos \phi = R/Z$$

13. (a) Write down the differences between a.c. and d.c.

(b) For a sinusoidal quantity, define (any three) – (i) instantaneous value, (ii) cycle, (iii) R.M.S. value, (iv) Peak factor. [Model Question]

Answer:

a) In direct current (DC), the electric charge (current) only flows in one direction. Electric charge in alternating current (AC), on the other hand, changes direction periodically. The voltage in AC circuits also periodically reverses because the current changes direction.

b) Refer to Chapter at a Glance.

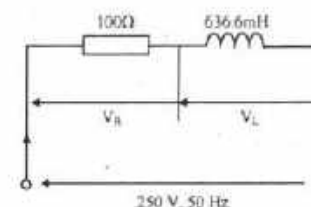
14. (a) What is apparent power? Define power factor.

(b) A coil of inductance 636.6 mH and negligible resistance is connected in series with a 100Ω resistor to a 250V, 50Hz supply. Calculate (i) the current in the circuit, (ii) the p.d. across each component, and the (iii) circuit phase angle. [Model Question]

Answer:

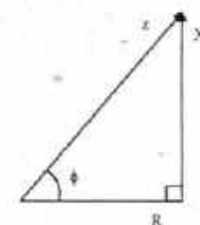
a) Refer to Chapter at a Glance.

b) The circuit is shown in the diagram below.



(a) Inductive reactance of coil, $X_L = 2\pi fL = 2\pi(50)(636.6 \times 10^{-3}) = 200\Omega$

(b) Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{100^2 + 200^2} = 223.6\Omega$ (from impedance triangle)



(c) Current, $I = \frac{V}{Z} = \frac{250}{223.6} = 1.118\text{ A}$

(d) Voltage across resistance, $V_R = IR = 1.118 \times 100 = 111.8\text{ V}$

Voltage across inductance, $V_L = IX_L = 1.118 \times 200 = 223.6\text{ V}$

(e) From impedance triangle, $\tan \phi = \frac{X_L}{R}$

from which, circuit phase angle, $\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \left(\frac{200}{100} \right) = 63.43^\circ$ lagging

Chapter 4: Transformer and Machines

Chapter at a glance:

4.1 Basic Working Principles

Principle of D.C. Machines

D.C. Machines are based on Faraday's Laws of electromagnetic induction. According to this law 'the voltage induced in a coil is proportional to rate of change of flux linking with the coil' i.e.

$$e = -N \cdot \frac{d\phi}{dt} \quad \dots (1)$$

where N = number of turns in a coil, and

$\frac{d\phi}{dt}$ = rate of change of flux linking with the coil.

The negative sign indicates that the current produced by it opposes the change in flux linkages.

Case-I: In case of a **generator**, when a conducting coils is rotated in a magnetic field, the flux linked with the coil changes due to motion of the coil, and due to this change in flux linkage, emf is induced in the coil (as per Faraday's laws). This induced emf is given by:

$$e = Blv \text{ volts} \quad \dots (2)$$

where B = flux density of the magnetic field (in wb/m^2),

ℓ = length of conductor in m,

and v = velocity of the conductor in m/s.

The induced emf is used i) to overcome the voltage drop in the resistance of the windings and ii) to supply the load. The electromagnetic torque produced in the moving current carrying conductor opposes the mechanical torque rotating the coil.

Case-II: In case of a **motor**, when a current carrying conductor is placed in a magnetic field, a mechanical force is exerted on the conductor (called Lorentz's force), which is given by:

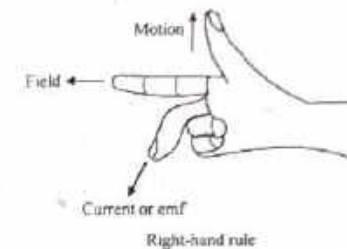
$$F = BIl \sin \theta \quad \dots (3)$$

where θ = angle between length (ℓ) and magnetic field (B).

In case of a generator the direction of induced emf or current is determined by Fleming's right hand rule. The thumb fore finger and middle finger of the right hand

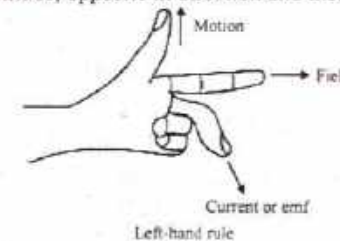
Transformer and Machines

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are stretched mutually perpendicular to each other, then if the thumb represents direction of motion of the conductor and fore-finger represents the direction of field, then middle finger will indicate the direction of induced emf.

In case of a motor, the rectangular coil of length ' ℓ ', width ' r ', carrying a current ' i ', the two arms of the conductor, according to Fleming's left hand rule are acted upon by two forces, equal in magnitude, opposite in direction and their lines of action are different.



and their lines of actions are different. Thus the two forces form a couple and the torque (twisting effect) is given by:-

$$T = F \cdot r = BIl \sin \theta \cdot r = BA l \sin \theta \quad \dots (4)$$

where A = area of coil = $\ell \times r$.

4.2 Working Principle of 3 Phase Induction Motor

The stator winding of a 3-phase induction motor is connected to 3-phase balanced supply. This 3-phase supply causes 3-phase current to circulate in the stator winding which produces 3-phase fluxes ϕ_a, ϕ_b, ϕ_c . Hence a rotating magnetic field is produced. This rotating field flux cuts the rotor conductors.

By Faraday's law of electromagnetic induction, a voltage is induced in the rotor conductors. Since the rotor circuit is closed, current flows through the circuit.

By Lenz Law, this current flowing in conductor produces a force and hence the conductor moves in the same direction as that of the rotating magnetic field so that it opposes the cause which produces it (cause is the rotating magnetic field) Thus the rotor rotates in

same direction as the rotating field but with lesser speed than the synchronous speed of magnetic field.

From the above said principle, it is also clear that a 3-phase induction motor is a self-starting motor. Since the whole operation is based on the induction principle, this motor is called as induction motor. Induction motor can be called as rotating transformers since it works on the mutual induction principle as that of the transformer. The stator is often referred to as the primary and rotor as short-circuited secondary.

4.3 Working Principle of Transformer

A transformer is a power converter that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils. A varying current in the primary winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force (EMF), or "voltage", in the secondary winding. This effect is called inductive coupling.

If a load is connected to the secondary winding, current will flow in this winding, and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding (V_s) is in proportion to the primary voltage (V_p) and is given by the ratio of the number of turns in the secondary (N_s) to the number of turns in the primary (N_p) as follows:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{----- (5)}$$

By appropriate selection of the ratio of turns, a transformer thus enables an alternating current (AC) voltage to be "stepped up" by making N_s greater than N_p , or "stepped down" by making N_s less than N_p . The windings are coils wound around a ferromagnetic core, air-core transformers being a notable exception.

4.4 Field of applications

Application of DC Motor

- ✓ DC series motors are used in the traction work like in train, tram, crane etc.
- ✓ DC shunt motors are used in constant speed application like in lathe machine.
- ✓ DC compound motors are used when high torque and constant speed is required.

Application of Transformer

- ✓ Transformers are used in AC Circuit to change the voltage and current level keeping the power same.
- ✓ There is no electrical connection between the primary and secondary, so it help to isolate the electrical circuit from the measuring or control circuit.

Application of induction Motor

- ✓ 1 phase induction motors are used in fan, mixer grinder, blower, hair dryer.
- ✓ 3 phase induction motors are used in industries, in lift, pumps.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. Voltage per turn in both primary and secondary winding of a transformer is [WBSCTE 2022]

(a) high in high voltage winding
(b) low in low voltage winding

(c) same
(d) cannot determine

Answer: (c)

2. The commutator of a d.c. generator— [Model Question]

(a) reverses the direction of the current in the armature,

(b) changes a.c. to d.c. within the armature,

(c) keeps the current in one direction in the load circuit,

(d) acts only as a sliding electrical contact.

Answer: (c)

3. With the star/delta starter the voltage across an induction motor winding is— [Model Question]

(a) 50%
(b) 57.7%
(c) 67.7%
(d) 75%

Answer: (b)

4. If the back e.m.f of a d.c. motor suddenly vanishes, then— [Model Question]

(a) the motor will run faster than the rated speed,

(b) the motor will burn,

(c) the motor will stop,

(d) the motor will run slower than the rated speed.

Answer: (b)

5. A lead acid storage battery rated at 120 amp-hours discharge rate will deliver a current of 5 amps for approximately— [Model Question]

(a) 24 hours,
(b) 12 hours
(c) 5 hours
(d) 120 hours

Answer: (a)

6. Which of the following d.c. generators will be in a position to build up without any residual magnetism in the poles? [Model Question]

(a) series generator (b) shunt generator
(c) compound generator (d) none of these

Answer: (a)

7. In a lead-acid battery the level of electrolyte should be— [Model Question]

(a) below the top level of the plates
(b) up to the top level of the plates
(c) above the top level of the plates
(d) container to be fully filled to exclude any air inside

Answer: (c)

8. Working Sp. Gr. Of the electrolyte of a lead acid storage battery is — [Model Question]

(a) 1.200 (b) 1.190 (c) 1.840

Answer: (c)

9. Output of a transformer is expressed in — [Model Question]

(a) KW (b) KVA (c) KVAR

Answer: (b)

10. What type of a.c. generator is used for electric welding? [Model Question]

(a) series (b) shunt (c) over compound (d) flat compound
(e) under compound (f) differential compound generator

Answer: (b)

11. How many terminals do you expect to find on the terminal box of a squirrel cage induction motor to be started by star-delta starter? [Model Question]

(a) six (b) four (c) five (d) eight

Answer: (a)

12. The specific gravity of the electrolyte of a lead-acid cell is — [Model Question]

(a) 2.1 (b) 2.0 (c) 1.50 (d) 1.25

Answer: (d)

13. Motor used in lathe machine is — [Model Question]

(a) d.c. series motor, (b) a.c. commutator motor,
(c) squirrel induction motor, (d) squirrel cage induction motor.

Answer: (d)

14. Electrolyte used in Nickel-Iron-Cadmium cell is — [Model Question]
(a) NaOH (b) KOH (c) Na_2SO_4 (d) K_2SO_4

Answer: (a)

15. When a single turn coil rotates in a uniform magnetic field the emf generated follows— [Model Question]

(a) cosine law (b) straight line law (c) sine law (d) tangent law

Answer: (b)

16. EMF generated in a rotating coil will be a perfect sine wave if the — [Model Question]

(a) magnetic flux in air gap follow sine law,
(b) flux distribution in the air gap is perfectly uniform,
(c) air gap is uniform,
(d) rotating coil size is uniform all along.

Answer: (a)

17. A single phase induction motor the rotor rotates because stator when connected to a.c. supply — [Model Question]

(a) creates rotating field
(b) creates varying magnetic field
(c) rotating field is created by split phasing the stator supply
(d) none of them

Answer: (c)

B. State the following statement as True or False

18. Direction of rotation of the motor armature or rotor can be determined by applying Fleming's Left Hand rule. [WBSCTE 2022]

Answer: True

19. Transformer core is constructed with insulated laminated silicon steel to reduce eddy current loss. [Model Question]

Answer: True

C. Fill in the Blanks:

20. A 10 KVA, 1100/400 V, 50 Hz single phase transformer has 100 turns in the secondary winding The number of turns in primary winding is — [WBSCTE 2022]

Answer: 275

Long Answer Type Questions

1. (a) Define a Transformer.

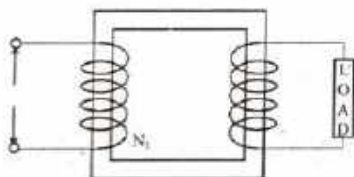
[WBSCTE 2022]

Answer:

A transformer is a power converter that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils. A varying current in the primary winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force (EMF), or "voltage", in the secondary winding. This effect is called inductive coupling.

If a load is connected to the secondary winding, current will flow in this winding, and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding (V_s) is in proportion to the primary voltage (V_p) and is given by the ratio of the number of turns in the secondary (N_s) to the number of turns in the primary (N_p) as follows:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$



By appropriate selection of the ratio of turns, a transformer thus enables an alternating current (AC) voltage to be "stepped up" by making N_s greater than N_p , or "stepped down" by making N_s less than N_p . The windings are coils wound around a ferromagnetic core, air-core transformers being a notable exception.

1. (b) A single phase 50 Hz transformer has 80 turns on the primary winding & 400 turns on the secondary winding. If the primary winding is connected to a 240 volt, 50 Hz supply, determine the e.m.f induced in the secondary winding.

[WBSCTE 2022]

Answer:

Here, V_1 = Voltage supply to primary coil = 240 V

V_2 = Voltage rating of secondary coil (Let)

N_1 = Number of turns of primary coil = 80

N_2 = Number of turns of secondary coil = 400

$\therefore$ e.m.f induced in the secondary winding,

$$\text{e.m.f or, } V_2 = V_1 \times \frac{N_2}{N_1} = 240 \times \frac{400}{80} = 1200 \text{ Volts}$$

The e.m.f. induced in the secondary winding is 1200 Volts.

2. Distinguish between primary and secondary cell.

[Model Question]

Answer:

Primary Cell: In primary cell the chemical reaction is not reversible. In primary cell the chemical reaction helps to produce an electromotive force which delivers the current in the load circuit. After working with that cell the cell cannot be used further. So the efficiency of that cell is less. The primary cells are mainly of low cost, low efficiency and low amp-hr capacity. The examples of that kind of cells are Weston Cell, Vpctic Cell etc.

Secondary Cell: In secondary cells the chemical reaction is reversible by choosing the proper chemicals. Chemical energy converts in electrical energy to give power to the load circuit as well as electrical energy can be converted to chemical energy to recharge the cell. When the stored power in the cell gets lowed we can charge the cell to restore the energy. The secondary cells are mainly of high cost, high efficiency and high Amp-hr capacity. The maintenance cost of that kind of cell is also high. The examples of that kind of cells are lead-acid cell and alkali cell.

3. What is the difference between e.m.f. and terminal voltage of a cell.

[Model Question]

Answer:

EMF is the electromotive force. It is the source of potential difference that creates current.

The terminal voltage is the voltage at the terminals of an emf source. It is less than the emf because of internal resistance (r).

If you measure the voltage across the terminals with no current flowing, the voltage will equal the emf. Do the same measurement when connected to a circuit (current flowing) and the terminal voltage will be less.

This is because the internal resistance will have a voltage drop ($V_r = I \cdot r$) due the current flowing, which would take away from the emf voltage ($V_T = E - I \cdot r$).

4. (a) Why a starter is necessary in an electric motor?

(b) What type of electric motors do you recommended for— (i) EOT crane, (ii) Ceiling fan

[Model Question]

Answer:

(a) In case DC motors, at the time of starting there is no back emf. (i.e. $E_b = 0$). Armature current, $[I_a = (V - E_b) / R_a]$. If $E_b = 0$, the value of I_a will be very high, which will damage the armature winding and hamper the line voltage also.

In DC motor starter a set of resistances are connected in series to limit the starting current, at the time of start the full resistances are in the circuit. When the motor speeds up the resistances are cut step by step.

For DC shunt motor a three point starter, four point starter is used.

(b) In EOT crane slip ring induction motor is used. In single phase ceiling fan single phase induction motor is used.

5. Explain the significance of back emf of D.C. motor. [Model Question]

Answer:

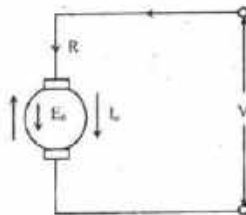
When the armature of the DC motor rotates in a magnetic field, conductor cuts the magnetic flux. For this reason an e.m.f. is induced in the armature conductors, which oppose the direction of the armature current according to the Lenz's law. This e.m.f. is known as back e.m.f. It is denoted by E_b .

$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$ When the machine is a reasonable amount of back e.m.f. is generated.

Armature current $I_a = \frac{V - E_b}{R}$, when machine is running at

rated speed the generated back e.m.f. oppose the supply voltage which help to limit the armature current.

Another importance of back e.m.f. is when the machine get loaded the speed of the machine decrease. As $E_b \propto N$ the back e.m.f. will also decrease then the armature current increases. If the armature current increases the torque of the machine increases (as $T \propto I_a$ [shunt motor] & $T \propto I_a^2$ [series motor]) which will help the motor to drive the increased load.



6. State the names of active materials used in a lead acid cell. [Model Question]

Answer:

The active materials used in lead acid cell are:

- ✓ Positive Plate: Lead per-oxide (PbO_2)
- ✓ Negative Plate: Lead (Pb)

✓ Electrolyte: Diluted Sulphuric Acid (H_2SO_4)

7. (a) How can you distinguish between the positive pole and negative pole negative pole storage battery by visual inspection? [Model Question]

Answer:

The positive terminal is slightly larger and in new storage cells the positive terminals are marked red, and negative terminal is marked black.

7. (b) What are the indications of a fully charged battery? [Model Question]

Answer:

Indication of Full Charged Battery:

- ✓ Colour of Plates: The colour of '+'Ve cables become dark chocolate and '-'Ve become grey when battery gets full charged.
- ✓ Specific Gravity of the Electrolyte: When fully charged the specific gravity is constant and about 1.23
- ✓ Voltage per cell will be about 2.6 volt when the battery gets fully charged.

8. (a) Where are squirrel cage induction motor used?

(b) A single phase transformer has 150 turns on the primary and 200 turns on the secondary. The load on the secondary is 6KW at 230 volts and 0.9 power factor. Determine the primary current and primary voltage of the transformer.

[Model Question]

Answer:

(a) Squirrel cage induction motor is used in those drives where moderate starting torque and more or less constant speed is required.

(b) Given,

Primary turns (N_1) = 150

Secondary turns (N_2) = 200

Secondary voltage (V_2) = 230 Volt

Load on secondary side = 6 KW at 0.9 p.f.

∴ Primary current (I_1) = $\frac{6 \times 1000}{230 \times 0.9} = 28.98$ Amp

We know,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \text{ or } \frac{V_1}{230} = \frac{150}{200} \therefore V_1 = 172.5 \text{ Volt}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \text{ or } \frac{I_1}{28.98} = \frac{200}{150} \therefore I_1 = 38.64 \text{ Amp}$$

$\therefore$ Primary voltage = 172.5 Volt

$\therefore$ Primary current = 38.64 Volt

9. (a) In a 500 KVA, 11000/400V single phase transformer, how much voltage shall have to be applied in primary so that full load current will flow in the secondary. Find out the full load secondary current.

(b) State the main parts of a d.c. motor. In which respect it differs with an a.c. motor? [Model Question]

Answer:

(a) Full load secondary current

$$I_2 = \frac{P}{V_2} = \frac{500 \times 1000}{400} \text{ Amp} = 1250 \text{ Amp}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$\therefore \frac{1250}{I_1} = \frac{11000}{400}$$

$$\text{or, } I_1 = 45.45 \text{ Amp}$$

$$\therefore \text{Primary voltage } V_1 = \frac{P}{I_1} = \frac{500 \times 1000}{45.45} \text{ Volt} = 11000 \text{ Volt}$$

(b) The main parts of DC motor are-

1. Armature winding and core
2. Field winding
3. Commutator
4. Brush

AC Induction motor is singly feed, only stator (field) is excited. DC motor is doubly energized, we have to supply in both the field and armature.

10. What types of electric motors are used for driving – (i) lathes, (ii) lifts? Give reasons. [Model Question]

Answer:

Mainly DC shunt motors and 3Ph induction motors are used in lathes because these motors are constant speed motors and having medium starting torque.

In lifts DC series motors are used because in lifts high starting torque is required so DC series motor is most suitable for this purpose.

11. (a) Calculate the full load primary and secondary currents of a 25 KVA, 1100/40V single phase transformer.

(b) States the name of four main parts of DC motors. [Model Question]

Answer:

(a) Rating of the transformer is 25 KVA = 25000 VA.

We know that $P = E_1 I_1 = E_2 I_2$

$$E_1 = 1100 \text{ volt}$$

$$\therefore I_1 = \frac{25 \times 10^3}{1100} \text{ Amp} = 22.73 \text{ Amp}$$

$$E_2 = 40 \text{ volt}$$

$$\therefore I_2 = \frac{25 \times 10^3}{40} \text{ Amp} = 625 \text{ Amp}$$

(b) The four main parts of DC motor are-

1. Armature winding and core
2. Field winding
3. Commutator
4. Brush

12. State the difference between the statically induced emf and dynamically induced emf. Give examples of each. [Model Question]

Answer:

Statically Induced Emf:

The emf induced in a coil due to change of flux linked with it (change of flux is by the increase or decrease in current) is called statically induced emf. Transformer is an example of statically induced emf. Here the windings are stationary; magnetic field is moving around the conductor and produces the emf.

Dynamically Induced Emf:

The emf induced in a coil due to relative motion of the conductor and the magnetic field is called dynamically induced emf.

Example: dc generator works on the principle of dynamically induced emf in the conductors which are housed in a revolving armature lying within magnetic field.

Chapter 5: Overview of Basic Semiconductor Devices

Chapter at a glance:

5.1 Semiconductor

The magic word semiconductor is composed of two words-**Semi** and **Conductor**. Semi means "not complete" while conductor means something, which can conduct electricity. Semiconductors are materials that conduct electric current, can be easily regulated, and can act as both insulators and conductors.

5.2 P-type and N-type semiconductor

Semiconductors are mainly classified into two categories: Intrinsic and Extrinsic. An intrinsic semiconductor material is chemically very pure and possesses poor conductivity. It has equal numbers of negative carriers (electrons) and positive carriers (holes). Where as an extrinsic semiconductor is an improved intrinsic semiconductor with a small amount of impurities added by a process, known as doping.

N-type doping:

The purpose of n-type doping is to produce an abundance of carrier electrons in the material. To help understand how n-type doping is accomplished, consider the case of silicon (Si). Si atoms have four valence electrons, each of which is covalently bonded with one of four adjacent Si atoms. If an atom with five valence electrons, such as (eg. phosphorus (P), arsenic (As), or antimony (Sb)), is incorporated into the crystal lattice in place of a Si atom, then that atom will have four covalent bonds and one unbonded electron. This extra electron is only weakly bound to the atom and can easily be excited into the conduction band. At normal temperatures, virtually all such electrons are excited into the conduction band. Since excitation of these electrons does not result in the formation of a hole, the number of electrons in such a material far exceeds the number of holes. In this case the electrons are the majority carriers and the holes are the minority carriers. Because the five-electron atoms have an extra electron to "donate", they are called donor atoms.

P-type doping:

The purpose of p-type doping is to create an abundance of holes. In the case of silicon a trivalent atom, such as boron, is substituted into the crystal lattice. The result is that an electron is missing from one of the four possible covalent bonds. Thus the atom can accept an electron from the valence band to complete the fourth bond, resulting in the formation of a hole. Such dopants are called acceptors. When a sufficiently large number of acceptors are added, the holes greatly outnumber the excited electrons. Thus, the holes are the majority carriers, while electrons are the minority carriers in p-type materials.

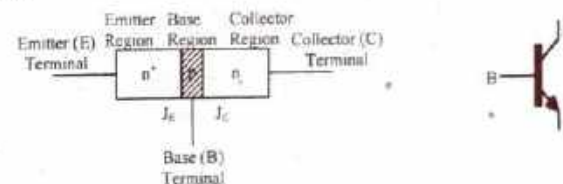
5.3 PN Junction

The p-n junction is a homo junction between a p-type and an n-type semiconductor. If we join (or fuse) n-type and p-type semiconductor materials together they behave in a very different way merging together and producing what is generally known as a "PN Junction".

5.4 Construction of PNP and NPN Transistors

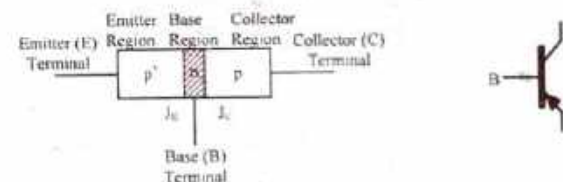
a) npn transistor

In an npn transistor, p-type semiconductor is sandwiched between two n-type semiconductors.



b) pnp transistor

In a pnp transistor, n-type semiconductor is sandwiched between two p-type semiconductors.

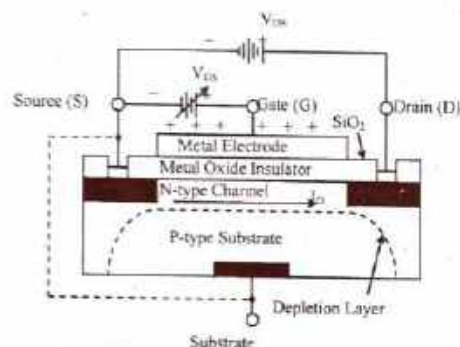


5.5 FET

A field-effect transistor (FET) is a type of transistor commonly used for weak-signal amplification (for example, for amplifying wireless signals). For FET, an electric field is established by the charges present that will control the conduction path of the output circuit without the need for direct contact between the controlling and controlled quantities. There are two types of FET. First one is the junction field effect transistor or JFET, which is a three terminal device. The second type is insulated gate field effect transistor (IGFET), more commonly known as metal oxide semiconductor transistor or MOSFET. The MOSFET is a four terminal device though it is used as three terminal devices in most applications.

5.6 MOSFET

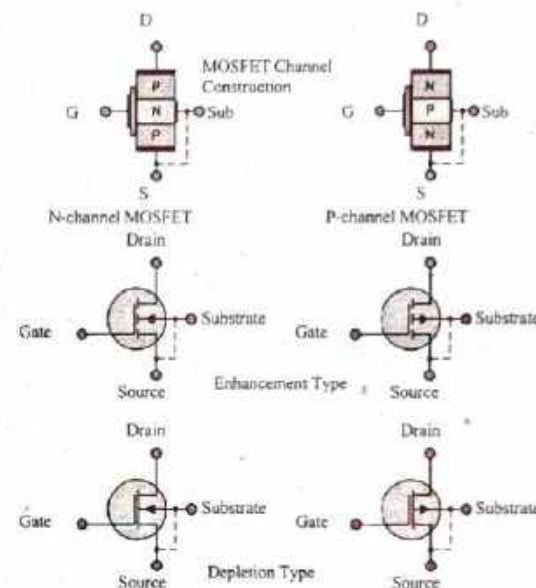
The MOSFET is a voltage controlled field effect transistor that differs from a JFET in that it has a "Metal Oxide" Gate electrode which is electrically insulated from the main semiconductor N-channel or P-channel by a very thin layer of insulating material usually silicon dioxide, commonly known as glass. This ultra thin insulated metal gate electrode can be thought of as one plate of a capacitor.



MOSFETs are available in two basic forms:

1. Depletion Type - the transistor requires the Gate-Source voltage, (V_{GS}) to switch the device "OFF". The depletion mode MOSFET is equivalent to a "Normally Closed" switch.
2. Enhancement Type - the transistor requires a Gate-Source voltage, (V_{GS}) to switch the device "ON". The enhancement mode MOSFET is equivalent to a "Normally Open" switch.

The symbols and basic construction for both configurations of MOSFETs are shown below.



Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. A diode is a/an [WBSCTE 2022]
 (a) bidirectional device
 (b) unidirectional device
 (c) both (a) and (b)
 (d) none of the above

Answer: (d)

2. Free electrons in p type material [WBSCTE 2022]
 (a) are majority carrier
 (b) are minority carriers
 (c) both (a) and (b)
 (d) none of the above

Answer: (b)

3. At absolute temperature an intrinsic semiconductor has [WBSCTE 2022]
 (a) few free electrons (b) few holes
 (c) many holes and free electrons (d) no holes and free electrons

Answer: (d)

4. Name of a breakdown in a lightly doped PN junction diode under reverse bias is – [Model Question]
 (a) Avalanche (b) Zener (c) Thermal

Answer: (a)

5. Current flowing through reverse bias PN junction diode can be measured by – [Model Question]
 (a) Ammeter (b) Milli-ammeter (c) Micro-ammeter

Answer: (c)

6. Free electrons in a p-type material – [Model Question]
 (a) are majority carriers (b) take no part in conduction
 (c) are minority carriers (d) exist in the same numbers as holes

Answer: (c)

7. Diffusion current in semiconductor diode flows due to – [Model Question]
 (a) potential gradient (b) applied voltage
 (c) concentration gradient (d) junction breakdown

Answer: (b)

8. In 1N 4007, the 'N' means – [Model Question]
 (a) number (b) 1V (c) junction (d) none

Answer: (c)

9. The injected charge on either side of p-n junction under forward bias forms a – [Model Question]
 (a) transition capacitance (b) storage capacitance
 (c) linear capacitance (d) none of the above

Answer: (c)

10. The reverse saturation current in a p-n junction diode increases with temperature because – [Model Question]
 (a) the minority carrier concentration increases
 (b) the majority carrier concentration increases
 (c) the barrier height decreases
 (d) all of the above

Answer: (a)

11. The cut in voltage for Si p-n junction diodes is about – [Model Question]
 (a) 0 V (b) 0.2 V (c) 1.3 V (d) 0.6 V

Answer: (d)

12. The Zener breakdown potential increases with the – [Model Question]
 (a) increase of doping percentage (b) decrease of doping percentage
 (c) increase of the width of the p-side (d) increase of the width of the n-side

Answer: (a)

13. In a junction diode – [Model Question]
 (a) the depletion capacitance increases with an increase of the reverse bias
 (b) the depletion capacitance decreases with an increase of the reverse bias
 (c) the diffusion capacitance increases with an increase of the forward bias
 (d) none

Answer: (b)

14. The reverse current in a diode is of the order of [Model Question]
 (a) kA (b) mA (c) μ A (d) A

Answer: (c)

15. If the doping level of a crystal diode is increased, the breakdown voltage [Model Question]
 (a) remains the same (b) is increased
 (c) is decreased (d) none of the above

Answer: (c)

16. The capacitance of a reverse biased PN junction [Model Question]
 (a) increases as reverse bias is increased
 (b) decreases as reverse bias is increased
 (c) increases as reverse bias is decreased
 (d) is insignificantly low

Answer: (b)

17. PN junction failure below 5V is caused primarily by [Model Question]
 (a) Avalanche breakdown
 (b) Zener breakdown
 (c) both (a) and (b)
 (d) none of the above

Answer: (b)

18. A MOSFET is essentially a – [Model Question]
 (a) current operated device (b) voltage operated device
 (c) power operated device (d) low input impedance device

Answer: (b)

19. The input impedance of a MOSFET is of the order of – [Model Question]
 (a) ohms (b) a few hundred ohms
 (c) kilo-ohms (d) several mega-ohms

Answer: (d)

20. In a CMOS configuration, there are – [Model Question]
 (a) two CMOS (b) three CMOS (c) four CMOS (d) no MOS

Answer: (a)

21. The oxide used in MOSFET is – [Model Question]
 (a) Silicon-dioxide (b) Aluminium oxide
 (c) Chromium oxide (d) None of these

Answer: (a)

22. Which type of metal is used in MOSFET – [Model Question]
 (a) Si (b) Al (c) Fe (d) Sb

Answer: (b)

23. Which of the following is the fastest switching device – [Model Question]
 (a) JFET (b) BJT (c) MOSFET (d) Triode

Answer: (c)

24. When drain voltage equals the pinch-off-voltage, then drain current _____ with the increase in drain voltage [Model Question]
 (a) decreases (b) increases
 (c) remains constant (d) none of the above

Answer: (c)

B. Fill in the blanks in the following statements:

25. Energy gap is absent (present/absent) in conductors. [Model Question]
 26. The power gain of a FET is lower than that of BJT. [Model Question]
 27. A MOSFET is also sometimes called as IGFET. [Model Question]

28. The input impedance of MOSFET is higher than that of JFET. [Model Question]

29. In CMOS, C stands for Complementary MOSFET. [Model Question]

30. For MOS, the metal layer is generally made up of aluminium. [Model Question]

C. State whether the following statements are True or False:

31. The energy gap for semiconductor is about 1 eV. [Model Question]

Answer: True

32. The normal current obtained with intrinsic semiconductor is about 1 MA. [Model Question]

Answer: False

33. Semiconductor resistivity varies inversely with temperature. [Model Question]

Answer: True

34. Band theory is employed to distinguish between insulators and conductors. [Model Question]

Answer: True

35. Conductivity of extrinsic semiconductor is less than intrinsic one. [Model Question]

Answer: False

36. Junction capacitance develops due to accumulation of majority carriers. [Model Question]

Answer: False

37. Holes are real fundamental particles. [Model Question]

Answer: False

38. Depletion region increases in volume with reverse bias. [Model Question]

Answer: True

39. The doping concentration varies smoothly over long distance across the junction. [Model Question]

Answer: True

Long Answer Type Questions

1. (a) Differentiate between Intrinsic & Extrinsic semiconductor.
(b) Define the term Doping. Give examples of material for creation of p & n – type material. [WBSCTE 2022]

Answer:

- a) The major differences between an intrinsic semiconductor and an extrinsic semiconductor –

Parameter	Intrinsic Semiconductor	Extrinsic Semiconductor
Definition	The semiconductor in its pure form is known as intrinsic semiconductor.	When a chemical impurity is added to an intrinsic semiconductor, then the resulting semiconductor is known as extrinsic semiconductor.
Types	There is no classification of intrinsic semiconductor.	Based on the impurity added, the extrinsic semiconductors are of two types viz. P-type semiconductor and N-type semiconductor.
Doping	In case of intrinsic semiconductor, there is no doping or addition of impurity.	In an extrinsic semiconductor, the doping is performed, i.e. a small amount of impurity is added in the pure semiconductor.
Charge carrier density	In an intrinsic semiconductor, the number of electrons is equal to number of holes.	In case of extrinsic semiconductors, the number of holes and electrons are not equal. In a P-type semiconductor, the holes are more than electrons while in an N-type semiconductor, the electrons are more than holes.
Conductivity	The electrical conductivity of intrinsic semiconductors is low.	The extrinsic semiconductors have high electrical conductivity.
Dependency of conductivity	The conductivity of intrinsic semiconductor is dependent on temperature only.	The conductivity of extrinsic semiconductor is dependent on temperature as well as amount of impurity added.
Conductivity at 0 Kelvin	The intrinsic semiconductor does not conduct at 0 Kelvin temperature.	The extrinsic semiconductor conducts at 0 Kelvin temperature.
Cause of charge carrier production	In the intrinsic semiconductors, the charge carriers are produced only	In case of extrinsic semiconductors, the charge carriers are produced due to thermal agitation as well as chemical impurities added to the

Parameter	Intrinsic Semiconductor	Extrinsic Semiconductor
	due to thermal agitation.	semiconductor.

b) 1st Part:

Doping is the process of adding impurities to intrinsic semiconductors to alter their properties. Trivalent and Pentavalent elements are used to dope silicon and germanium. When a intrinsic semiconductor is doped with trivalent impurity it becomes a p-type semiconductor.

2nd Part:

Silicon (Si) and germanium (Ge) are the materials for creation of p & n type transistor.

2. (a) Why is transistor termed as current controlled device?

- (b) Explain with diagram the principle of operation of an NPN transistor.

[WBSCTE 2022]

Answer:

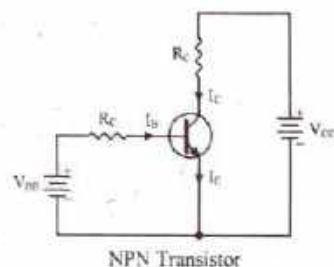
a) A transistor is a semiconductor used to amplify electronic signals or switch electronic signals. The bipolar junction transistor is one of the most commonly used transistors. A transistor has three parts: emitter, base, and collector. A transistor contains two semiconductor types namely n-type and p-type. Together they either form an NPN type transistor or a PNP type transistor. In a bipolar junction transistor, the collector current is due to the flow of electrons or holes that comes from the highly doped emitter part of the transistor, that passes the base part where these carriers are lightly doped, that then diffuses to the collector part of the transistor. The ratio of the output current to the input current is defined as the current amplification factor. It is given by the ratio of the collector current to the base current.

$$\beta = \frac{I_c}{I_b}$$

The output current is dependent on the input currents, or the collector current is controlled by the base current. So, a bipolar junction transistor is a current controlled device. The output characteristics of a bipolar junction transistor are dependent on the base current.

b) Circuit Diagram of NPN Transistor:

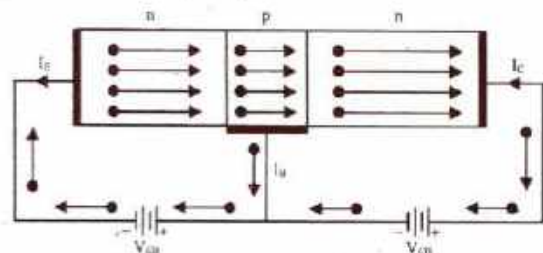
The circuit diagram of the NPN transistor is shown in the figure below. The collector and the base circuit is connected in reverse biased while the emitter and base circuit is connected in forward biased. The collector is always connected to the positive supply, and the base is in negative supply for controlling the ON/OFF states of the transistor.



NPN Transistor

Working of NPN Transistor:

The forward bias is applied across the emitter-base junction, and the reverse bias is applied across the collector-base junction. The forward biased voltage V_{BE} is small as compared to the reverse bias voltage V_{CB} .



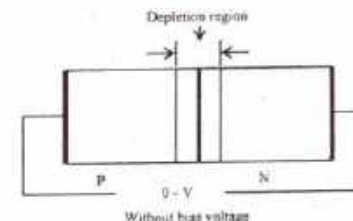
The emitter of the NPN transistor is heavily doped. When the forward bias is applied across the emitter, the majority charge carriers move towards the base. This causes the emitter current I_E . The electrons enter into the P-type material and combine with the holes.

The base of the NPN transistor is lightly doped. Due to which only a few electrons are combined and remaining constitutes the base current I_B . This base current enters into the collector region. The reverse bias potential of the collector region applies the high attractive force on the electrons reaching collector junction. Thus attract or collect the electrons at the collector.

The whole of the emitter current is entered into the base. Thus, the emitter current is the sum of the collector and the base current.

3. With the help of a neat sketch briefly explain the operation of P-N junction diode in No bias mode. [Model Question]

Answer:



A p-n junction with no external inputs represents equilibrium between carrier generation, recombination, diffusion and drift in the presence of the electric field in the depletion region. Despite the presence of the electric field, which creates an impediment to the diffusion of carriers across the electric field, some carriers still cross the junction by diffusion. Most majority carriers which enter the depletion region move back towards the region from which they originated. However, statistically some carriers cross the junction. Once a majority carrier crosses the junction, it becomes a minority carrier. It will continue to diffuse away from the junction and can travel a distance on average equal to the diffusion length before it recombines. The current caused by the diffusion of carriers across the junction is called **diffusion current**.

Minority carriers which reach the edge of the diffusion region are swept across it by the electric field in the depletion region. This current is called the **drift current**. In equilibrium the drift current is limited by the number of minority carriers which are thermally generated within a diffusion length of the junction.

In equilibrium, the net current from the device is zero. The electron drift current and the electron diffusion current exactly balance out (if they did not there would be a net buildup of electrons on either one side or the other of the device). Similarly, the hole drift current and the hole diffusion current also balance each other out.

4. (a) Why conductivity of extrinsic semiconductor may be greater than intrinsic semiconductor? [Model Question]

Answer:

Conductivity of a semiconductor is given by:

$$\sigma = N \mu e$$

Where, μ is the mobility

The conductivity of an intrinsic semiconductor is due to its own charge carriers. At room temperature, there are no free charge carriers. As the temperature increases, the covalent bond breaks, thereby creating the electron hole pair increasing the conductivity. But, for an extrinsic semiconductor, a dopant is already added, which gives charge carriers even at room temperature. The conductivity of extrinsic semiconductors is due to impurities. But as the temperature rises, thermal breakdown creates more electron hole pair. At sufficiently high temperatures, the concentration of dopant becomes negligible to the

thermally generated carriers, so the conductivity of both intrinsic and extrinsic semiconductor becomes comparable. Hence, the conductivity of extrinsic semiconductor may be greater than intrinsic conductor depending on temperature.

➤ 4. (b) Name four techniques of Fabrication of P-N junction. [Model Question]

Answer:

The techniques of Fabrication of P-N junction are:

1. Grown Junction method
2. Diffused Junction method
3. Fused or alloy Junction method
4. Epitaxial growth method

➤ 5. (a) Draw the block diagram of a P-N junction? [Model Question]

Answer:

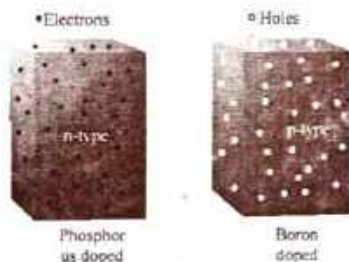


Fig: 1 Doped silicon

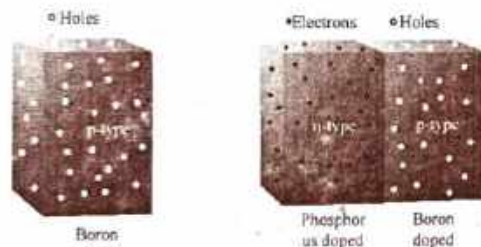


Fig: 2 n-type and p-type materials brought

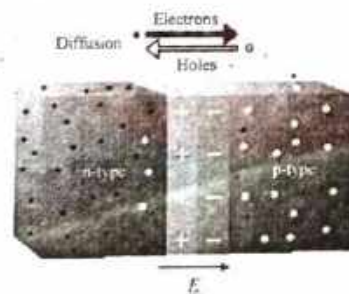


Fig: 3

➤ 5. (b) What is depletion region? [Model Question]

Answer:

The depletion region, also called depletion layer, depletion zone, junction region or the space charge region, is an insulating region within a conductive, doped semiconductor material where the mobile charge carriers have

diffused away, or have been forced away by an electric field. The only elements left in the depletion region are ionized donor or acceptor impurities. The 'depletion region' is so named because it is formed from a conducting region by removal of all free charge carriers, leaving none to carry a current.

➤ 6. (a) What do you understand by semiconductor?

(b) Discuss some important properties of semiconductors. [Model Question]

Answer:

a) The magic word semiconductor is composed of two words-Semi and Conductor. Semi means "not complete" while conductor means something, which can conduct electricity. Semiconductors are materials that conduct electric current, can be easily regulated, and can act as both insulators and conductors. These qualities have made semiconductors useful in the electronics field since its inception.

Semiconductor devices are all around us. They can be found in just about every commercial product we touch, from the family car to the pocket calculator. Today semiconductor devices are ubiquitous in a wide range of industries, including computers, communications, aerospace, manufacturing, agriculture, and healthcare. Semiconductors have made electronic devices – such as MP3 players, HDTVs / TVs, CD players, computers, and cell phones – smaller, cheaper, faster, and more reliable.

b) Properties of Semiconductor:

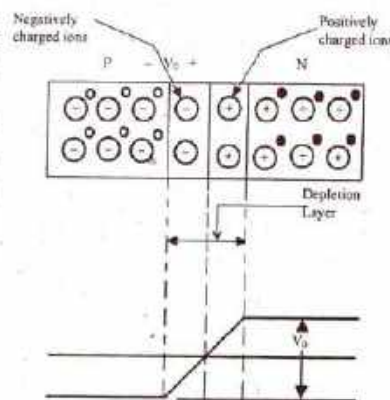
- The resistivity of a semiconductor is less than an insulator but more than a conductor.
- Semiconductors have negative temperature co-efficient of resistance i.e. the resistance of a semiconductor decreases with the increase in temperature and vice-versa. For example, germanium is actually an insulator at low temperatures but it becomes a good conductor at high temperatures.
- When a suitable metallic impurity (e.g. arsenic, gallium etc.) is added to a semiconductor, its current conducting properties change appreciably.

7. Explain the formation of a potential barrier in a p-n junction and show the polarity of the Barrier potential. [Model Question]

Answer:

Formation of Potential Barrier in a P-N junction: There are two types of extrinsic semiconductors - p-type and n-type. The p-type semiconductor is having negative acceptor ions and positively charged holes. Whereas, the n-type semiconductor is having positive donor ions and negatively charged electrons.

When these two pieces are joined together and suitably treated, they form a p-n junction. The moment they form a p-n junction, some of the conduction electrons from n-type material diffuse over to the p-type material and undergo electron-hole recombination with the holes available in the valence band. Simultaneously holes from p-type material diffuse over to the n-type material and undergo hole-electron combination with the electrons available in the conduction band. This process is called diffusion. In this process, some of the free electrons move across the junction from n-type to p-type, leaving behind positive donor ions as they are robbed of the free electrons. This establishes a positive charge on the n-side of the junction.



8. Explain the construction and operation of E-MOSFET and DE-MOSFET. Draw the necessary diagrams. [Model Question]

Answer:

Enhancement MOSFET (E-MOSFET)

Symbol

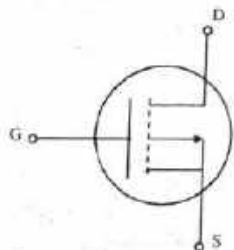


Fig (a): p-channel enhancement type MOSFET

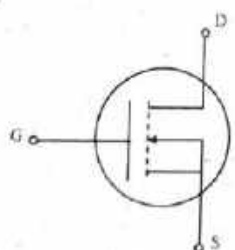


Fig (b): n-channel enhancement type MOSFET

Construction

The following figure shows the construction of a n-channel MOSFET. Two highly doped n⁺ regions are diffused into a lightly doped p-type substrate. One of these n⁺ regions works as source and the other one as drain. A thin layer of an insulating material viz., silicon dioxide (SiO₂) is deposited over the surface.

The gate metal, namely Aluminum is deposited over SiO₂ layer, for making connection for the gate, source and drain.

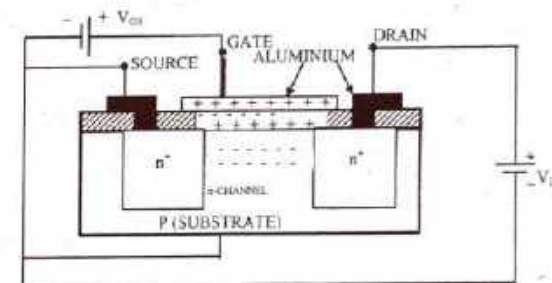


Fig: Enhancement type (n-channel) MOSFET

Operation

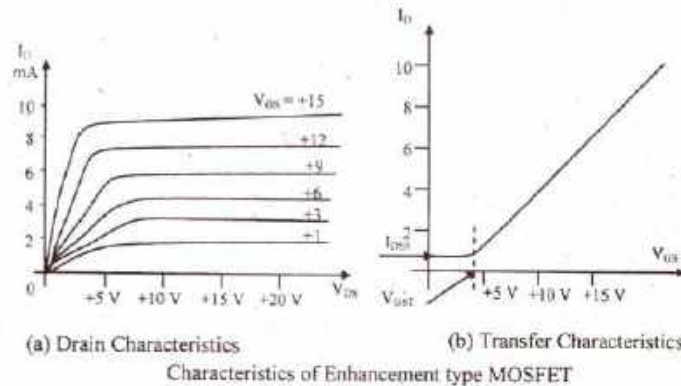
The substrate is grounded (connected to the source). If we apply a positive voltage to the gate, an electric field will be developed perpendicular to the SiO₂ surface. The +ve charge on the gate will induce -ve charge in the near end of the SiO₂ layer and +ve charge in the far end of the SiO₂. Finally, it will induce negative charge in the p-type substrate, as shown. These induced negative charges (also called induced n-channel), which are minority carriers in the p-type substrate, causes current flow between source and drain.

If the magnitude of +ve voltage on the gate increases, the induced -ve charge in the semiconductor also increases. The drain current is "enhanced" (means: increased) by the +ve gate voltage. Hence such a device is called Enhancement type MOSFET.

Characteristics

In Enhancement type MOSFETs, the drain current increases by making V_{GS} more positive. This is shown in the drain characteristics, Fig (a).

The Fig.(b) shows the transfer characteristics, which gives the relationship between drain current and gate voltage. When V_{GS} = 0, the drain current I_{DSS} is very small, only a few nano-amperes. The drain-current starts increasing only after a minimum value of V_{GS} is exceeded; this value of V_{GS} is called the threshold voltage V_{GS(th)}. The drain characteristic in this condition is governed by V_{DS} as in the case of JFET through the modulation of depletion region.



Depletion MOSFET (DE-MOSFET)

Symbol

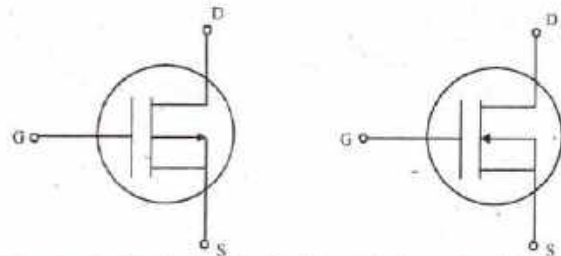


Fig (a): p-channel depletion type MOSFET Fig (b): n-channel depletion type MOSFET

Construction

Fig. below shows the construction of depletion type of MOSFET with n-type channel. Two highly doped n^+ regions work as source and the other one as drain. Between these two n^+ regions, an n-type of channel is diffused. A thin layer of SiO_2 is deposited over the surface. The gate, source and drain connections are made by depositing aluminum over SiO_2 layer.

Operation

If the gate voltage is made negative, there will be +ve charges (induced) in the near side of SiO_2 layer. This, in turn, will induce +ve charges in the channel (n-type). The n-type channel has electrons as the majority carriers. Therefore the induced +ve charges will make the channel less conductive. As a result, the drain current will reduce.

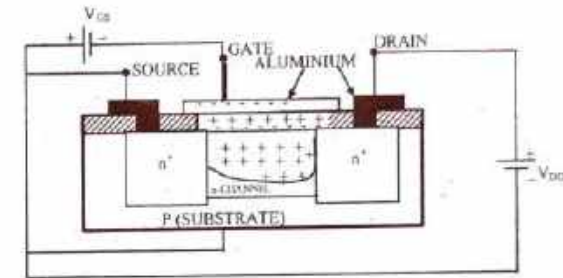


Fig. Depletion type (n-channel) MOSFET

As the gate voltage is made more negative the drain current will be lesser. The negative gate voltage causes an effective depletion (i.e., reduction) of majority carriers; hence it has the name depletion MOSFET. On the other hand if +ve voltage is applied to the gate it induces -ve charges in the channel, thereby increasing the conductivity and is equivalent to enhancement mode of operation.

Characteristics

Fig.(a) shows the drain characteristics of the depletion type FET. The set of curves for values below $V_{GS} = 0$ are for depletion mode (similar to JFET). The set of curves with values above $V_{GS} = 0$ are for enhancement mode.

Fig. (b) shows the transfer characteristics. On the left-hand side, is the depletion mode, where V_{GS} is -ve. On the right hand side is the enhancement mode where V_{GS} is +ve. At $V_{GS} = 0$, the value of I_d is equal to I_{DSS} . As a value of $V_{GS} = V_{GS(OFF)}$, the I_d reduces to a negligible value; this value $V_{GS(OFF)}$ corresponds to pinch-off voltage V_p of a JFET.

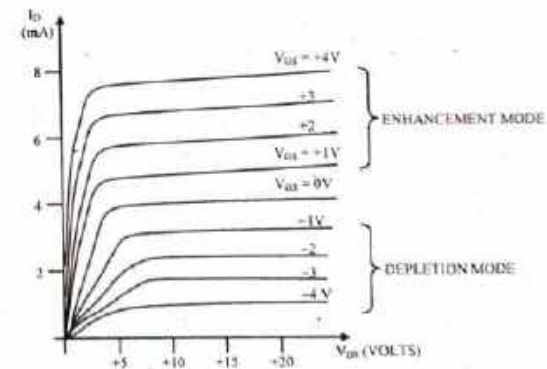


Fig (a): Drain Characteristics

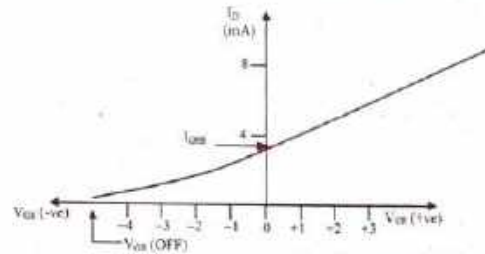


Fig (b): Transfer Characteristics

9. Enumerate the differences between a Bipolar Junction Transistor (BJT) and a Junction Field Effect Transistor (JFET) in respect of construction, doping and operation. Draw a schematic symbol of a P-channel JFET. [Model Question]

Answer:

Comparison between the BJT and the FET

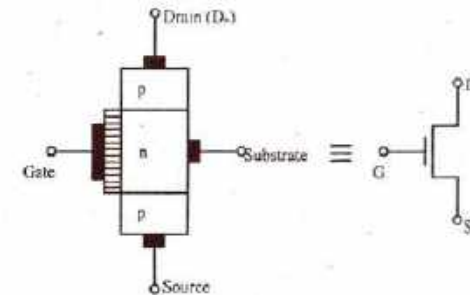
- | BJT | FET |
|---|---|
| 1. Two types of carriers (electrons and holes) are required. | 1. Only one type of carrier (electron or hole) is required. |
| 2. Carriers move through the base by diffusion process. | 2. Carriers move through the channel by drift process. |
| 3. The BJT has a comparatively lower switching speed due to the diffusion process. | 3. The FET has a higher switching speed due to the drift process; the drift of the carrier is faster than diffusion. |
| 4. The BJT is not a thermally stable device. | 4. The FET has a negative temperature coefficient of high-current operations, i.e., the current decreases as temperature increases. Due to the particular feature, a uniform temperature distribution and protection against breakdown can be achieved. |
| 5. In case of IC fabrication, the BJT requires more space than the FET. | 5. In case of IC fabrication the FET requires lesser space than the BJT. |
| 6. The BJT is a current-controlled device. | 6. The FET is a voltage-controlled device. |
| 7. The BJT offers low input impedance. | 7. The FET offers high input impedance; therefore, it can be used as a buffer. |
| 8. BJT is much noisier than FET. | 8. FET is less noisy. |
| 9. The BJT has offset voltage. | 9. The FET has no offset voltage. |
| 10. The BJT can also be used as a switch; it is taken to be in the OFF state when operating in the cut off region, and in the ON state when it is operating in the saturation region. | 10. The FET is particularly useful for its operation as a controlled switch, operating in both the conducting and the non-conducting zones. |

10. What is CMOS?

[Model Question]

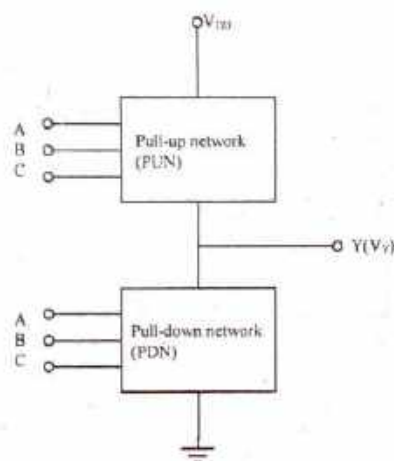
Answer:

CMOS is most popular technology for the implementation of digital systems. The small size, cost of fabrication and low power dissipation of MOSFETs enable extremely high levels of integration of both logic and memory circuits.



The CMOS logic gate consists of two networks. The pull-up network (PUN) constructed of PMOS transistors and the pull-down network (PDN) constructed of NMOS transistors, (shown in the above figure). The two networks are operated by the input variables, in a complementary fashion. Thus, for the three-input gate represented in the figure, the PDN will conduct for all input combinations that require a low output ($Y = 0$) and will then pull the output node down to ground, causing a zero voltage to appear at the output $V_Y = 0$. Simultaneously, the PUN will be off and no direct dc path will exist between V_{DD} and ground. On the other hand, all input combinations that call for a high output ($Y = 1$) will cause the PUN to conduct, and the PUN will then pull the output node up to V_{DD} , establishing an output voltage $V_Y = V_{DD}$. Simultaneously, the PDN will be cut-off and again no current path between V_{DD} and ground will exist in the circuit.

Now, since the PDN comprises NMOS transistors and since an NMOS transistor conducts when the signal at its gate is high, the PDN is activated (i.e., conducts) when the inputs are high. In a dual manner, PUN comprises PMOS transistors and a PMOS transistor conducts when the input signal at its gate is low; thus the PUN is activated when the inputs are low.



Examples of CMOS logic circuits

We can implement any logic gates (NOT, NAND/AND, NOR/OR, XOR/XNOR etc.) or combinational circuits using CMOS technology.

Chapter 6: Overview of Analog Circuits

Chapter at a glance:

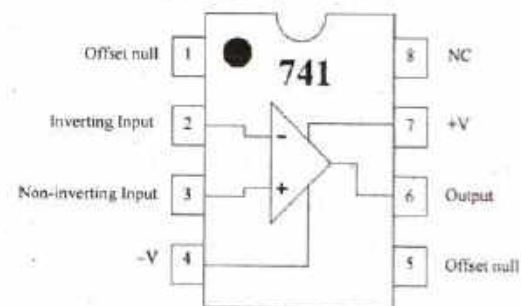
6.1 Introduction

One of the most versatile and widely used electronic devices in linear application is the operational amplifier, most often referred to as operational amplifier. An operational amplifier (Op-amp) is a linear integrated circuit (IC) that has a very high differential voltage gain, high input impedance and low output impedance. It can amplify signals having a frequency range from 0 Hz to 1 MHz approximately. Most commonly uses are in instrumentation and control system.

Operational amplifiers are popular because they are low cost, easy to use, and fun to work. Operational amplifiers are being used in number of electronic application such as in fields like audio and radio communication, instrumentation systems, control systems, bio-medical electronics, and industrial electronics.

An Operational Amplifier can be used as a adder, subtractor, integrator, differentiator, comparator, voltage follower, etc.

6.2 IC-741



6.3 Feedback amplifiers

The important characteristic of an amplifier is its voltage gain, input and output impedance and bandwidth. These parameters are constant for a given amplifier. In order to vary these values of amplifier, parameter feedback technique is used.

The basic concept is that the output signal (current or voltage) is sampled and a portion of it is mixed with the input. If the feedback voltage opposes the input voltage, then it is called **negative feedback**. If the feedback voltage assists the input voltage, it is called **Positive feedback**.

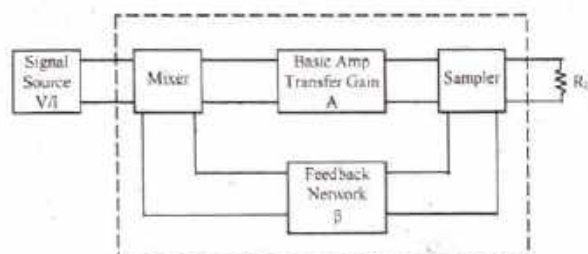


Fig: Block Diagram of feedback Network

The figure above depicts block diagram of a feedback amplifier. The overall gain of feedback amplifier is given by, $A_f = \frac{A}{1 \pm A\beta}$

where A = gain of basic amplifier

A_f = Gain with feedback

β = feedback factor.

[Here, '+' sign is for negative feedback and '-' sign for positive feedback].

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

- An ideal OP-AMP has the following characteristics- (R_{in} = input resistance, A = open loop gain, R_{out} = output resistance) [WBSCTE 2022]

(a) $R_{in} = \infty$, $A = \infty$ and $R_{out} = 0$	(b) $R_{in} = 0$, $A = \infty$ and $R_{out} = 0$
(c) $R_{in} = \infty$, $A = 0$ and $R_{out} = \infty$	(d) $R_{in} = 0$, $A = \infty$ and $R_{out} = \infty$

Answer: (c)

- The expression of voltage gain in a non-inverting amplifier using ideal OP-AMP is (R_f = feedback resistance, R_i = input resistance) [WBSCTE 2022]

(a) $-R_f/R_i$	(b) R_f/R_i	(c) $1+R_f/R_i$	(d) $-(1+R_f/R_i)$
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Answer: (c)

- The initial slope of a sine wave is directly proportional to – [Model Question]

(a) slew rate	(b) voltage gain	(c) frequency	(d) capacitance
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Answer: (c)

- The voltage follower has a – [Model Question]

(a) closed-loop voltage gain of unity	(b) small open-loop voltage gain
(c) closed-loop bandwidth of zero	(d) large closed-loop output impedance

Answer: (a)

- An instrumentation amplifier has a high – [Model Question]

(a) output impedance	(b) power gain	(c) CMRR	(d) supply voltage
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Answer: (c)

- In an op-amp integrator, the current through the input resistor flows into the – [Model Question]

(a) inverting input	(b) non-inverting input
(c) bypass capacitor	(d) feedback capacitor

Answer: (b)

- A comparator with a trip point of zero is sometimes called a [Model Question]

(a) threshold detector	(b) Zero-crossing detector
(c) positive limit detector	(d) half-wave detector

Answer: (b)

- The non-linear application of an OPAMP is – [Model Question]

(a) adder	(b) subtractor
(c) voltage to current converter	(d) comparator

Answer: (d)

- CMRR may be defined as- [Model Question]

(a) $\frac{A_d}{A_{cm}}$	(b) A_{cm}	(c) $\frac{A_{cm}}{A_d}$	(d) A_d
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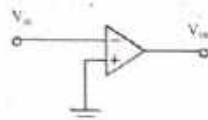
Answer: (a)

- The CMRR of an Op-Amp is [Model Question]

(a) impedance dependent	(b) voltage dependent
(c) Frequency dependent	(d) independent of these

Answer: (c)

11. If the i/p to the circuit of the figure is a sine wave, the o/p will be [Model Question]



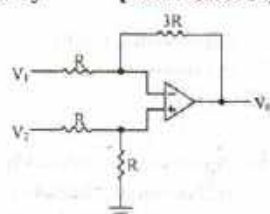
- (a) a half-wave rectifier sine wave
(b) a full-wave rectifier sine wave
(c) a triangular wave
(d) a square wave

Answer: (a)

12. The value of V_0 is given for the following circuit is given by [Model Question]

- (a) $-3V_1 + 2V_2$
(b) $-3V_2$
(c) $1.5V_2 - 2.25V_1$
(d) $2V_2 - 3V_1$

Answer: (d)



B. Answer the following questions:

13. Write one application of OPAMP as differential mode. [Model Question]

Answer:

One application of OPAMP as differential mode is light activated relay.

14. What is a zero-crossing detector? [Model Question]

Answer:

Zero Crossing Detector using 741 IC

The zero crossing detector circuit is an important application of the op-amp comparator circuit. It can also be called as the sine to square wave converter. Any one of the inverting or non-inverting comparators can be used as a zero-crossing detector. The only change to be brought in is the reference voltage with which the input voltage is to be compared, must be made zero ($V_{ref} = 0V$). An input sine wave is given as V_{in} . These are shown in the circuit diagram and input and output waveforms of an inverting comparator with a 0V reference voltage.

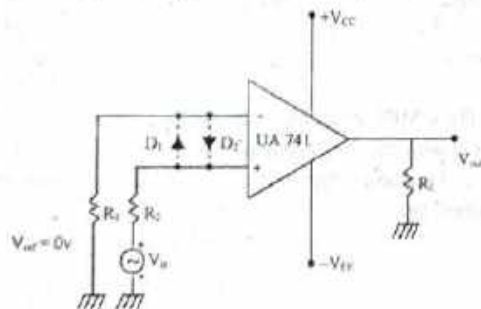


Fig: Zero Crossing Detector Using UA 741 op-amp IC

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Long Answer Type Questions

1. Draw the pin diagram of IC741 OP AMP and describe function of each pin? [WBSCTE 2022]

Answer:

1st part:

General Purpose Op-Amp 741:

The IC 741 is high performance monolithic op-amp IC. It is available in 8 pin, 10 pin or 14 pin configuration. It can operate over a temperature of $-55^{\circ}C$ to $125^{\circ}C$.

Features:

- No frequency compensation required
- Short circuit protection provided
- Offset voltage null capability
- Large common mode and differential voltage range.
- No latch up.

Internal Schematic:

The Figure shows the schematic diagram of the IC 741 op-amp that has become an industry standard. This equivalent circuit illustrates the internal structure of the 741 IC op-amp. The diagram helps to understand all the internal stages along with their capabilities and the limitation. The most important point about this equivalent circuit is that all the 741 models have this same equivalent circuit. The data sheet of IC includes absolute maximum ratings, electrical specifications, the performance curves and typical applications.

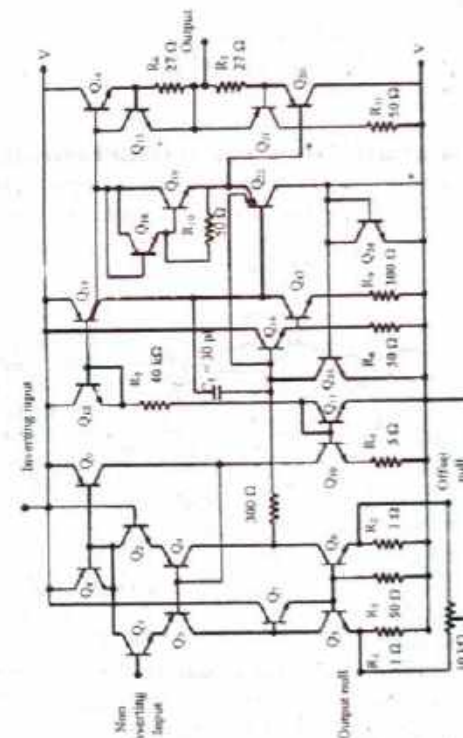


Fig: Internal schematic of IC 741

2nd part:

IC 741 Op Amp (Operational Amplifier)

The 741 Op Amp IC is a monolithic integrated circuit, comprising of a general purpose Operational Amplifier. It was first manufactured by Fairchild semiconductors in the year

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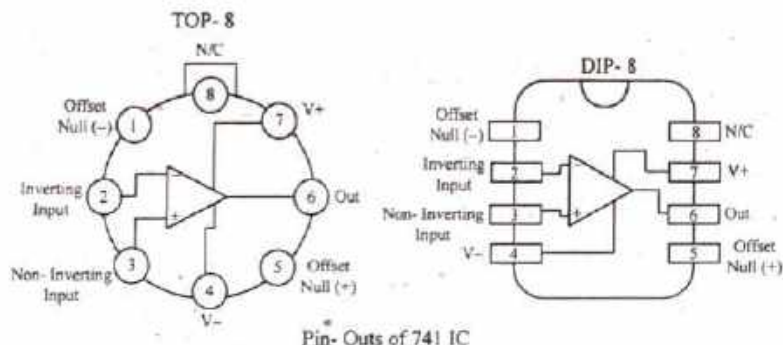
1903. The number 741 indicates that this operational amplifier IC has 7 functional pins, 4 pins capable of taking input and 1 output pin.

IC 741 Op Amp can provide high voltage gain and can be operated over a wide range of voltages, which makes it the best choice for use in integrators, summing amplifiers and general feedback applications. It also features short circuit protection and internal frequency compensation circuits built in it. This Op-amp IC comes in the following form factors:

- 8 Pin DIP Package
- TO5-8 Metal can package
- 8 Pin SOIC

Pin out of IC 741 Op Amp and their Functions

The below figure illustrates the pin configurations and internal block diagram of IC 741 in 8 pin DIP and TO5-8 metal can package.



The functions of different pins of 741 IC:

- **Pin4 & Pin7 (Power Supply):** Pin7 is the positive voltage supply terminal and Pin4 is the negative voltage supply terminal. The 741 IC draws in power for its operation from these pins. The voltage between these two pins can be anywhere between 5V and 18V.
- **Pin6 (Output):** This is the output pin of IC 741. The voltage at this pin depends on the signals at the input pins and the feedback mechanism used. If the output is said to be high, it means that voltage at the output is equal to positive supply voltage. Similarly, if the output is said to be low, it means that voltage at the output is equal to negative supply voltage.
- **Pin2 & Pin3 (Input):** These are input pins for the IC. Pin2 is the inverting input and Pin3 is the non-inverting input. If the voltage at Pin2 is greater than the voltage at Pin3, i.e., the voltage at inverting input is higher, the output signal stays low.

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Similarly, if the voltage at Pin3 is greater than the voltage at Pin2, i.e., the voltage at non-inverting input is high, the output goes high.

- **Pin1 & Pin5 (Offset Null):** Because of high gain provided by 741 Op-Amp, even slight differences in voltages at the inverting and non-inverting inputs, caused due to irregularities in manufacturing process or external disturbances, can influence the output. To nullify this effect, an offset voltage can be applied at pin1 and pin5, and is usually done using a potentiometer.
- **Pin8 (N/C):** This pin is not connected to any circuit inside 741 IC. It's just a dummy lead used to fill the void space in standard 8 pin packages.

Specifications

The following are the basic specifications of IC 741:

- **Power Supply:** Requires a Minimum voltage of 5V and can withstand upto 18V
- **Input Impedance:** About 2 megaohms
- **Output impedance:** About 75 ohms
- **Voltage Gain:** 200,000 for low frequencies
- **Maximum Output Current:** 20mA
- **Recommended Output Load:** Greater than 2 kilohms
- **Input Offset:** Ranges between 2mV and 6mV
- **Slew Rate:** 0.5V/microsecond (It is the rate at which an Op-Amp can detect voltage changes)

The high input impedance and very small output impedance makes IC 741 a near ideal voltage amplifier.

- ➔ 2. Draw the circuit diagram of a non inverting amplifier with feedback using op-amp & derive the expression of gain. [WBSCTE 2022]

Answer:

Figure (a) depicts the circuit diagram of a non-inverting amplifier. The input voltage v_i is applied to the non-inverting terminal. Since the voltage gain of the OP AMP is infinite, the potential of the point G is also v_i . The current flowing into the OP AMP is negligible, its input impedance being very large. Hence, applying Kirchhoff's current law at the point G we obtain

$$\frac{v_o - v_i}{R_f} = \frac{v_i}{R_i} \quad \dots (1)$$

$$\text{or, } \frac{v_o}{v_i} = 1 + \frac{R_f}{R_i} \quad \dots (2)$$

Which is the voltage gain of the amplifier system. The voltage gain is greater than unity by a factor R_f/R_i . As the gain is positive, there is no phase difference between the input voltage v_i and the output voltage v_o . The input impedance of the circuit is high and the output impedance is low.

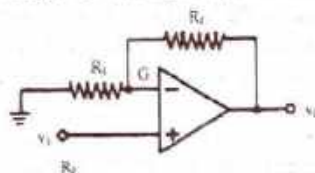


Fig: (a) Non-inverting amplifier

In the circuit of Fig (a), if $R_f = 0$ and $R_i = \infty$, the circuit reduces to that of Fig: (b). Equation (2) shows that the voltage gain in this case is unity. Therefore, the circuit of Fig: (b) is referred to as a unity-gain buffer or a voltage follower. This circuit offers a high input impedance, and a low output impedance, and therefore can be employed as an impedance matching device between a high-impedance source and a low impedance load.

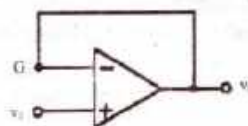


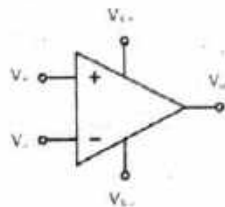
Fig: (b) A voltage follower

3. What is the differential amplifier? Describe the use of an op-amp as an integrator [Model Question]

Answer:

A differential amplifier is a type of electronic amplifier that amplifies the difference between two input voltages but suppresses any voltage common to the two inputs. It is an analog circuit with two inputs V_{in}^+ and V_{in}^- and one output V_{out} in which the output is ideally proportional to the difference between the two voltages

$V_{out} = A(V_{in}^+ - V_{in}^-)$, where A is the gain of the amplifier.

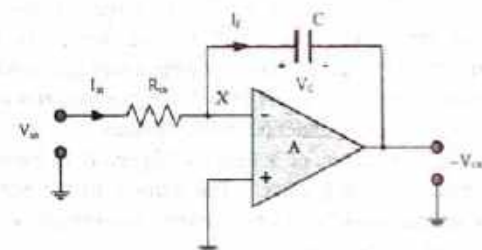


The Op-amp Integrating Amplifier

In the previous tutorials we have seen circuits which show how an operational amplifier can be used as part of a positive or negative feedback amplifier or as an adder or subtracted type circuit using just pure resistances in both the input and the feedback loop. But what if we were to change the purely resistive (R_f) feedback element of an inverting amplifier to that of a frequency dependant impedance, (Z) type complex element, such as a Capacitor, C . What would be the effect on the op-amps output voltage over its frequency range.

By replacing this feedback resistance with a capacitor we now have an **RC Network** connected across the operational amplifiers feedback path producing another type of operational amplifier circuit commonly called an **Op-amp Integrator** circuit as shown below.

Op-amp Integrator Circuit



As its name implies, the **Op-amp Integrator** is an Operational Amplifier circuit that performs the mathematical operation of **Integration** that is we can cause the output to respond to changes in the input voltage over time as the op-amp integrator produces an *output voltage which is proportional to the integral of the input voltage*.

In other words the magnitude of the output signal is determined by the length of time a voltage is present at its input as the current through the feedback loop charges or discharges the capacitor as the required negative feedback occurs through the capacitor.

When a step voltage, V_{in} is firstly applied to the input of an integrating amplifier, the uncharged capacitor C has very little resistance and acts a bit like a short circuit allowing maximum current to flow via the input resistor, R_{in} as potential difference exists between the two plates. No current flows into the amplifiers input and point X is a virtual earth resulting in zero output. As the impedance of the capacitor at this point is very low, the gain ratio of X/R_{in} is also very small giving an overall voltage gain of less than one, (voltage follower circuit).

As the feedback capacitor, C begins to charge up due to the influence of the input voltage, its impedance X_c slowly increase in proportion to its rate of charge. The

capacitor charges up at a rate determined by the RC time constant, (τ) of the series RC network. Negative feedback forces the op-amp to produce an output voltage that maintains a virtual earth at the op-amp's inverting input.

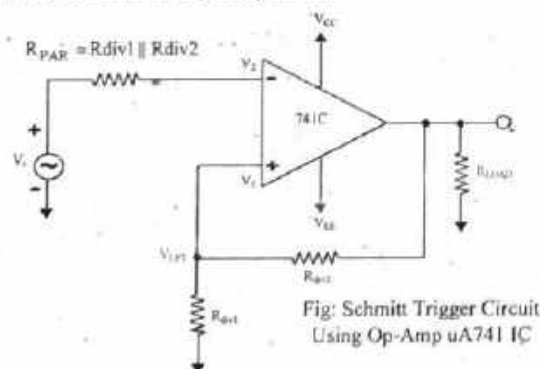
Since the capacitor is connected between the op-amp's inverting input (which is at earth potential) and the op-amp's output (which is negative), the potential voltage, V_c developed across the capacitor slowly increases causing the charging current to decrease as the impedance of the capacitor increases. This results in the ratio of X_c/R_{in} increasing producing a linearly increasing ramp output voltage that continues to increase until the capacitor is fully charged.

4. Draw schematically an op-amp Schmitt trigger and explain its operation.
Name an application of it. [Model Question]

Answer:

A Schmitt trigger circuit is also called a regenerative comparator circuit. The circuit is designed with a positive feedback and hence will have a regenerative action which will make the output switch levels. Also, the use of positive voltage feedback instead of a negative feedback, aids the feedback voltage to the input voltage, instead of opposing it. The use of a regenerative circuit is to remove the difficulties in a **zero-crossing detector circuit** due to low frequency signals and input noise voltages.

Shown below is the circuit diagram of a Schmitt trigger. It is basically an inverting comparator circuit with a positive feedback. The purpose of the Schmitt trigger is to convert any regular or irregular shaped input waveform into a square wave output voltage or pulse. Thus, it can also be called a squaring circuit.



As shown in the circuit diagram, a voltage divider with resistors R_{div1} and R_{div2} is set in the positive feedback of the 741 IC op-amps. The same values of R_{div1} and R_{div2} are used to get the resistance value $R_{par} = R_{div1} || R_{div2}$ which is connected in series with the input voltage. R_{par} is used to minimize the offset problems. The voltage across R_1 is feedback to

the non-inverting input. The input voltage V_i triggers or changes the state of output V_{out} every time it exceeds its voltage levels above a certain threshold value called Upper Threshold Voltage (V_{up}) and Lower Threshold Voltage (V_{lp}).

Let us assume that the inverting input voltage has a slight positive value. This will cause a negative value in the output. This negative voltage is feedback to the non-inverting terminal (+) of the op-amp through the voltage divider. Thus, the value of the negative voltage that is feedback to the positive terminal becomes higher. The value of the negative voltage becomes again higher until the circuit is driven into negative saturation ($-V_{sat}$). Now, let us assume that the inverting input voltage has a slight negative value. This will cause a positive value in the output. This positive voltage is feedback to the non-inverting terminal (+) of the op-amp through the voltage divider. Thus, the value of the positive voltage that is feedback to the positive terminal becomes higher. The value of the positive voltage becomes again higher until the circuit is driven into positive saturation ($+V_{sat}$). This is why the circuit is also named a regenerative comparator circuit.

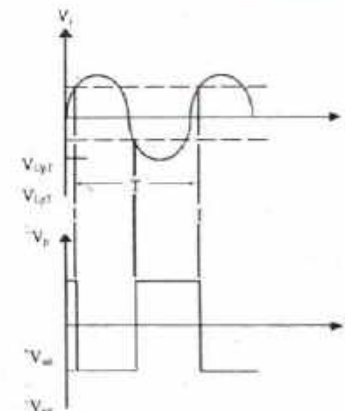


Fig: Schmitt Trigger Input and Output Waveform

When $V_{out} = +V_{sat}$, the voltage across R_{div1} is called Upper Threshold Voltage (V_{up}). The input voltage, V_{in} must be slightly more positive than V_{up} in order to cause the output V_o to switch from $+V_{sat}$ to $-V_{sat}$. When the input voltage is less than V_{up} , the output voltage V_{out} is at $+V_{sat}$.

Upper Threshold Voltage, $V_{up} = +V_{sat}(R_{div1} / [R_{div1} + R_{div2}])$

When $V_{out} = -V_{sat}$, the voltage across R_{div1} is called Lower Threshold Voltage (V_{lp}). The input voltage, V_{in} must be slightly more negative than V_{lp} in order to cause the output V_o to switch from $-V_{sat}$ to $+V_{sat}$. When the input voltage is less than V_{lp} , the output voltage V_{out} is at $-V_{sat}$.

Lower Threshold Voltage, $V_{lp} = -V_{sat}(R_{div1} / [R_{div1} + R_{div2}])$

If the value of V_{upt} and V_{lpt} are higher than the input noise voltage, the positive feedback will eliminate the false output transitions. With the help of positive feedback and its regenerative behaviour, the output voltage will switch fast between the positive and negative saturation voltages.

5. (a) Define slew rate of OPAMP.

(b) Draw the circuit diagram of an instrumentation amplifier and explain its operation. [Model Question]

Answer:

Slew rate: The slew rate is defined as, the maximum rate of change of output voltage caused by a step input and usually specified in $V/\mu S$.

Mathematically,

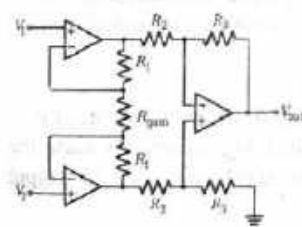
$$\text{Slew Rate} = \left. \frac{dV_o}{dt} \right|_{\text{maximum}} \quad V/\mu S$$

Ideally, the value of slew rate should be infinity.

As the name implies OPAMP can be used to perform various kinds of mathematical, logical and other types of operations. There are some examples of OPAMP circuits performing various operations.

(b) An instrumentation (or instrumentational) amplifier is a type of differential amplifier that has been outfitted with input buffer amplifiers, which reduce the need for input impedance matching. This amplifier is particularly suitable for use in measurement and test equipment. Additional characteristics include very low DC offset, low drift, low noise, very high open-loop gain, very high common-mode rejection ratio, and very high input impedances. Instrumentation amplifiers are used where great accuracy and stability of the circuit both short and long-term are required.

Although the instrumentation amplifier is usually shown schematically identical to a standard OPAMP, the electronic instrumentation amp is almost always internally composed of 3 op-amps. These are arranged so that there is one op-amp to buffer each input (+, -), and one to produce the desired output with adequate impedance matching for the function.



The most commonly used instrumentation amplifier circuit is shown in the figure. The gain of the circuit is

$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}} \right) \frac{R_3}{R_2}$$

The rightmost amplifier, along with the resistors labelled R_2 and R_3 is just the standard differential amplifier circuit, with gain = R_3 / R_2 and differential

input resistance = $2R_1$. The two amplifiers on the left are the buffers. With R_{gain} removed,

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they are simple unity gain buffers; the circuit will work in that state, with gain simply equal to R_3 / R_2 and high input impedance because of the buffers. The buffer gain could be increased by putting resistors between the buffer inverting inputs and ground to shunt away some of the negative feedback; however, the single resistor R_{gain} between the two inverting inputs is a much more elegant method: it increases the differential-mode gain of the buffer pair while leaving the common-mode gain equal to 1.

This increases the common-mode rejection ratio (CMRR) of the circuit and also enables the buffers to handle much larger common-mode signals without clipping than would be the case if they were separate and had the same gain. Another benefit of the method is that it boosts the gain using a single resistor rather than a pair, thus avoiding a resistor-matching problem and very conveniently allowing the gain of the circuit to be changed by changing the value of a single resistor. A set of switch-selectable resistors or even a potentiometer can be used for R_{gain} , providing easy changes to the gain of the circuit, without the complexity of having to switch matched pairs of resistors.

The ideal common-mode gain of an instrumentation amplifier is zero. In the circuit shown, common-mode gain is caused by mismatches in the values of the equally numbered resistors and by the mis-match in common mode gains of the two input op-amps. Obtaining very closely matched resistors is a significant difficulty in fabricating these circuits, as is optimizing the common mode performance of the input op-amps.

An instrumentation amp can also be built with two op-amps to save on cost and increase CMRR, but the gain must be higher than two (+6 dB).

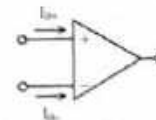
6. Define the terms in connection with OP-AMP (a) input bias current (b) output offset voltage (c) CMRR. [Model Question]

Answer:

a) Input Bias current:

Ideally, no current flows into the input terminals of an op amp. In practice, there are always two input bias currents.

Values of I_B range from 60 fA (about one electron every three microseconds) in the AD549 electrometer, to tens of microamperes in some high speed op amps. Op amps with simple input structures using bipolar junction transistors (BJT) or FET long-tailed pair have bias currents that flow in one direction. More complex input structures (bias-compensated and current feedback op amps) may have bias currents that are the difference between two or more internal current sources, and may flow in either direction. Bias current is a problem to the op amp user because it flows in external impedances and produces voltages, which add to system errors. Consider a non-inverting unity gain buffer driven from a source impedance of 1 MΩ. If I_B is 10 nA, it will introduce an additional 10 mV of error. This degree of error is not trivial in any system. Or, if the designer simply forgets about I_B and uses capacitive coupling, the



Or, if the designer simply forgets about I_B and uses capacitive coupling, the

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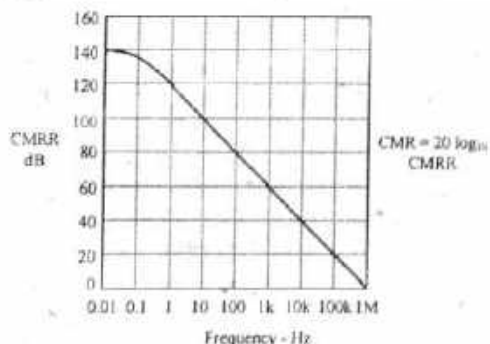
circuit won't work—at all! Or, if I_B is low enough, it may work momentarily while the capacitor charges, giving even more misleading results. The moral here is not to neglect the effects of I_B in any op amp circuit. The same admonition goes for in-amp circuits.

b) Input Offset Voltage:

The input offset current, I_{OS} , is the difference between I_{B-} and I_{B+} , or $I_{OS} = I_{B+} - I_{B-}$. Note also that I_{OS} is only meaningful where the two individual bias currents are fundamentally reasonably well-matched, to begin with. This is true for most voltage feedback (VFB) op amps. However, it wouldn't for example be meaningful to speak of I_{OS} for a current feedback (CFB) op amp, as the currents are radically un-matched. It should be noted that rail-to-rail input stages comprised of two parallel stages have bias currents that change direction as the common-mode voltage passes through the transition region. Bias and offset currents for these devices are especially difficult to specify, other than simply giving a maximum positive/negative value.

c) CMRR:

If a signal is applied equally to both inputs of an op amp, so that the differential input voltage is unaffected, the output should not be affected. In practice, changes in common mode voltage will produce changes in output. The op amp common-mode rejection ratio (CMRR) is the ratio of the common-mode gain to differential-mode gain. For example, if a differential input change of Y volts produces a change of 1 V at the output, and a common-mode change of X volts produces a similar change of 1 V, then the CMRR is X/Y . When the common-mode rejection ratio is expressed in dB, it is generally referred to as common-mode rejection (CMR)—please note that there is very little consistency in this throughout the semiconductor industry with regards to the use of dB or ratio values for CMR or CMRR. Typical low frequency CMR values can be between 70 dB and 120 dB, but at higher frequencies, CMR deteriorates. In addition to a CMRR numeric specification, many op amp data sheets show a plot of CMR versus frequency.



CMRR produces a corresponding output offset voltage error in op amps configured in the non inverting mode. Inverting mode operating op amps will have less CMRR error. Since both inputs are held at a ground (or virtual ground), there is no CM dynamic voltage.

7. What are the different parameters of Operational Amplifier? Explain.

[Model Question]

Answer:

The different parameters of Operational Amplifier are:

Input Offset Voltage (V_{IO})

It is the voltage that must be applied between two input terminals to balance the operational amplifier i.e. to null the out-put as shown in figure 1. Here V_{IO} denotes input offset voltage.

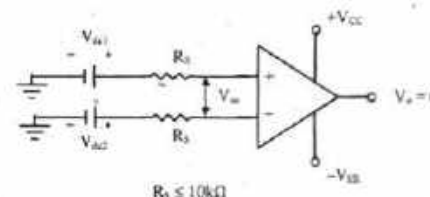


Fig: 1 Input offset voltage

Input Offset current (I_{IO})

The difference between the separate current entering input terminals of a balanced operational amplifier is called a input offset currents I_{IO} .

$$I_{IO} = |I_{B1} - I_{B2}|$$

Where the output voltage is 0 V and the temperature is 25°C.

I_{B1} = Current into non-inverting input

I_{B2} = Current into inverting input

Ideally, if both transistors in the differential pair have exactly the same β 's, both current will be equal $I_{B1} = I_{B2}$. In practice, small difference will exist between the two bias currents. This difference is called input offset current I_{IO} .

For operational amplifiers with BJT inputs, I_B is typically in the range from 50 to 100 nA, where as I_{IO} will usually be around 10 to 20 nA. These currents are low, but they still can have a significant effect on the output of the operational amplifier. The range of I_B is from 1pA or more for general-purpose operational amplifier to 1pF or less for operational amplifiers that have field-effect transistors at the input.

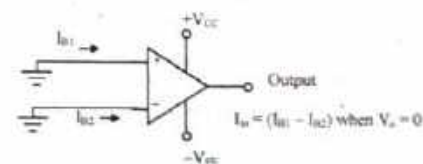


Fig: 2 Input offset current

Input bias Current (I_B)

It is an average of currents into inverting and non-inverting input terminals of operational amplifier with output voltage zero. $I_B = \frac{I_{B1} + I_{B2}}{2}$ when, $V_o = 0$.

Input offset voltage drift

It is the ratio of change in input offset voltage (ΔV_{io}) to change in temperature (ΔT).

$$\text{Input offset voltage drift} = \frac{\Delta V_{io}}{\Delta T}$$

Input offset current drift

It is the ratio of change in input offset current to change in temperature.

$$\text{Input offset current drift} = \frac{\Delta I_{io}}{\Delta T}$$

Output offset voltage

It is the difference between the DC voltage present at the output terminal when two input terminals are grounded.

Supply Voltage Rejection Ratio (SVRR)

It is the ratio of change in input offset voltage V_{io} to the variation in supply voltage. Generally SVRR is expressed in μV per volt or in decibels.

Slew Rate (SR)

It is defined as the maximum rate of change of output voltage with respect to time.

$$SR = \left. \frac{dV_o}{dt} \right|_{\text{maximum}}$$

$\frac{dV}{dt}$ = Rate of change of output voltage per unit time under large signal condition ($\text{V}/\mu\text{s}$).

Slew rate gives the information about how fast the output of op-amp can change with the change in input. In a.c. application slew rate is very important parameter.

Output resistance (R_o)

It is the equivalent resistance measured between output terminal of op-amp and ground. Generally the value of R_o is low.

Power Consumption (P_c)

It is the amount of power that operational amplifier consumes for its proper operation.

Full power bandwidth

This is the maximum frequency at which full output voltage swing can be obtained.

8. Describe the function of an Op-Amp on (i) an inverter, (ii) an adder, (iii) an integrator (iv) a differentiator and (v) an amplifier. [Model Question]

Answer:

(i) **Inverter:** The figure 1 shows a basic inverting amplifier with an input resistor R_i and a feedback resistor R_f . The non-inverting input terminal is grounded. The input voltage is e_i and the output voltage is e_o . Since the gain A of the operational amplifier (A is also called the open loop gain) is very large. The bias currents are ideally zero as operational amplifier offer infinite input resistance. Also different between terminal voltages i.e. $V_1 - V_2 = E_d$ is ideally zero. That is, two voltages are equal. The voltage V_1 at the inverting input terminal is very small, in fact, it is very close to the ground potential i.e. although the point G is not actually connected to ground, it is held virtually at ground potential, irrespective of the magnitudes at the potentials e_i and e_o . Hence the node is said to be virtual ground. Let the current ' i ' be flowing through resistance R_i . As amplifier is not drawing any current, all current will flow through the feedback resistance R_f .

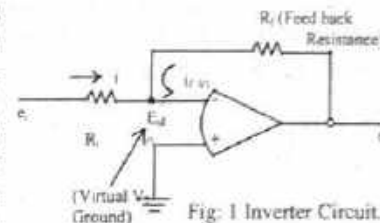


Fig. 1 Inverter Circuit.

Now applying Kirchhoff's law at node G .

$$i + i_f = 0 \quad \dots(1)$$

$$\frac{e_i}{R_i} + \frac{e_o}{R_f} = 0; \quad e_o = -\frac{R_f}{R_i} e_i$$

$$= \frac{\text{Feedback Resistance}}{\text{Input Resistance}} \times \text{Input Voltage}$$

$$\frac{e_o}{e_i} = -\frac{R_f}{R_i} \quad \text{or, } e_o = -\frac{R_f}{R_i} e_i \quad \dots(2)$$

Thus the voltage gain is given by the ratio of the feedback resistance R_f to the input resistance R_i . The negative sign indicates that the output voltage is inverted with respect to the input voltage.

(ii) **Adder:** An operational amplifier can be used as an adder. An adder or a summing amplifier using an operational amplifier is shown in the figure 2.

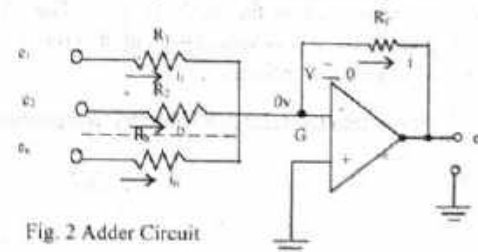


Fig. 2 Adder Circuit

Let us consider $e_1, e_2, \dots, e_n$ inputs are applied through $R_1, R_2, \dots, R_n$.

Hence the point G is a virtual ground.

Now considering the idea of virtual ground, we can easily see that the sum of the currents through the resistance $R_1, R_2, \dots, R_n$ is equal to the current I through R_f . So applying Kirchhoff's current law, we get $i_1 + i_2 + i_3 + \dots + i_n = i$... (3)

Now,

$$i_1 = \frac{e_1 - 0}{R_1} = \frac{e_1}{R_1}$$

$$i_2 = \frac{e_2 - 0}{R_2} = \frac{e_2}{R_2}$$

$$i_n = \frac{e_n}{R_n} \text{ and also } \frac{0 - e_o}{R_f} = i = -\frac{e_o}{R_f}$$

$$\therefore \frac{e_o}{R_f} = -\left(\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} + \dots + \frac{e_n}{R_n}\right) = -\sum_{i=1}^n \frac{e_i}{R_i} \quad \dots(4)$$

If $R_1 = R_2 = R_3 = \dots = R_n = R_f$

$$\therefore e_o = -(e_1 + e_2 + \dots + e_n) = -\sum_{i=1}^n e_i \quad \dots(5)$$

So it is clear that, the output is the sum of all the inputs, i.e., the output voltage e_o is numerically equal to the algebraic sum of the input voltage e_1 through e_n . Hence the name of the circuit is given.

(iii) Integrator: The operational amplifier can be used as an integrator. The circuit diagram is shown in the figure 3. Here the output voltage is proportional to the time integral of the input voltage. Hence this circuit is called an integrator.

Integrators are used in ramp or sweep generators etc. Here C is introduced in the feedback path. The circuit is most easily analyzed in the following way. Since G is virtual ground, its ground potential is zero and the gain of the operational amplifier is infinite.

The input current $i_1 = \frac{e_1}{R}$ flows into the feedback capacitor and produces the voltage e_o .

$$i_1 = \frac{e_1 - 0}{R} = \frac{e_1}{R} \quad \dots(6)$$

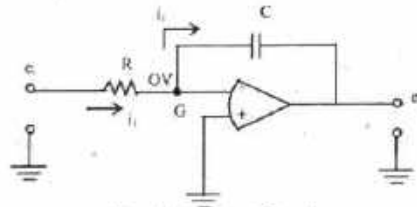


Fig. 3 Integrator Circuit

$$\text{and } 0 - e_o = \frac{1}{C} \int i_1 dt \quad \dots(7)$$

$$\text{Now From eqn. (7), } e_o = -\frac{1}{C} \int i_1 dt$$

Now putting the value of i_1 from eqn. (6) we have

$$e_o = -\frac{1}{RC} \int e_1 dt = -\frac{1}{RC} \left[\int e_1 dt + [I.C.]_{t=0} \right] \rightarrow \text{Integral constant} \quad \dots(8)$$

Now it can be seen that the output potential is proportional to the time integral of the input. $\frac{1}{RC}$ is called time constant. If $\frac{1}{RC} = 1$ (By making $R = 1M\Omega$ & $C = 1\mu F$), we call it real time integration. If $\frac{1}{RC} = 2$, then we get double value. Time Constant is the slope of the curve.

(iv) Differentiator: The operational amplifier can be used as a differentiator. The circuit diagram is shown in the figure 4. Here the output voltage is proportional to the time differential of the input voltage. Hence the circuit is called a "DIFFERENTIATOR". The circuit is most easily analyzed in the following way. Here C is introduced in the input path. Since G is the virtual ground its ground potential is infinite gain of the operational amplifier. Input current 'i' flows into the feedback resistor and produces a voltage e_o .

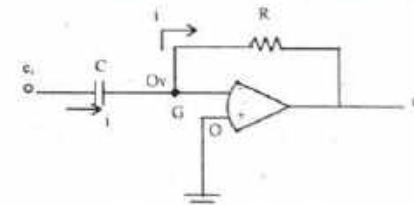


Fig. 4: Differentiator circuit

Now,

$$Q = Ce_1 \text{ or } i = \frac{dQ}{dt} = C \frac{de_1}{dt}$$

$$\int i dt = ce_1 \text{ or } i = C \frac{d}{dt}(e_1) \quad \dots(9)$$

$$\text{Again, } \frac{0 - e_o}{R} = i \quad \dots(10)$$

Therefore, from eqn. (9) and (10) we have $-\frac{e_o}{R} = C \frac{d}{dt}(e_1)$

$$\text{or, } e_o = -RC \frac{d}{dt}(e_i) \quad \dots (1)$$

Now it can be seen that the output potential is proportional to the differentiation of the input. Here RC is called time constant.

(v) Differential Amplifier with one operational amplifier (Subtractor):

The differential amplifiers are used in instrumentation and industrial applications to amplify difference between two input signals. Figure (a) shows the differential amplifier with one operational amplifier. The circuit has two inputs V_1 and V_2 . Now applying Kirchhoff's current law we have,

$$\frac{V_1 - V_3}{R_1} = \frac{V_3 - V_o}{R_2} \quad \dots (1)$$

$$\frac{V_2 - V_3}{R} = \frac{V_3}{R} \quad \dots (2)$$

Now from equation (2) we have,

$$\frac{V_2}{R} + \frac{V_3}{R} = \frac{V_3}{R}$$

$$\text{or, } V_3 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_2}{R_2}$$

$$\text{or, } V_3 = \frac{V_2}{R_1} \left(\frac{R_1 R_2}{R_1 + R_2} \right) = V_2 \left(\frac{R_2}{R_1 + R_2} \right) \quad \dots (3)$$

Now from equation (1) we have

$$\frac{V_1 - V_3}{R_1} = \frac{V_3 - V_o}{R_2} = \frac{V_3}{R_2} - \frac{V_o}{R_2}$$

$$\text{or, } \frac{V_o}{R_2} = \frac{V_1}{R_2} - \left(\frac{V_1 - V_3}{R_1} \right) = V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{V_1}{R_1} = V_1 \left(\frac{R_1 + R_2}{R_1 R_2} \right) - \frac{V_1}{R_1}$$

Now putting the value of V_3 from equation (3) we have,

$$\frac{V_o}{R_2} = V_2 \left(\frac{R_2}{R_1 + R_2} \right) \left(\frac{R_1 + R_2}{R_1 R_2} \right) - \frac{V_1}{R_1}$$

$$\text{or, } V_o = \frac{R_2}{R_1} (V_2 - V_1) = -\frac{R_2}{R_1} (V_1 - V_2) = A_D V_{12} \quad \dots (4)$$

$$\left[\because A_D = -\frac{R_2}{R_1}, V_{12} = V_1 - V_2 \right]$$

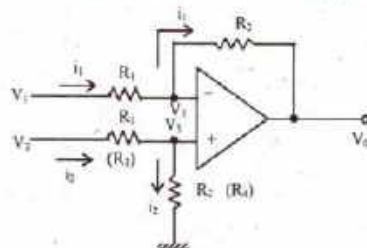


Fig. A Subtractor circuit

Voltage Gain: (A_D)

Therefore, the voltage gain of the operational amplifier is:

$$A_D = \frac{V_o}{V_{12}} = -\frac{R_2}{R_1} \quad \dots (5)$$

Note that the gain of the differential amplifier is the same as that of the inverting amplifier.

Input Resistance (R_i)

The input resistance R_i of the differential amplifier is the resistance determined into either one of the two input terminals with other grounded.

Therefore, $V_2 = 0V$, the circuit is an inverting amplifier the input resistance at which,

$$R_{i2} \cong R_1 \quad \dots (6)$$

Similarly in the case of non-inverting amplifier, the input resistance

$$R'_{i2} \cong R_2 + R_1 \quad \dots (7)$$

In equation (6) and (7), it is obvious that the input resistances seen by the signal sources V_1 and V_2 are not same. This problem can be corrected and both input resistances can be made equal if we modify the basic differential amplifier at figure. If the value of both R_1 and $(R_3 + R_4)$ can be made larger than the source resistances so that the loading of the signal sources does not occur.

9. (a) What are the properties of feedback amplifiers?

(b) Draw a current-series feedback amplifier circuit and derive its overall gain.

(c) What type of feedback is necessary for oscillation and why?

(d) What are the advantages of Wien-bridge oscillator? [Model Question]

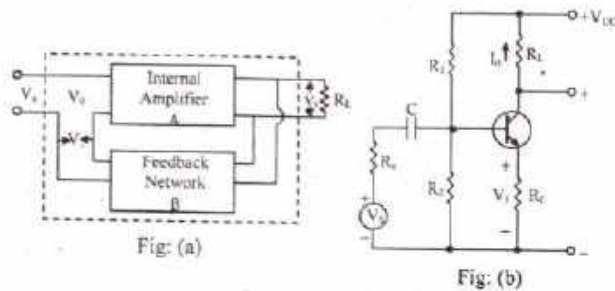
Answer:

a) Properties of feedback amplifier:

- Can increase or decrease input impedance (depending on type of feedback)
- Can increase or decrease output impedance (depending on type of feedback)
- Reduces distortion (increases linearity)
- Increases the bandwidth
- Desensitizes gain to component variations
- Can control step response of amplifier

b) Current-Series Feedback:

In Fig. (a) a block diagram of a current-series feedback topology is shown. As can be seen it is a series-series (SS) circuit.



Here a part of the Fig. (a) output circuit is made to feed back a proportional voltage in series with the input. Since it is a series pickup and a series feedback, both input and output impedances of the amplifier are increased due to feedback. The circuit shown in Fig. (b) shows one of current-series feedback. Here the feedback voltage is the voltage V_f across the unbypassed emitter resistor R_e . Neglecting the base current we get, $V_f = -R_e I_o$, where I_o is the output collector current. The sampled signal is the current I_o and the feedback signal is the voltage V_f . Hence the circuit is referred to as the current series feedback. Here the feedback is negative.

c) The use of positive feedback is useful for producing oscillators.

The condition for positive feedback is that a portion of the output is combined in phase with the input. For an amplifier with positive feedback the gain is given by the expression below.

The large open loop gain of an op-amp makes it inevitable that the condition $A_o B = 1$ will be reached, and the gain expression

$$V_{out} = A_o V'$$

$$V_{out} = A_o (V_{in} + B V_{out}) \quad [\text{as, } V' = V_{in} + B V_{out}]$$

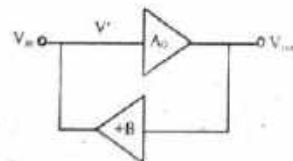
$$V_{out} = [1 - A_o B] = A_o V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_o}{1 - A_o B}$$

$$V_{out} = V_{in} \left(\frac{A_o}{1 - A_o B} \right)$$

$$\text{If, } V_{out} = V_{in} \frac{A_o}{1 - 1}$$

V_{out} becomes infinite.



Practically speaking, the gain which applies at low signal amplitudes will be reduced until the output amplitude reaches some constant value. However, that limiting value will be independent of input, allowing above expression.

d) Wien Bridge Oscillator's advantages:

1. Provides a stable low distortion sinusoidal output over a wide range of frequency.
2. The frequency range can be selected simply by using decade resistance boxes.
3. The frequency of oscillation can be easily varied by varying capacitances C_1 and C_2 simultaneously. The overall gain is high because of two transistors.

Chapter 7: Overview of Digital Electronics

Chapter at a glance:

7.1 Boolean Operation (Boolean Theorems & Identification)

7.1.1 Commutative law

1. $A+B = B+A$
2. $A.B = B.A$
3. $A \oplus B = B \oplus A$
4. $A \odot B = B \odot A$

7.1.2 Associative law

1. $A+(B.C) = (A+B).C$
2. $A.(B.C) = (A.B).C$
3. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

Note: NAND & NOR function are not associative.

7.1.3 Distributive law

1. $(B+C).A = A.B+A.C$
2. $A+(B.C) = (A+B).(A+C)$

$a(b+c) = ab + ac$ This is called the Distributive Law. Distributivity is a property of binary operations that generalizes the distributive law.

7.1.4 Double Negation law

1. $\overline{\overline{A}} = A$

7.1.5 Identity law

1. $A.A = A$
2. $A+A = A$
3. $A+\overline{A} = 1$
4. $A.\overline{A} = 0$

7.1.6 Redundancy law

1. $A+A.B = A$
2. $A.(A+B) = A$

7.2 Demorgan's Theorem

Statement 1:

It is state that complement of the sum of variable is equal to the product of the complements of the variable.

Proof: i.e. $\overline{A+B} = \overline{A}.\overline{B}$

A	B	$\overline{A}$	$\overline{B}$	A+B	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

From the truth table we see that $\overline{A+B} = \overline{A}.\overline{B}$

Statement 2:

It state that the complement of the product of variable is equal to the sum of the complements of the variable.

Proof:

i.e. $\overline{A.B} = \overline{A}+\overline{B}$

A	B	$\overline{A}$	$\overline{B}$	A.B	$\overline{A.B}$	$\overline{A}+\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

From the truth table we see that $\overline{A.B} = \overline{A}+\overline{B}$

The various standard forms for logical functions are:

- (i) Standard Sum of Product Form.
- (ii) Standard Product of Sum Form.

7.3 Logic Gate

Logic gate is an electronic circuit, which makes logic decisions.

- It has one output and one or more inputs.
- The output signal appears only for certain combinations of input signals.

Types of Gates

- OR Gate, AND Gate, NOT Gate, NOR Gate, NAND Gate, EX-OR Gate /XOR Gate / Exclusive OR Gate,
- XNOR Gate / EX-NOR Gate/ Exclusive NOR Gate.

There are just three basic gates- AND gate, OR gate, and NOT gate.

Truth Table: This table gives the output of all possible input combinations.

7.3.1 OR Gate

The OR gate, which is composed of two or more inputs and a single output, performs logical addition.

Symbol of OR gate:  Logic Equation: $Y = A + B$	Truth table: <table><tr><th>A</th><th>B</th><th>A+B</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	A+B	0	0	0	0	1	1	1	0	1	1	1	1
A	B	A+B														
0	0	0														
0	1	1														
1	0	1														
1	1	1														

7.3.2 AND Gate

The AND gate, which is composed of two or more inputs and a single output, performs logical multiplication

Symbol of AND gate:  Logic Equation: $Y = A.B$	Truth table: <table border="1"><tr><th>A</th><th>B</th><th>A.B</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	A.B	0	0	0	0	1	0	1	0	0	1	1	1
A	B	A.B														
0	0	0														
0	1	0														
1	0	0														
1	1	1														

7.3.3 NOT Gate

The inverter (NOT circuit) performs a basic logic function called inversion or complementation.

Symbol of NOT gate: Logic Equation: $A = \bar{A}$	Truth table: <table> <tr> <th>A</th><th>$\bar{A}$</th></tr> <tr> <td>0</td><td>1</td></tr> <tr> <td>1</td><td>0</td></tr> </table>	A	$\bar{A}$	0	1	1	0
A	$\bar{A}$						
0	1						
1	0						

7.3.4 NOR Gate

The NOR gate, which is composed of two or more inputs and a single output, also has a universal property. The term NOR is formed by the concatenation NOT-OR and implies an OR function with an inverted output.

Symbol of NOR gate  Logic Equation: $Y = \overline{A + B}$	Truth table: <table><tr><th>A</th><th>B</th><th>$\overline{A + B}$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	$\overline{A + B}$	0	0	1	0	1	0	1	0	0	1	1	0
A	B	$\overline{A + B}$														
0	0	1														
0	1	0														
1	0	0														
1	1	0														

7.3.5 NAND Gate

The NAND, which is composed of two or more inputs and a single output, is a very popular logic element because it may be used as a universal function. The term NAND is formed by the concatenation NOT-AND and implies an AND function with an inverted output.

Symbol of NAND gate:  Logic Equation: $Y = \overline{A \cdot B}$	Truth table: <table><tr><th>A</th><th>B</th><th>$\overline{A \cdot B}$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	$\overline{A \cdot B}$	0	0	1	0	1	1	1	0	1	1	1	0
A	B	$\overline{A \cdot B}$														
0	0	1														
0	1	1														
1	0	1														
1	1	0														

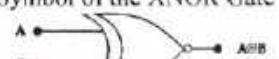
7.3.6 EX-OR Gate

The exclusive-OR or XOR gate has two or more inputs. In general, an XOR gate gives an output value of 1 when there are an odd number of 1's on the inputs to the gate.

Symbol of the XOR Gate  Logic Equation: $Y = A \oplus B = \overline{A}B + A\overline{B}$	Truth Table: <table><tr><th>A</th><th>B</th><th>$A \oplus B$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	$A \oplus B$	0	0	0	0	1	1	1	0	1	1	1	0
A	B	$A \oplus B$														
0	0	0														
0	1	1														
1	0	1														
1	1	0														

7.3.7 EX-NOR Gate

The exclusive-NOR or XNOR gate has two or more inputs. The output is equivalent to inverting the output from the exclusive-OR gate described above. In general, an XNOR gate gives an output value of 1 when there are an even number of 1's on the inputs to the gate.

Symbol of the XNOR Gate  Logic Equation: $Y = A \odot B = \overline{A \oplus B} = \overline{A}B + A\overline{B}$	Truth Table: <table><tr><th>A</th><th>B</th><th>$\overline{A \oplus B} = A \odot B$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	$\overline{A \oplus B} = A \odot B$	0	0	1	0	1	0	1	0	0	1	1	1
A	B	$\overline{A \oplus B} = A \odot B$														
0	0	1														
0	1	0														
1	0	0														
1	1	1														

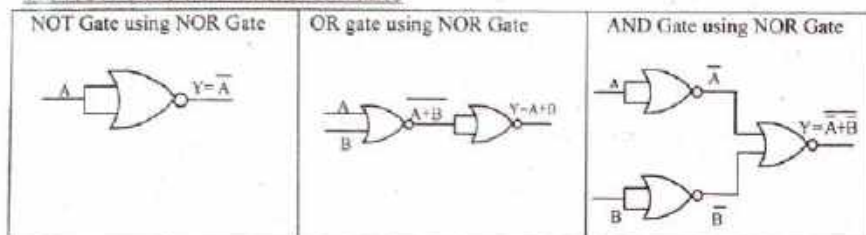
7.4 Universal Gates

By this gate all the Boolean function can be implemented i.e. all the digital system can be implemented. NAND Gate and NOR Gate are two universal gates.

NAND Gate as a Universal Gate

NOT Gate using NAND Gate: 	AND gate using NAND Gate: 	OR Gate using NAND Gate:
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7.5 NOR Gate as a Universal Gate



7.6 K-map

The Karnaugh map is a tool to facilitate management of Boolean algebraic expressions. A Karnaugh map is unique in that only one variable changes value between squares, in other words, the rows and columns are ordered according to the principles of Gray code.

- Basically K-map is a graphical method for minimizing the Boolean expression. There is a one to one map between truth table rows with respective minterms or maxterms.

7.6.1 Steps for Solving K-map

Step 1 Convert Boolean expression in the sum of minterms or maxterms.

Step 2 Find No. of cell in K-map by using formula cell No. 2^n , n is the No. of variable in Boolean expression.

Variable	2	3	4
Rows	2	2	4
Columns	2	4	4

Step 3 Draw the K-map and write down the number in following manners.

AB \ CD	$\bar{C}\bar{D}(00)$	$\bar{C}D(01)$	$CD(11)$	$C\bar{D}(10)$
$\bar{A}\bar{B}(00)$	0	1	3	2
$\bar{A}B(01)$	4	5	7	6
$AB(11)$	12	13	15	14
$A\bar{B}(10)$	8	9	11	10

Step 4 Write 1 for minterms or 0 for the maxterms in the respective cell which presents in the standard SOP or POS of the Boolean expression. e.g. Let a Boolean expression in a SOP form be $(A B C D) = \sum m(0, 1, 6, 7, 11, 14)$

AB \ CD	00	01	11	
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Step 5 Grouping

The most important thing in a K-map is grouping. We have to choose octets, quads & pairs in such a manner, which can form the largest group.

Octet: An octet is a group of 8 adjacent 1 (for SOP) or 0 (for POS).

Quad: A quad is a group of 4 adjacent 1 (for SOP) or 0 (for POS).

Pair: A pair is a group of 2 horizontal or vertical adjacent of 1 after completing the largest possible grouping if any 1 (for minterms) and 0 (for maxterms) remains encircle each of them.

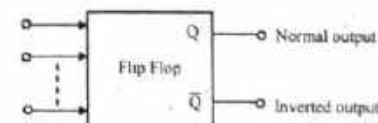
Step 6 Eliminate any redundant group.

Step 7 Write the Boolean equation corresponding to the encircled group.

7.7 Flip-Flop

Flip-flop is made up of logic gates and it permits information to be stored in it. Figure shows a general type of symbol used for a flip-flop.

Block diagram



Flip Flop symbol

Explanation

Output States

$Q=1, \bar{Q}=0$: It is called High or 1 state also called Set state.

$Q=0, \bar{Q}=1$: It is called Low or 0 state also called CLEAR or Reset state.

7.8 Latch

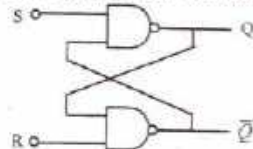
A latch is the most basic type of flip-flop circuit. It can be constructed using NAND or NOR gates.

Type of Latches

1. NAND gate latch
2. NOR gate latch.

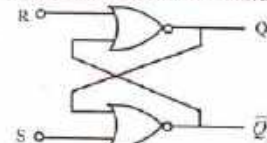
Logic diagram

NAND Gate Latch / S R Latch



Logic diagram

NOR Gate Latch / R S Latch

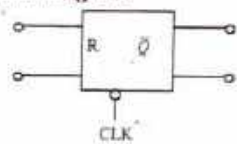


Logic diagram

7.9 RS Flip-Flop

RS Flip-flop used as restoring information. It has two stable states, which can be achieved by giving proper inputs to R and S inputs. The flip-flop will assume one of its two stable states depending upon any asymmetry in the circuit.

Block diagram



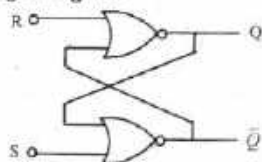
Block diagram

Truth Table

Inputs		Outputs		Mode
S	R	Q	$\bar{Q}$	
0	0	Q_0	$\bar{Q}_0$	No change
0	1	0	1	Reset
1	0	1	0	Set
1	1	-	-	Prohibited

(Q_0 is the o/p of previous state)

Logic diagram

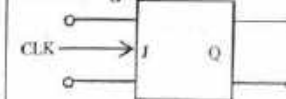


Logic diagram

7.10 JK Flip-Flop

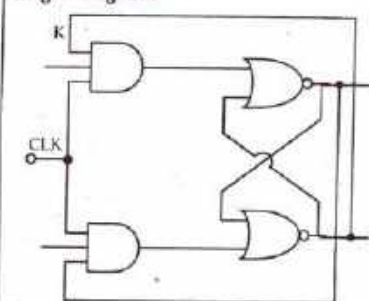
JK flip-flop is modification of SR flip-flop. Here we overcome the race condition which occur in JK flip-flop. JK flip-flop is called an universal flip-flop because the other flip-flops like D, R-S and T can be derived from it.

Block diagram



Block diagram of JK Flip-Flop

Logic diagram



Logic diagram

Truth Table

J	K	C	Q	$\bar{Q}$	Operation
0	0	Rising edge	Q_0	$\bar{Q}_0$	Hold (no change)
0	1	Rising edge	0	1	Reset
1	0	Rising edge	1	0	Set
1	1	Rising edge	$\bar{Q}_0$	Q_0	Toggle

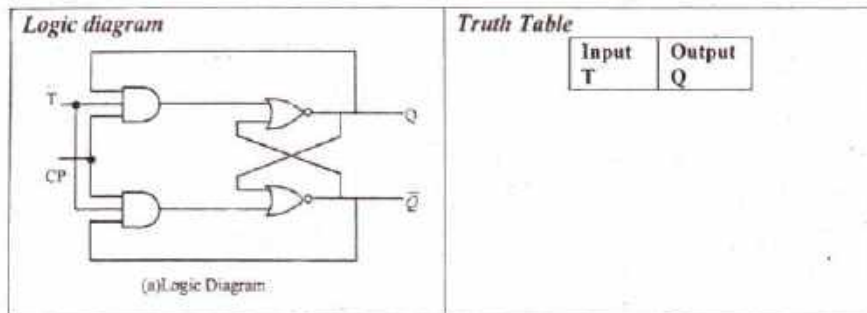
7.11 D Flip-Flop

Basic Idea

D Flip-Flop is a flip-flop with a delay (D) equal to exactly one cycle of the clock. In D flip-flop input to R is through an inverter from S, so that input to R is always the complement of S and never same.

7.12 T Flip-Flop

A T Flip-flop has a single control input, labeled T for toggle. When T is HIGH, the flip-flop toggles on every new clock pulse. When T is LOW, the flip-flop remains in whatever state it was before. Although T flip-flop are not widely available commercially, it is easy to convert a J-K flip-flop to the functional equivalent of a T flip-flop by just connecting J and K together and labeling the common connection as T. Thus when $T = 1$, we have $J = 1$ $K = 1$, and flip-flop toggles. When $T = 0$ we have $J = K = 0$, and there is no change.



7.13 Counter

A counter is a special kind of register, designed to count the number of clock pulses arriving at an input.

In the digital equipments, counters are used for measurement frequency and time period, sequencing of equipment operation, frequency division and manipulation. Counters are composed of flip-flop or ICs.

Applications of counter

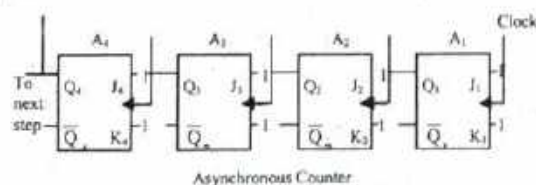
Counter can be used to count clock pulses. Few important applications are given below:

- Measurement of frequency
- Digital voltmeter
- Direct counting of objects moving at a high speed
- Measurement of time interval between two events
- A/D converters used to count and store digital output, which is obtained from analog signal.
- Measurement of time interval between two events.

7.13.1 Asynchronous Counter

When the flip-flops are connected serially and output of preceding flip-flop clocks the succeeding flip-flop, it is asynchronous counter as the change of states occurs one after another. In asynchronous counter the time delay is high than the synchronous counter, so frequency is low than the sequential counter.

Block diagram

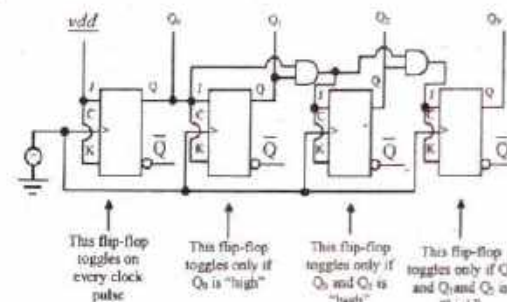


7.13.2 Synchronous Counter

In synchronous counter all flip-flops are clocked simultaneously with the clock's input connected in parallel to clock pulse.

This increases the speed of operation of the counter. The pulse to be counted is applied at the clock input terminals.

Block Diagram



7.14 Logical Families

Digital systems are mostly designed by combining various logic functions. All these logic circuits are available in IC modules and are divided into many 'families'. Each family is classified by abbreviations that indicate the type of logic circuit used. They are of the following

- Resistance – Transistor Logic (RTL),
- Diode Transistor Logic (DTL),
- Transistor – Transistor Logic (TTL),
- Emitter Coupled Logic (ECL),
- Integrated – Injection Logic (I²C),
- Complementary metal oxide semiconductor (CMOS) etc.

Short Answer Type Questions

- Which of these sets of logic gates are known as universal gates? [WBSCTE 2022]
 (a) XOR, NAND, OR (b) OR, NOT, XOR
 (c) NOR, NAND, XNOR (d) NOR, NAND

Answer: (c)

- A digital circuit that can store only 1 bit information [WBSCTE 2022]
 (a) NOR gate (b) XOR gate (c) Flip flop (d) AND gate

Answer: (c)

3. The logical sum of two or more than two logical products is termed as [WBSCTE 2022]
 (a) OR operation (b) POS (c) SOP (d) NAND operation
Answer: (c)
4. The dual of the Boolean theorem $A.(B+C)$ is – [Model Question]
 (a) $A + (B.C) = A.B + A.C$ (b) $A.(B+C) = (A+B)(A+C)$
 (c) $A + (B.C) = (A+B).(A+C)$ (d) none of these
Answer: (c)
5. The universal gate is- [Model Question]
 (a) NAND gate (b) OR gate
 (c) AND gate (d) None of the above
Answer: (a)
6. The inverter is - [Model Question]
 (a) NOT gate (b) OR gate
 (c) AND gate (d) None of the above
Answer: (a)
7. The inputs of a NAND gate are connected together. The resulting circuit is – [Model Question]
 (a) OR gate (b) AND gate
 (c) NOT gate (d) None of the above
Answer: (d)
8. The NOR gate is OR gate followed by- [Model Question]
 (a) AND gate (b) NAND gate
 (c) NOT gate (d) None of the above
Answer: (c)
9. The NAND gate is AND gate followed by [Model Question]
 (a) NOT gate (b) OR gate
 (c) AND gate (d) None of the above
Answer: (a)
10. When signed numbers are used in binary arithmetic, then which one of the following notations would have unique representation of zero – [Model Question]
 (a) magnitude (b) 1's complement (c) 2's complement (d) none
Answer: (a)

11. A logic circuit that provides a HIGH output for both inputs HIGH or both inputs LOW is a(n) – [Model Question]
 (a) EX-NOR gate (b) OR gate (c) EX-OR gate (d) NAND gate
Answer: (a)
12. A logic circuit that provides a HIGH output if one input or the other input, but not both, is a HIGH, is a(n) [Model Question]
 (a) EX-NOR gate (b) OR gate
 (c) EX-OR gate (d) NAND gate
Answer: (c)
13. Identify the type of gate below from the equation $X = A \oplus B = \overline{A}B + A\overline{B}$. [Model Question]
 (a) EX-NOR gate (b) OR gate
 (c) EX-OR gate (d) NAND gate
Answer: (c)
14. Which type of gate can be used to add two bits? [Model Question]
 (a) EX-OR (b) EX-NOR (c) EX-NAND (d) NOR
Answer: (a)
15. Why is an exclusive-NOR gate also called an equality gate? [Model Question]
 (a) The output is false if the inputs are equal
 (b) The output is true if the inputs are opposite
 (c) The output is true if the inputs are equal
Answer: (b)
16. A NAND gate is called a universal gate because – [Model Question]
 (a) many computers use this gate,
 (b) any function can be realized by this gate,
 (c) all minimization techniques are applicable to this gate
Answer: (b)
17.  [Model Question]
 This figure is the symbol of a –
 (a) OR gate, (b) NOR gate, (c) XOR gate.
Answer: (c)
18. The minimum number of NAND gates required to realize a XOR gate is – [Model Question]
 (a) 4 (b) 5 (c) 6
Answer: (b)

19. The first contribution to logic was made by – [Model Question]
 (a) George Boole (b) Copernicus (c) Aristotle (d) Shannon
 Answer: (a)
20. Which logic gate is similar to the function of two series switches? [Model Question]
 (a) AND (b) OR (c) XOR (d) NAND
 Answer: (a)
21. The minimum number of NAND gates required to implement $A+AB'+AB'C$ is equal to – [Model Question]
 (a) 0 (b) 1 (c) 2 (d) 4
 Answer: (a)
22. The Boolean expression $AB+AB'+AC+AC$ is unaffected by the value of the Boolean variable – [Model Question]
 (a) A (b) B (c) C (d) all of these
 Answer: (d)
23. $A+B$ can be implemented by using – [Model Question]
 (a) AND gates only (b) XOR gates only
 (c) NAND gates only (d) none of the above
 Answer: (c)
24. The dual of $x.0=0$ is – [Model Question]
 (a) $x+1=1$ (b) $x+0=x$ (c) $x+x=x$ (d) $x.0=0$
 Answer: (a)
25. Which of the following gate can be used as a controlled inverter? [Model Question]
 (a) AND (b) OR (c) XOR (d) NAND
 Answer: (c)
26. Any combinational circuit can be designed using – [Model Question]
 (a) only NAND (b) AND and OR (c) only AND
 Answer: (a)
27. Exponent in floating point number can be represented by – [Model Question]
 (a) Sign-magnitude form, (b) 1's complement from,
 (c) 2's complement from, (d) biased form,
 Answer: (a)

28. A subtractor is not usually present in a computer because – [Model Question]
 (a) It is expensive (b) It is not possible to design it
 (c) The adder will take care of subtraction (d) None of the above
 Answer: (c)
29. Which of the following 4-bit combination is invalid BCD? [Model Question]
 (a) 0101 (b) 1010 (c) 1001 (d) 0011
 Answer: (b)
30. The expression $(\bar{A} + AB)$ can be written as – [Model Question]
 (i) $\bar{A}$ (b) B (c) $\bar{A} + B$ (d) A
 Answer: (c)
31. The dual of $(AB + CD)$: [Model Question]
 (a) $(A + B)(C + D)$ (b) $(A + B + C + D)$
 Answer: (a)
32. Laws of tautology states - [Model Question]
 (a) $A.A=A$ and $A=\bar{A}=A$ (b) $\bar{A}.A=A$ and $A+A=\bar{A}$
 (c) $A.A=A$ and $A+A=A$
 Answer: (c)
33. Minimization of Boolean logical expressions helps to reduce – [Model Question]
 (a) Space (b) Cost (c) Number of gate (d) All of these
 Answer: (d)
34. Which of the following does not represent Boolean algebra operation? [Model Question]
 (a) AND (b) OR (c) NOT (d) XOR
 Answer: (d)
35. Tri-state buffer has – [Model Question]
 (a) 3 output states (b) 2 output states (c) 1 output states (d) no output states
 Answer: (a)
36. A multiplexer with 4-bit data select input is a – [Model Question]
 (a) 4:1 multiplexer (b) 8:1 multiplexer
 (c) 16:1 multiplexer (d) 32:1 multiplexer
 Answer: (c)

37. The number of clock pulses required to completely load serially a 8-bit shift register is — [Model Question]
 (a) 8 (b) 4 (c) 7 (d) 5

Answer: (a)

38. The minimum number of flip-flops required for a synchronous decade counter is — [Model Question]
 (a) 2 (b) 1 (c) 10 (d) 4

Answer: (d)

39. Race around condition occurs in J-K flip-flop if [Model Question]
 (a) $J=0, K=0$ (b) $J=0, K=1$ (c) $J=1, K=0$ (d) $J=1, K=1$

Answer: (d)

40. Which flip-flop does not have Race problem?— [Model Question]
 (a) SR (b) JK (c) T (d) JK master-slave

Answer: (d)

B. State whether the following statements are True or False:

41. $A + (B.C) = (A + B) (A + C)$. [Model Question]

Answer: True

42. $A + AB = A$ [Model Question]

Answer: True

43. $\overline{A+B+C} = \overline{A}.\overline{B}.\overline{C}$. [Model Question]

Answer: False

44. According to Boolean laws $A + A = 1$ [Model Question]

Answer: False

45. The dual of $A + 1 = 1$ is $A' + 0 = 0$ [Model Question]

Answer: False

[dual is $A.0 = 0$]

46. In 2's complement representation, MSB is the sign bit. [Model Question]

Answer: True

47. Decimal number 90 equals 01011010 in 2's complement [Model Question]

Answer: True

48. A 2's complement adder can add and subtract. [Model Question]

Answer: True

49. A 4-variable Karnaugh map for SOP form has 16 cells. [Model Question]
 Answer: True

50. In a Karnaugh map, adjacent pair eliminates single variable. [Model Question]
 Answer: True

51. Max-term expression directly yields OR-AND logic circuit. [Model Question]
 Answer: True

52. A NOR gate is equivalent to bubble input AND gate. [Model Question]
 Answer: True

53. A NAND gate is equivalent to bubble input OR gate. [Model Question]
 Answer: True

54. If any input is low in an AND gate, the output is high. [Model Question]
 Answer: False

55. In a NOT gate the output is equal to magnitude to input. [Model Question]
 Answer: False

56. A two input XNOR gate recognizes these words which has similar inputs. [Model Question]
 Answer: True

57. XOR gate is used as a controlled inverter. [Model Question]
 Answer: True

58. NAND and NOR gates are called universal gates. [Model Question]
 Answer: True

59. Universal gates consume more power than other gates. [Model Question]
 Answer: False

60. XOR gate is used both for parity generating and checking purposes [Model Question]
 Answer: True

61. A flip-flop is a memory element [Model Question]
 Answer: True

62. In a JK Master Slave, the race condition does not occur. [Model Question]
 Answer: True

63. In SR latch, $R = 1$, $S = 1$ is called the race condition [Model Question]
 Answer: True

64. A T flip-flop is obtained by connecting J and K inputs of JK flip flop. [Model Question]
 Answer: True

65. A shift register can convert serial data in parallel data and vice versa. [Model Question]
 Answer: True

66. A J-K master-slave flip-flop has two clocked flop flops. [Model Question]
 Answer: True

67. Divide-by-2 is an inherent capability of D flip-flop. [Model Question]
 Answer: False

68. In a D flip-flop 'D' stands for duration. [Model Question]
 Answer: False

C. Fill in the blanks in the following statements:

69. The minimum number of flip-flops required for a decade counter is 4. [Model Question]

70. A ripple counter is Asynchronous sequential circuit [Model Question]

71. The minimum number of flip-flops required for a Mod 11 counter is 4. [Model Question]

72. A modulus counter can be used for frequency division. [Model Question]

D. Answer the following questions:

73. What is the function of clock in Mod-N counter? [Model Question]
 Answer:

We can use Mod-N counter as a frequency divider. A mod-N counter has N repeating states and its output is taken to be 1 only during one state and hence its frequency is F/N .

74. What are differences between positive logic and negative logic? [Model Question]
 Answer:

There are two types of representations used in digital systems, the positive logic and the negative logic representations

In positive logic representation Bit 1 represents Logic high and Bit 0 represent a Logic low as shown. High is represented by +5 Volts and low is represented by -5 Volts or 0 Volts.

In Negative logic representation Bit 1 represents logic low and Bit 0 represents logic high as shown. In terms of voltage level, bit 1 can be represented as +5V and bit 0 can be represented as 0 V or -5 Volts.

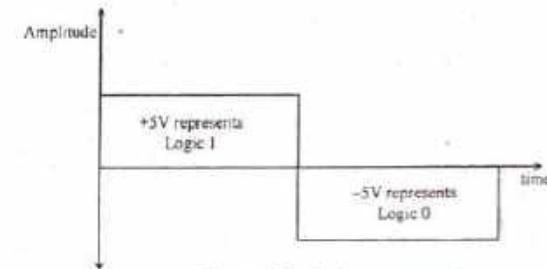


Fig: Positive logic representation

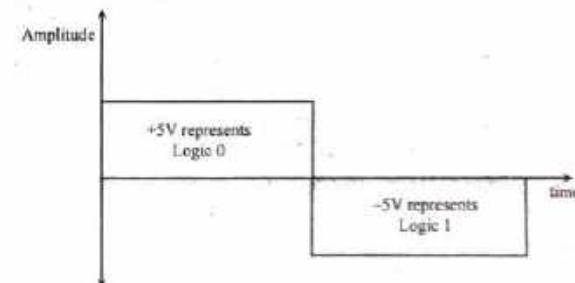


Fig: Negative logic representation

75. Which input values in a J-K flip flop causes race around condition in it? [Model Question]

Answer:

No race is found for any input combinations only for $J = 1$ and $K = 1$ output toggles.

76. What are the asynchronous inputs to flip-flop? [Model Question]

Answer:

In the case of S-R F/F the asynchronous inputs are S (set) and R (Reset). In the case of J-K F/F the asynchronous inputs are J and K, in D-F/F the asynchronous input is D.

77. For synchronous operation of flip-flop, what will be values of asynchronous inputs? [Model Question]

Answer: 1 and 0 continuously.

78. What is the characteristic equation of a T-flip flop? [Model Question]

Answer:

$$Q_{i+1} = TQ' + T'Q$$

Where Q_{i+1} = final O/P

Q_i = output at i^{th} clock.

79. The initial state of a MOD-16 down counter is 0110. After 37 clock pulses, what will be state of the counter? [Model Question]

Answer:

As it is a MOD-16 counter the output repeats after 16 clock. So the output after 37 clock pulse is same as $(37-2 \times 16) = 5$ clock.

As it is down counter so the output is 0001.

80. A 4-bit binary ripple counter uses flip-flops with propagation delay time of 25 ns. What will be the maximum possible time required for change of state? [Model Question]

Answer:

As the counter is of 4-bit so total number of F/F is 4, so maximum possible delay is $(4 \times 25) = 100$ ns.

81. What is an asynchronous sequential circuit. [Model Question]

Answer:

An asynchronous sequential circuit is that circuit whose O/P is a function of present inputs and past outputs and each block does not associated with a single clock. The functional blocks do not operate at the same time.

82. What is the purpose of latch? [Model Question]

Answer:

The purpose of the latch is to keep the output at the same level after the removal of the input from the input terminal. The output of the latch changes only when a new input arrives on the input terminal.

83. What is race around condition? [Model Question]

Answer:

When we put $J=1$ and $K=1$ in a level triggered J-K flip flop, and the clock is high the output, Q toggles to 0 and 1 continuously; and it becomes uncertain to predict the output. This condition is known as Race around condition.

84. Write down the name of two applications of flip-flop? [Model Question]

Answer:

- Flip-flops are used in counters.
- Flip-flops are used in shift-registers.

85. What is decade counter? [Model Question]

Answer:

A decade counter is a counter that has 10 states.

86. Which type of multivibrator can be used as flip-flop? [Model Question]

Answer:

Bistable Multivibrator

87. How many flip-flops are required to construct a MOD-32 asynchronous counter? [Model Question]

Answer:

5 flip-flops are required to construct a MOD-32 asynchronous counter.

88. How many flip-flops are required to implement a Mod 126 counter? [Model Question]

Answer:

7 flip-flops are required to implement a Mod 126 counter.

Long Answer Type Questions

1. (a) State Demorgans's theorem.

(b) Draw the NOR gate & write down its truth table.

[WBSCTE 2022]

Answer:

a) De Morgan has suggested two theorems which are extremely useful in Boolean Algebra. The two theorems are discussed below.

Theorem 1

The left hand side (LHS) of this theorem represents a NAND gate with inputs A and B, whereas the right hand side (RHS) of the theorem represents an OR gate with inverted inputs.

Theorem 2

The LHS of this theorem represents a NOR gate with inputs A and B, whereas the RHS represents an AND gate with inverted inputs.

b) The NOR gate, which is composed of two or more inputs and a single output, also has a universal property. The term NOR is formed by the concatenation NOT-OR and implies an OR function with an inverted output.

Symbol of NOR gate		Truth table:		
	$\overline{A+B}$	A	B	$A+B$
		0	0	1
		0	1	0
		1	0	0

Logic Equation: $Y = A + B$

1	0	0
1	1	0

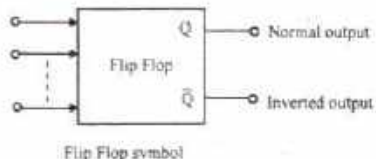
➤ 2. Define Flip-flop.

[WBSC TE 2022]

Answer:

Flip-flop is made up of logic gates and it permits information to be stored in it. Figure shows a general type of symbol used for a flip-flop.

Block diagram



Explanation

Output States

$Q=1, \bar{Q}=0$: It is called High or 1 state also called Set state.

$O=0, \overline{O}=1$: It is called Low or 0 state also called CLEAR or Reset state.

3. (a) Draw the logic diagram of an Ex-OR gate and discuss its operation.

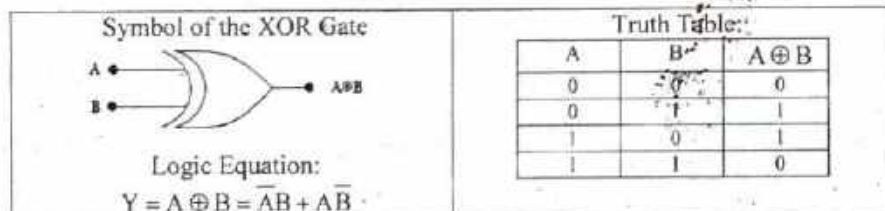
(b) Prove that:

$$\overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} = C$$

[Model Question]

Answer:

a) The exclusive-OR or XOR gate has two or more inputs. In general, an XOR gate gives an output value of 1 when there are an odd number of 1's on the inputs to the gate.



b) $\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C = (\text{L.H.S.})$
 $= \overline{A}\overline{C}(\overline{B} + B) + AC(\overline{B} + B)$
 $= \overline{A}\overline{C} \cdot 1 + AC \cdot 1 \quad [\because A + \overline{A} = 1, B + \overline{B} = 1]$
 $= \overline{A}\overline{C} + AC = C = \text{R.H.S.}$
 $\therefore \overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C = C \quad [\text{Proved}]$

[Model Question]

➤ 4. Prove the following:

(ii) $A + \bar{A}.B + A.\bar{B} = A + B$

$$(ii) (A + B)(\bar{A} + C) = AC + \bar{A}B$$

Answer:

(i) **LHS** : $(A + \bar{A})(A + B) + A\bar{B}$ [By distributive law]

$$\Rightarrow (A+B)+A\bar{B}, \text{ Since } A + \bar{A} = 1$$

$$\Rightarrow A + B + \overline{A}\overline{B}$$

$$\Rightarrow A + A\bar{B} + B$$

$$\Rightarrow A(1 + \bar{B}) + B \text{ } [\because 1 + \bar{B} = 1]$$

$$\Rightarrow A + B = RHS$$

(ii) LHS : $A\bar{A} + \bar{A}B + AC + BC$

$$\overline{A}B + AC + BC \quad [\because A.\overline{A} = 0]$$

$$\bar{A}B + AC + BC(\bar{A} + A)$$

$$\bar{A}B + \bar{A}BC + AC + ABC$$

$$\overline{AB} + \overline{ABC} + AC + ABC$$

$$\bar{A}B(1+C) + AC(B+1)$$

$$\overline{AB} + AC = RHS$$

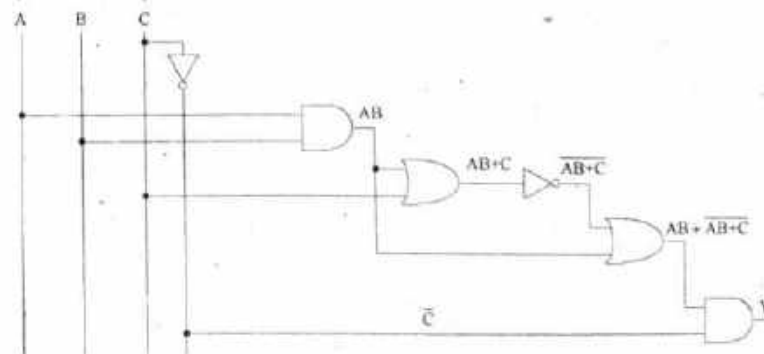
➤ 5. Realize the Boolean expression using basic gates.

[Model Question]

$$Y = \overline{C}(AB + \overline{AB} + C)$$

Answer:

$$Y = \bar{C}(AB + \overline{AB} + C)$$



6. Given the logic equation $F = ABC + \overline{BCD} + \overline{ABC}$. Make a truth table for the same.

[Model Question]

Answer:

$$F = ABC + \overline{BCD} + \overline{ABC}$$

Put the value of A,B,C from the truth table and get the output F

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

7. Prove the following :

$$(i) (AB + C + D)(\overline{C} + D)(\overline{C} + D + E) = ABC + D$$

[Model Question]

Answer:

$$(AB + C + D)(\overline{C} + D)(\overline{C} + D + E)$$

$$= (ABC + C\overline{C} + \overline{C}D + ABD + CD + D)(\overline{C} + D + E)$$

$$[\because C\overline{C} = 0 \text{ \& } ABD + CD + D = D]$$

$$= (ABC + \overline{C}D + D)(\overline{C} + D + E)$$

$$= \{ABC + D(\overline{C} + 1)\}(\overline{C} + D + E)$$

$$= (ABC + D)(\overline{C} + D + E)$$

$$= ABC + \overline{C}D + ABCD + D + ABC\overline{E} + ED$$

$$= ABC(1 + D + E) + D(\overline{C} + 1 + E)$$

$$= ABC + D \quad [\because 1 + D + E + 1 + E + 1 = 1]$$

$$= RHS$$

$$(ii) \overline{A}BC + \overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC = AB + BC + CA$$

Answer:

$$\overline{A}BC + \overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC$$

$$= \overline{A}BC + \overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC + ABC + ABC$$

$$[\because ABC + ABC + ABC = ABC]$$

$$= ABC + A\overline{B}\overline{C} + \overline{A}BC + \overline{A}\overline{B}C + ABC + A\overline{B}\overline{C}$$

$$= AB(C + \overline{C}) + BC(A + \overline{A}) + AC(B + \overline{B}) \quad [\because C + \overline{C} = 1, A + \overline{A} = 1 \text{ \& } B + \overline{B} = 1]$$

$$= AB + BC + CA = R.H.S.$$

8. (a) Make a Karnaugh map of the function $F(A, B, C, D) = \sum m(5, 8, 9, 11, 13, 15) + d(7, 12)$ and

(b) Explain the applications of parity bit.

[Model Question]

Answer:

a)

AB \ CD	CD			
	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	2	3	15	14
10	8	9	11	10

$$Y = \overline{A}\overline{C} + AD + BD$$

$$= A(\overline{C} + D) + BD$$

b) A parity bit, or check bit, is a bit added to a string of binary code that indicates whether the number of 1-bits in the string is even or odd. Parity bits are used as the simplest form of error detecting code. There are two variants of parity bits: even parity bit and odd parity bit.

9. Implement the following function by NAND gates only.

[Model Question]

$$Y = \overline{A}BC + A\overline{B}C + A$$

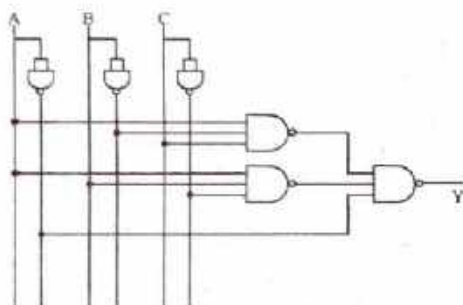
Answer:

$$\text{We know that } \overline{\overline{x} \cdot \overline{y}} = \overline{\overline{x}} + \overline{\overline{y}} = x + y$$

$$\text{So, } Y = \overline{A}BC + A\overline{B}C + A$$

$$= \overline{A}BC + A\overline{B}C + A$$

$$= \overline{A}BC \cdot \overline{A}BC \cdot A$$



10. Show that a positive logic NAND operator is equivalent to a negative logic NOR operation and vice versa. [Model Question]

Answer:

In the positive logic low voltage signal is represented by 0 & high voltage signal is represented by 1 but in negative logic high voltage signal is represented by 0 & low voltage signal is represented by 1.

The truth table of positive logic NAND gate is: The truth table of negative logic NOR gate is:

A	B	$Y_{NAND} = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$Y = \overline{\overline{A} + \overline{B}}$
1	1	0
1	0	1
0	1	1
0	0	1

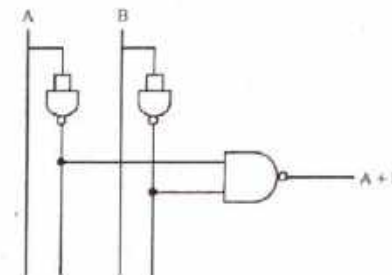
So it can be seen that the both truth table are same, so positive logic NAND operator is equivalent to a negative logic NOR operator.

11. Construct the OR gate equivalent circuit using NAND gates. [Model Question]

Answer:

Implementing OR Using only NAND Gates

An OR gate can be replaced by NAND gates as shown in the figure (The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters).



12. Implement the following function by NAND gate: [Model Question]

$$Y = \overline{A}BC + AB(\overline{C} + A)$$

Answer:

$$Y = \overline{A}BC + AB(\overline{C} + A)$$

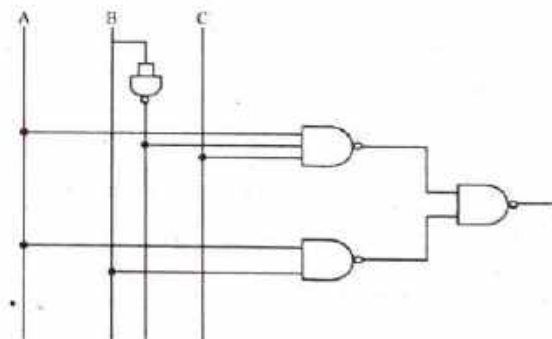
$$= \overline{A}BC + AB\overline{C} + AB$$

$$= \overline{A}BC + AB(\overline{C} + 1)$$

$$= \overline{A}BC + AB$$

We know that, $\overline{\overline{X} \cdot \overline{Y}} = \overline{\overline{X}} + \overline{\overline{Y}} = X + Y$

Using this we can say that –



13. Draw the logic circuit of SR flip flop using NAND gates and explain working. [Model Question]

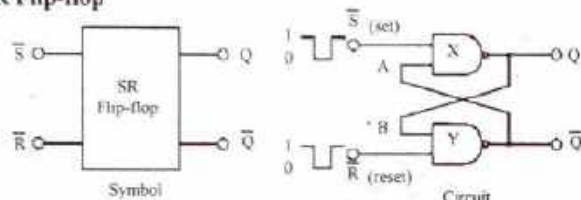
Answer:

The NAND Gate SR Flip-Flop

The simplest way to make any basic single bit set-reset SR flip-flop is to connect together a pair of cross-coupled 2-input NAND gates as shown, to form a Set-Reset Bistable also known as an active LOW SR NAND Gate Latch, so that there is feedback from each

output to one of the other NAND gate inputs. This device consists of two inputs, one called the *Set*, S and the other called the *Reset*, R with two corresponding outputs Q and its inverse or complement Q (not-Q) as shown below.

The Basic SR Flip-flop



The Set State

Consider the circuit shown above. If the input R is at logic level "0" ($R = 0$) and input S is at logic level "1" ($S = 1$), the NAND gate Y has at least one of its inputs at logic "0" therefore, its output Q must be at a logic level "1" (NAND Gate principles). Output Q is also fed back to input "A" and so both inputs to NAND gate X are at logic level "1", and therefore its output Q must be at logic level "0". Again NAND gate principals. If the reset input R changes state, and goes HIGH to logic "1" with S remaining HIGH also at logic level "1", NAND gate Y inputs are now $R = "1"$ and $B = "0"$. Since one of its inputs is still at logic level "0" the output at Q still remains HIGH at logic level "1" and there is no change of state. Therefore, the flip-flop circuit is said to be "Latched" or "Set" with $Q = "1"$ and $Q = "0"$.

Reset State

In this second stable state, Q is at logic level "0", (not $Q = "0"$) its inverse output at Q is at logic level "1", ($Q = "1"$), and is given by $R = "1"$ and $S = "0"$. As gate X has one of its inputs at logic "0" its output Q must equal logic level "1" (again NAND gate principles). Output Q is fed back to input "B", so both inputs to NAND gate Y are at logic "1", therefore, $Q = "0"$. If the set input, S now changes state to logic "1" with input R remaining at logic "1", output Q still remains LOW at logic level "0" and there is no change of state. Therefore, the flip-flop circuits "Reset" state has also been latched and we can define this "set/reset" action in the following truth table.

Truth Table for this Set-Reset Function

State	S	R	Q	Description
Set	1	0	1	Set $Q \gg 1$
	1	1	0	no change
Reset	0	1	0	Reset $Q \gg 0$
	1	1	1	no change
Invalid	0	0	0	memory with $Q = 0$
	0	0	1	memory with $Q = 1$

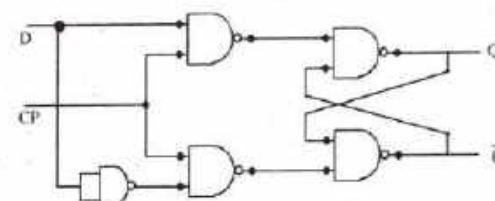
14. What is a D-flip-flop? Draw its circuit diagram and explain its operation. Explain how a S-R flip flop can be converted to D-flip flop. [Model Question]

Answer:

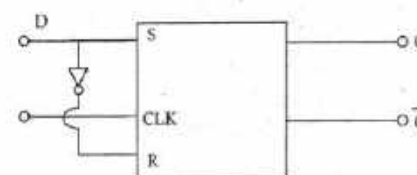
Basic Idea

D Flip-Flop is a flip-flop with a delay (D) equal to exactly one cycle of the clock. In D flip-flop input to R is through an inverter from S, so that input to R is always the complement of S and never same.

Logic diagram



Logic diagram using NAND gates



D flip-flop from the S-R flip-flop

Truth Table

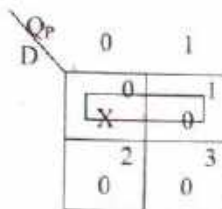
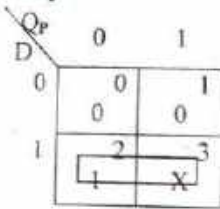
Input	Output	Comments
0	0	RESET
1	1	SET

As shown in the figure, S and R are the actual inputs of the flip flop and D is the external input of the flip flop. The four combinations, the logic diagram, conversion table, and the K-map for S and R in terms of D and Q are shown below.

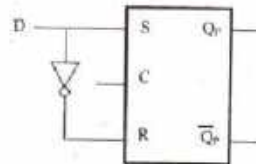
Conversion Table

D Input	Outputs		S-R Inputs	
	Q _n	Q _{n+1}	S	R
0	0	0	0	×
0	1	0	0	1
1	0	1	1	0
1	1	1	×	0

K-maps



Logic Diagram

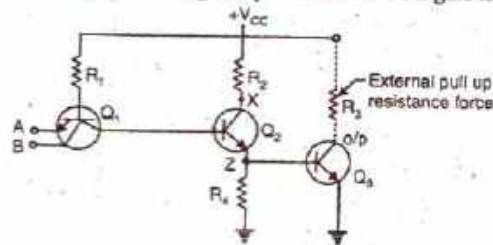


15. Explain with diagram the working of 2 input TTL NAND gate.

[Model Question]

Answer:

The circuit diagram of 2-input NAND gate open-collector TTL gate is as shown:



Working:

Case 1: When $A = 0, B = 0$

When both inputs A and B are low, both functions of Q1 are forward biased and Q2 remains off. So no current flows through R4 and Q3 is also off and its collector voltage is equal to V_{cc} i.e. $Y = 1$

Case 2: When $A = 0, B = 1$ and

Case 3: When $A = 1, B = 0$

When one input is high and other is low, then one junction is forward biased so Q2 is off and Q3 is also off. So collector voltage is equal to V_{cc} i.e. $Y = 1$

Case 4: When $A = 1, B = 1$

When both inputs are high, Q1 is turned off and Q2 turned 'ON' Q3 goes into saturation and hence $Y = 0$. The open-collector output has main advantage that wired AND is possible in it.

16. (a) What is meant by edge triggering? Show how the race around condition is eliminated in master-slave flip-flop?

(b) What is ring counter?

[Model Question]

Answer:

a) **Edge Triggering:** In edge triggering the circuit becomes active at negative or positive edge of the clock signal. For example if the circuit is positive edge triggered, it will take input at exactly the time in which the clock signal goes from low to high. Similarly input is taken at exactly the time in which the clock signal goes from high to low in negative edge triggering. But keep in mind after the input; it can be processed in all the time till the next input is taken.

Race-around condition: The truth table below of the JK flip-flop presupposes that the inputs do not alter during a clock pulse ($CP = 1$). However, because of the feedback connection of the outputs to the inputs, the inputs may change during a clock pulse. Consider, for example, that the inputs are $J = 1, K = 1$ and $Y = 0$, when a clock pulse, as shown in Figure below, is applied simultaneously.

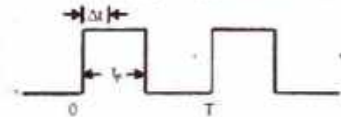


Fig: A clock pulse ($t_p > \Delta t$)

After a time $\Delta t (< t_p)$, the output will change to $Y = 0$, when a Δt equals the propagation delay through the two NAND gates in series (see row seven of Table below). Thus we obtain $J = 1, K = 1$, and $Y = 1$. Considering row eight of Table below, we find that the output Y changes back to 0 after another interval of time Δt . Clearly the output will oscillate back and forth between 0 and 1 when $t_p \gg \Delta t$. Therefore, at the end of the clock pulse, the output Y remains uncertain. This phenomenon is called a race-around condition.

The race-around condition does not occur if $t_p < \Delta t < T$. This inequality is difficult to achieve with IC components which offer propagation delays much less than t_p . A practical solution of the problem is offered by the master-slave configuration, described below.

Table: Detailed truth table of JK flip-flop

J_n	K_n	Y_n	$\bar{Y}_n$	S_n	R_n	Y_{n-1}	Last column
0	0	0	1	0	0	Y_n	Y_n
0	0	1	0	0	0	Y_n	Y_n
1	0	0	1	1	0	1	1
1	0	1	0	0	0	Y_n	Y_n
0	1	0	1	0	0	Y_n	0
0	1	1	0	0	1	0	0
1	1	0	1	1	0	1	$\bar{Y}_n$
1	1	1	0	0	1	0	0

Master-slave JK flip-flop:

The master-slave JK flip-flop, shown in Figure below, is a cascade of two RS flip-flops where the output of the second, called the slave, is feedback to the first, called the master. Positive clock pulses are applied directly to the master and the same after inversion are applied to the slave. The master is enabled when $P_p=1, C_p=1$ and $CP=1$. Then its operation follows Table 1.

Table 1: Reduced truth table of JK flip-flop

J_n	K_n	Y_{n+1}
0	0	Y_n
1	0	1
0	1	0
1	1	$\bar{Y}_n$

Also, since $\overline{CP}=0$, the slave is inhibited and so cannot change state. Thus Y_n remains invariant during the time t_p . Consequently, the race-around condition does not occur.

When $CP=0$, the master is inhibited and the slave is enabled since $\overline{CP}=1$. The slave being an RS flip-flop, then operates in accordance with Table 2. When $S=Y_n=1$ and $R=\bar{Y}_n=0$, then $Y=1$ and $\bar{Y}=0$. Similarly, when $S=Y_n=0$ and $R=\bar{Y}_n=1$, then $Y=0$ and $\bar{Y}=1$. That is, the value of Y_n is transmitted to the output Y in the time interval between the clock pulses.

Table 2: Truth table for clocked RS flip-flop

R_n	S_n	Y_{n+1}
0	0	Y_n
0	1	1
1	0	0
1	1	unpredictable

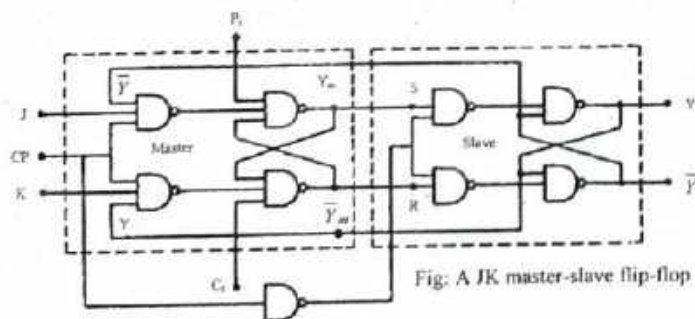


Fig: A JK master-slave flip-flop

b) A ring counter is a type of counter composed of a type of circular shift register. The output of the last shift register is fed to the input of the first register. The hamming distance of an Overbeck counter is 2; the hamming distance of a Johnson counter is 1.

17. What is a modulus counter? Draw the logic diagram of a 4 bit binary ripple counter that trigger on the positive-edge transition and also draw its waveform. [Model Question]

Answer:

a) 1st part:

The number of states through which the counter passes before returning to the starting state is called the modulus of the counter. Hence the modulus of a counter is equal to the total number of distinct states (counts) including zero that a counter can restore. In other words, the number of input pulses the causes the counter to reset to its initial count is called the modulus of the counter.

2nd part:

A 4 bit binary ripple counter can count up to a maximum of 2 states i.e. 16 states, that is why it is also known by other names like four stage or modulo-16 (Mod-16) or divide by 16 ripple counter. It uses four, negative edge triggered type J-K flip-flops. Initially all the four flip-flops A, B, C and D are in logic '0' state. That is Q output of A, B, C and D flip-flops are all in logic '0' state as shown in Fig (a).

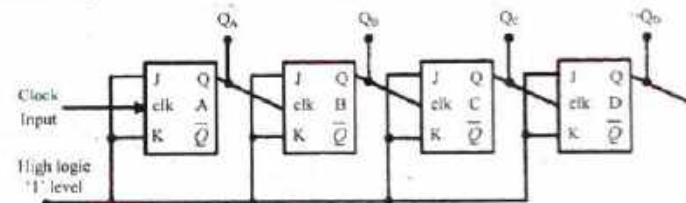


Fig: (a) 4-bit (Mod-16) binary ripple counter

$$Q = Q_D Q_C Q_B Q_A = 0001$$

Now when second clock pulse is applied to flip-flop A, output Q_A changes state from '1' to '0'. Due to this change of state of Q going from '1' to '0', a negative going pulse is created at Q_A , which is connected to clock input to flip-flop B. Thus this negative going pulse triggers the flip-flop B, changing the state of Q from its previous or original state '0' to '1'. At this instant the outputs Q_C and Q_D of flip flops C and D respectively continue to remain in their logic '0' state, and thus the counter reads.

$$Q = Q_D Q_C Q_B Q_A = 0010$$

The counter will continue to count the input clock pulses in the binary form as explained above up to state till Q_D, Q_C, Q_B and Q_A all become high i.e. logic level '1', and counter will read $Q = Q_D Q_C Q_B Q_A = 1111$, which in decimal form means that it will count upto 15

clock pulses. On arrival of the 16th clock pulse, all the four flip-flops A, B, C and D will go to '0' and the counter will once again repeat its counting from 0000 to 1111 i.e. from 0 to 15. thus in this ripple counter, it is seen that output of the first flip-flop is driving the clock of the second flip-flop and output of the second flip-flop drives the clock of the third flip-flop and so on.

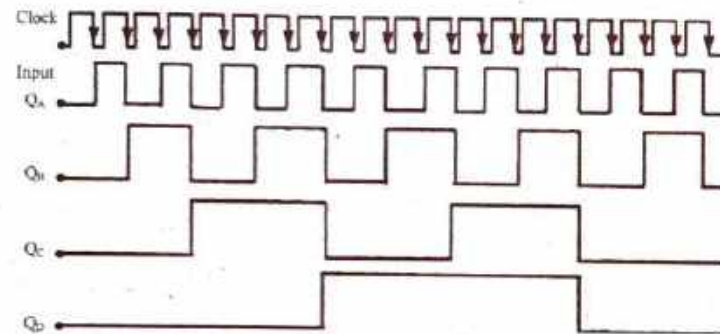
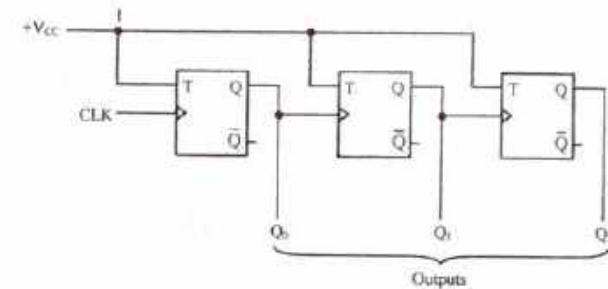


Fig: (b) Modulo-16 (mod-16) ripple counters timing diagram

18. What is ripple counter? Draw a 3 bit ripple counter its waveforms. What is ring counter? [Model Question]

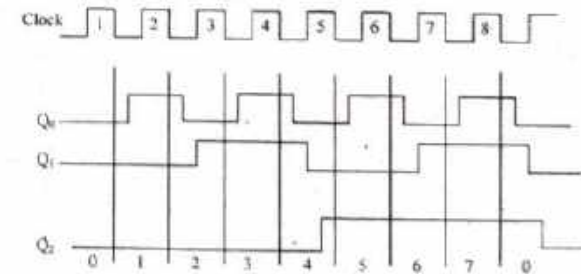
Answer:

A ripple counter is a counter where all the flip flops are not driven by the same clock pulse. Here the clock pulse is applied to the first flip flop. And the successive flip flop is triggered by the output of the previous flip flop. So by this property it is very much clear that ripple counter has cumulative settling time, which limits its speed of operation. As the first stage of the counter changes its state first with the application of the clock pulse to the flip flop and the successive flip flops change their states in turn causing a ripple through effect of the clock pluses. As the signal propagates through the counter in a ripple fashion, it is called a ripple counter. In bellow you will find the block diagram of 3-bit ripple counter.



By 3-bit ripple counter we can count 0-7. Because we know by 3 bit we can represents minimum 0 (000) and maximum 7 (111). The clock inputs of the three flip flops are connected in cascade. The T input of each flip flop is connected to a constant 1, which means that the state of the flip flop will toggle at each negative edge of its clock. Thus the clock input of the first flip flop is connected to the Clock line. The other two flip flops have their clock inputs driven by the Q output of the preceding flip flop. Therefore, they toggle their state whenever the preceding flip flop changes its state from Q = 1 to Q = 0, which results in a negative edge of the Q signal. So as we take the output from Q₀, Q₁, Q₂ then we get the count sequence with different counter state as mention below on table.

Counter State	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1



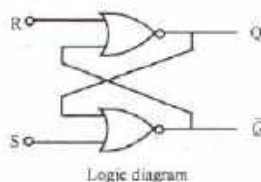
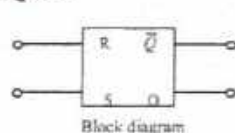
A ring counter is a Shift Register (a cascade connection of flip-flops) with the output of the last flip flop connected to the input of the first. It is initialized such that only one of the flip flop output is 1 while the remainder is 0. The 1st bit is circulated so the state repeats every n clock cycles if n flip-flops are used. The "MOD" or "MODULUS" of a counter is the number of unique states. The MOD of the n flip flop ring counter is n .

➤ 19. Distinguish between latch and flip-flop. Construct a basic latch circuit and explain its operation with truth table. [Model Question]

Answer:

Both flip-flop and latch are bi-stable and are used to store binary data but in flip-flop state change only occurs on a clock edge or pulse whereas in latch, change state occurs without being clocked. So it can be concluded that Flip Flops are clocked but latches are not.

S-R latch is the most basic form of a memory element that is used for storing (also known as latching) of a single bit. S-R latch has two inputs SET(S), RESET(R) and two outputs Q , $\bar{Q}$ which are complements each other. It can easily be synthesized by using two 2-input NOR gates.



In SR latch, S and R stand for *set* and *reset*. It can be constructed from a pair of cross-coupled NOR logic gates. The stored bit is present on the output marked Q.

While the S and R inputs are both low, feedback maintains the Q and $\bar{Q}$ outputs in a constant state, with $\bar{Q}$ the complement of Q . If S (*Set*) is pulsed high while R (*Reset*) is held low, then the Q output is forced high, and stays high when S returns to low; similarly, if R is pulsed high while S is held low, then the Q output is forced low, and stays low when R returns to low.

SR Flip Flop operation							
Characteristic table				Excitation table			
S	R	Q_{next}	Action	Q	Q_{next}	S	R
0	0	Q	hold state	0	0	0	x
0	1	0	Reset	0	1	1	0
1	0	1	Set	1	0	0	1
1	1	X	not allowed	1	1	X	0

The $R = S = 1$ combination is called a **restricted combination** or a **forbidden state** because, as both NOR gates then output zeros, it breaks the logical equation $Q = \text{not } Q$. The combination is also inappropriate in circuits where *both* inputs may go low *simultaneously* (i.e. a transition from *restricted* to *keep*). The output would lock at either 1 or 0 depending on the propagation time relations between the gates (a race condition). In certain implementations, it could also lead to longer ringings (damped oscillations) before the output settles, and thereby result in undetermined values (errors) in high-frequency digital circuits. Although this condition is usually avoided, it can be useful in some applications.

To overcome the restricted combination, one can add gates to the inputs that would convert $(S,R) = (1,1)$ to one of the non-restricted combinations. That can be:

- $Q = 1 (1,0)$ – referred to as an *S-latch*
- $Q = 0 (0,1)$ – referred to as an *R-latch*
- Keep state $(0,0)$ – referred to as an *E-latch*

Alternatively, the restricted combination can be made to *toggle* the output. The result is the JK latch.

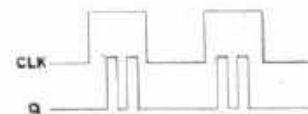
Characteristic: $Q+ = R'Q + R'S$ or $Q+ = R'Q + S$.

➤ 20. What is race around in J-K flip-flop? How can it be avoided? [Model Question]

Answer:

Race-Around Condition In J-K Flip Flop

Practically, we don't get toggling. Since, clock pulse is more than the propagation delay, so within one clock pulse the output will keep on toggling again and again and it may become indeterminate. This is known as race around condition. Race Around condition occurs because of the feedback connection.



There are three ways to avoid Race-Around Condition:

1. If $T_{on} < T_{pd}$

i.e. clock pulse is less than the propagation delay. Practically T_{pd} is of the order of nanoseconds or picoseconds and therefore, it is only a theoretical possibility to get clock pulse lower than propagation delay.

2. Another way is by using Edge Triggering

In edge triggering output is affected only at the time of presence of edge i.e. only during the rising or falling edge of a clock pulse. When the input clock pulse makes a +ve going transition (or a -ve going transition), the value of input is transferred to output i.e. Q. Changes in input when clock is maintained at a steady 1 value do not affect Q. Moreover, a -ve pulse transition does not affect the output and nor does when clock pulse is 0. Hence the edge triggered flip flop eliminates any feedback problem in sequential circuit. The race around condition can be eliminated by using master slave JK flipflop.

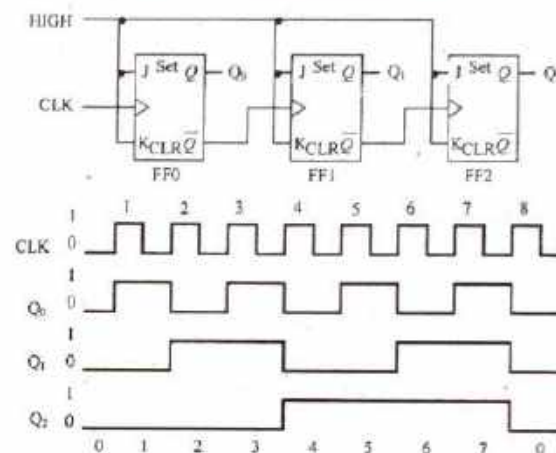
21. (a) What are the advantages and disadvantages of synchronous and asynchronous counters?
(b) How a D-flip-flop can be used to design a 3-bit binary asynchronous counter?
[Model Question]

Answer:

a)

Synchronous Counter	Asynchronous Counter
(i) With increase in number of states, the logic circuit becomes complicated.	(i) Logic circuit is simple.
(ii) There is no connection between output of preceding FF and CLK of next one.	(ii) Output of the preceding FF is connected to clock of the next FF.
(iii) All FF receive clock signal simultaneously.	(iii) All FFs are not clocked simultaneously.
(iv) Propagation delay (P.D) = $(n)_{FF} \times (t_d)_{gate}$. It is much shorter than that of asynchronous counter.	(iv) P.D = $n \times t_d$ where n = number of FFs and t_d is p.d per FF.
(v) Maximum frequency of operation is high due to shorter propagation delay.	(v) Low because of the long propagation delay.

b) The following is a three-bit asynchronous binary counter and its timing diagram for one cycle. It works exactly the same way as a two-bit asynchronous binary counter mentioned above, except it has eight states due to the third flip-flop.

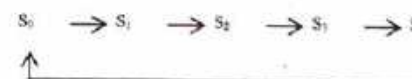


22. Design a modulo-6 synchronous counter.

[Model Question]

Answer:

A MOD - 6 counter will go through the following states

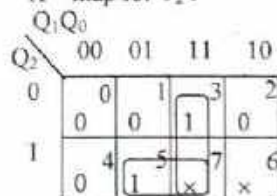


The flip flops will be required. The truth table and the flip flop inputs are shown in table below:-

Present State			Next State			Flip-flop Inputs		
Q ₂	Q ₁	Q ₀	Q ₂	Q ₁	Q ₀	T ₂	T ₁	T ₀
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	0	0	0	1	0	1

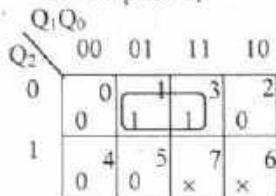
The K-maps for the flip flop inputs are:

K-map for T_2 :



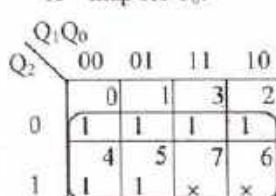
$$T_2 = Q_2 Q_0 + Q_1 Q_0$$

K-map for T_1 :



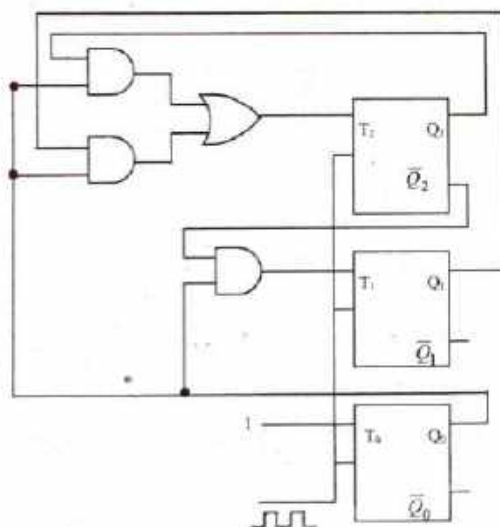
$$T_1 = \bar{Q}_1 Q_0$$

K-map for T_0 :



$$T_0 = 1$$

The circuit of MOD -6 counter is as shown.



➤ 23. Design a 3-bit up-down synchronous counter.

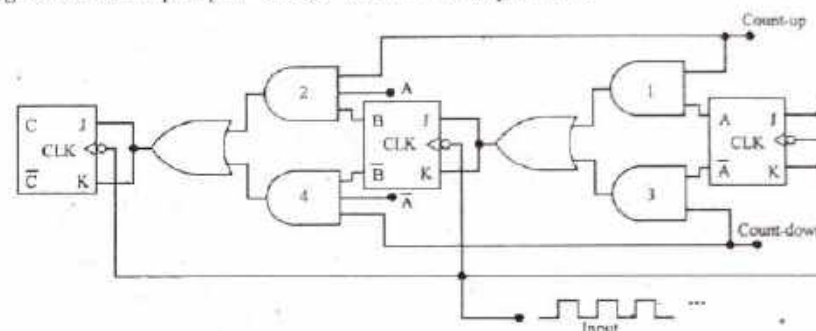
[Model Question]

Answer:

3-bit binary Up/Down Counters:

The similarities between the implementation of a binary up counter and a binary down counter leads to the possibility of a binary up/down counter, which is a binary up counter and a binary down counter combined into one. Since the difference is only in which

output of the flip-flop to use, the normal output or the inverted one, we use two AND gates for each flip-flop to "choose" which of the output to use.



➤ 24. Design a Mod-10 ripple counter.

[Model Question]

Answer:

The first step is to determine how many flip-flops to use. (This could be on a single IC, but we will use JK flip-flops.)

Will 3 flip-flops do it?

$2^3 = 8$ states, not quite there

$2^4 = 15$ states, more than enough states

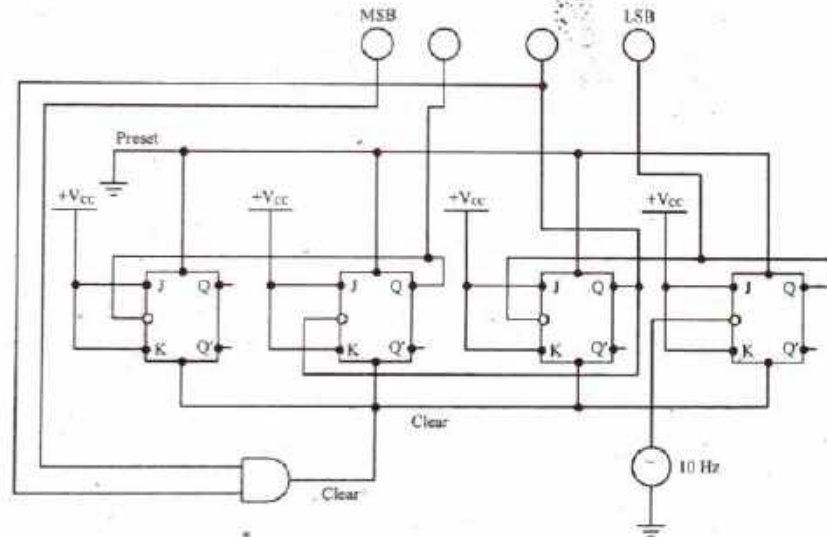
The trick is to find a way not to use all of those states. There must be a way to “force” the counter to stop counting at 9 and roll over to 0.

Looking at a truth table, the counter should run from 0000₂ to 1001₂.

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0

1	1	0	1
1	1	1	0
1	1	1	1

Since the counter has to display 1001_2 , the next number (1010_2) will be used to reset the counter to zero. Since the asynchronous inputs are active high, an AND gate will be used. The two flip-flops where a "1" occurs will be tied to an AND gate, and the output will be tied to the "clear" input.



25. (a) What is Modulus Counter?

(b) Draw the logic diagram of a binary ripple counter and draw its waveform.

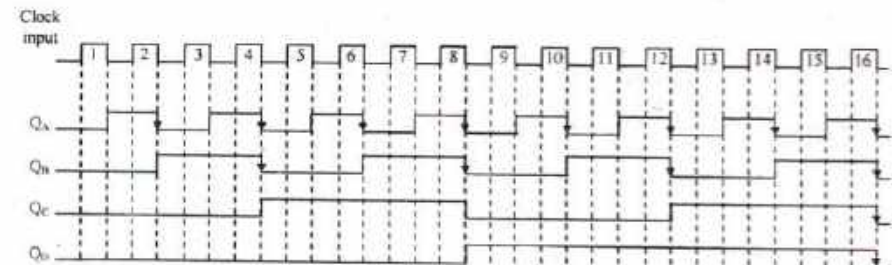
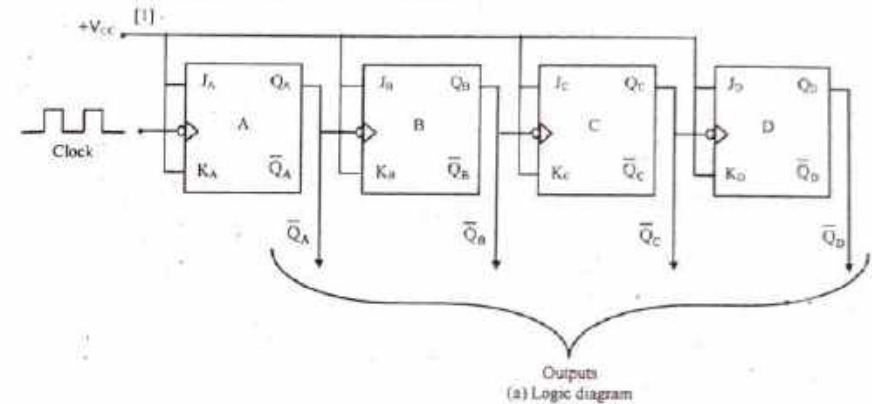
(c) What factors determine whether a Counter operates as a Count-up or Count-down Counter? **[Model Question]**

Answer:

a) A counter which goes through all the possible states before restarting is called the full modulus counter. A counter in which the maximum number of states can be changed is called the variable modulus counter.

b) Fig. below shows a 4-bit binary ripple counter. As shown, a binary ripple counter is constructed using clocked JK flip-flops. The system clock, a square wave, drive flip-flop A. The output of A drives flip-flop B, the output of B drives flip-flop C, and the output of C drives flip-flop D. The overall propagation delay time of the counter is the sum of

the individual delays of flip-flops. All the J and K inputs are connected to V_{CC} (1), which means that each flip-flop toggles on the negative edge of its clock input.



Consider initially all flip-flops to be in the logical 0 state (i.e. $Q_A = Q_B = Q_C = Q_D = 0$). A negative transition (1 to 0) in clock input which drives flip-flop A causes Q_A to change from logical 0 to logical 1. Flip-flop B does not change its state since it also requires negative transition at its clock input, i.e. it requires its clock input (Q_A) to change from logical 1 to logical 0. With the arrival of the second clock pulse to flip-flop A, Q_A goes from 1 to 0. This change of state creates the negative going edge needed to trigger flip-flop B, and thus Q_B goes from 0 to 1. Before the arrival of the sixteenth clock pulse, all the flip-flops are in the logical 1 state. Clock pulse 16 causes Q_A, Q_B, Q_C and Q_D to go to logical 0 state in turn.

Table 1 shows the sequence of binary states that the flip-flops will follow as clock pulses are applied continuously. An n -bit binary counter repeats the counting sequence for every 2^n (n = number of flip-flops) clock pulses and has discrete states from 0 to $2^n - 1$.

Table 1

State	Q_D	Q_C	Q_B	Q_A
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1
0	0	0	0	0

c) Counters can operate in two modes: count-up or count-down. In the count-up mode, the counter starts at 0 (or any other value), and then the count value is incremented by 1 whenever a new pulse has arrived. In the count-down mode, on the other hand, the counter is loaded with a starting value, and this value gets decremented with the arrival of each new pulse. Due to the finite size of counters (i.e. maximum number of counts for that counter). Similarly, a count-down counter will overflow when the count goes below 0. When a counter overflows, the counter is reloaded with the starting value, and the counting process repeats.

ENGINEERING MECHANICS

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4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1
0	0	0	0	0

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Unit 1: Basics of Mechanics and Force System

Unit at a glance:

1.1 Fundamentals and Force

1. Terminology

- **Particle:** It is defined as an object whose mass is concentrated at a point. This assumption is made when the size of a body is negligible and is irrelevant to the description of motion of the body.
- **Rigid body:** A body in which the particles do not change their relative positions under the action of any external force is called as rigid body. No body is perfectly rigid.
- **Statics:** It is the branch of applied mechanics, which deals with bodies at rest.
- **Dynamics:** It is the branch of applied mechanics, which deals with bodies in motion.
- **Kinematics:** It is the branch of dynamics which studies motion without considering mass of the body and force causing motion.
- **Kinetics:** It is the branch of dynamics, which deals with bodies in motion considering mass and force causing motion.

2. Vector Quantity

A vector quantity is a quantity that has both magnitude and direction. Velocity, acceleration, and force are examples of vector quantities.

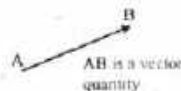
3. Scalar Quantity

A scalar quantity is a quantity that has magnitude, but no direction. Time, mass, volume, and speed are examples of scalar quantities.

Definition

A **Vector** quantity is defined as a quantity, which satisfies

- The qualification of possessing a certain **magnitude**, including a null value.
- Having a certain orientation in space, called its **direction** and
- A **sense**, which qualifies its nature of action (moving toward or away from a given point).



4. Vector Notations

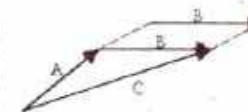
$\overrightarrow{AB}$, $\vec{AB}$, $\hat{AB}$, $\vec{AB}$ etc.

Magnitude of a vector represented by $|AB|$ or $|\vec{AB}|$

5. Vector Addition

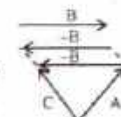
The sum of two vectors, A and B , is a vector C , which is obtained by placing the initial point of B on the final point of A , and then drawing a line from the initial point of A to the final point of B , as illustrated in Panel 4. This is sometimes referred to as the "Tip-to-Tail" method.

The operation of vector addition as described here can be written as $C = A + B$



6. Vector Subtraction

The difference of two vectors, $A - B$, is a vector C that is, $C = A - B$ or $C = A + (-B)$. Thus vector subtraction can be represented as a vector addition.



7. Dimensional Homogeneity

The qualitative description of physical variable is known as dimension while the quantitative description is known as unit. We come across several relations among the physical quantities. Some of the terms may be having dimensions and some others may be dimensionless.

However in any equation dimensions of the terms on both sides must be the same. This is called dimensional homogeneity. The branch of mathematics dealing with dimensions of quantities is called dimensional analysis.

There are two systems of dimensional analysis viz. *absolute system* and *gravitational system*.

8. Force

Force is a physical quantity, which changes or tends to change the state of rest or uniform motion of a body. Force is a vector quantity.

The unit of force is Newton (N) in SI system.

9. System of Forces

A group of forces or a set of forces acting together on a body is form a system of forces. The characteristics of different force systems are explained in Table.

Table 1.1: Characteristics and Representation of Force systems

Force System	Representation	Characteristics
1. Coplanar Forces		Lines of action of all forces lie on a single plane
2. Non-Coplanar Forces		Lines of action of forces are not lying on a single plane
3. Concurrent Forces		Lines of action of all forces pass through a single point
4. Non-Concurrent Forces		Lines of action of all forces do not pass through a single point
5. Collinear Forces		Lines of action of all forces pass through a single line.
a) Like Collinear Force		Lines of action of all forces pass through a single line in the same direction
b) Unlike Collinear Force		Lines of action of all forces pass through a single line in different directions
6. Parallel Forces		Lines of action of all forces are parallel to each other
a) Like Parallel Forces		Lines of action of all forces are parallel to each other in the same direction
b) Unlike Parallel Forces		Lines of action of all forces are parallel to each other in different directions
7. Non-Parallel Forces		Lines of action of all forces are not parallel to each other

10. Few terminologies

- **Active force** $\Rightarrow$ applied externally
- **Reactive force** $\Rightarrow$ counter balancing force generated by the medium or supports on which action is taking place

- **Free body diagram** $\Rightarrow$ The body isolated from the surroundings, all constraints and supports are replaced by their reactions and all active forces and moments are shown
- **Equivalent system** $\Rightarrow$ The effect of the two systems on the rigid body is identical.

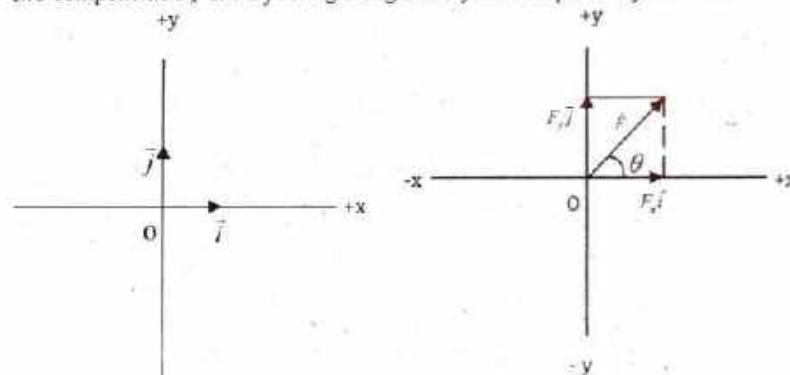
1.2 Resolution of forces

1. Definition

Breaking down a single vector into 2 or more vectors is known as the resolution of forces.

The process of splitting the force into components is called the **resolution** of a force whereas the process of finding the resultant of any number of forces is called the **composition** of forces. The resolution of forces helps in determining the resultant of a number of forces acting on a body as it reduces vector addition to algebraic addition.

A force making an angle θ with respect to x axis as shown in Figure can be resolved into two components F_x and F_y acting along x and y axes respectively. If $\vec{i}$ and



$\vec{j}$ are the unit vectors acting along x and y axes respectively then the force $\vec{F}$ can be expressed as $\vec{F} = F_x \vec{i} + F_y \vec{j}$

Where F_x and F_y are the magnitudes of the components along x and y axes.

Refer Figure, F_x and F_y are determined as,

$$F_x = F \cos \theta$$

$$F_y = F \sin \theta \text{ and } \tan \theta = \frac{F_y}{F_x}$$

The magnitude of the force can also be expressed as

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

1.3 Composition of Forces

Analytical method

a) Parallelogram theorem:

"If two forces, acting at a point be represented in magnitude, direction and sense by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude, direction and sense by the diagonal of the parallelogram passing through that point."

b) Triangle Law of Forces

If two forces acting on a body are represented one after another by the sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last point.

c) Law of Polygon of Forces

"If a number of forces acting at a point are such that they can be represented in magnitude and direction by the sides of an open polygon taken in order, then their resultant is represented in magnitude and direction by the closing side of the polygon but taken in the opposite order".

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

- Action and reaction – [WBSCTE 2006]
(a) always act on same body (b) never act on the same body
(c) action act but reaction never act (d) none of the above

Answer: (a)

- The forces which meet at one point but their lines of action do not lie on one plane, are called [WBSCTE 2006]
(a) coplanar concurrent forces (b) coplanar non-concurrent forces
(c) non-coplanar concurrent forces (d) intersecting forces

Answer: (c)

- The tension in the wire ropes supporting a lift – [WBSCTE 2006]
(a) is more when the lift is moving downwards
(b) is less when the lift is moving upward
(c) is more when the lift is moving upward
(d) remains constant irrespective of the lift movement

Answer: (a)

- The resultant of a force acting on a body will be zero, if the body – [WBSCTE 2006]
(a) rotates (b) does not rotate
(c) rotates with uniform acceleration (d) rotates with uniform de-acceleration

Answer: (c)

B. Fill in the blanks in the following statements:

- The vector quantities are those quantities which have both direction and magnitude. [WBSCTE 2005, 2009]
- Scalar Quantity has Magnitude only. [WBSCTE 2006, 2009]
- The resultant of two forces P and Q act at angle θ is equal to
 $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ [WBSCTE 2007, 2014]
- The Lami's theorem is applicable only three coplanar Concurrent forces. [WBSCTE 2007, 2014]
- For coplanar concurrent forces Lami's theorem states that if three coplanar concurrent forces acting on a body keep it in equilibrium then each force is proportional to the sine of the angle between the other. [WBSCTE 2011]
- A force is completely defined by the product of mass and acceleration. [WBSCTE 2012]
- The expression for normal acceleration is given by V^2/R [WBSCTE 2012]
- A Rigid body is one that the body does not deform or the distance between any two points of the body does not change under the action of a applied force. [WBSCTE 2013, 2014]
- Bow's notations are related to analysis by Graphical Solution of Forces. [WBSCTE 2013, 2014]
- Frictional force depends upon the nature of material of the surfaces in contact and on their roughness or smoothness. [WBSCTE 2014]
- Vector quantity has magnitude and direction. [WBSCTE 2015]
- The units which are used for measurement of fundamental quantities i.e. mass, length and time, are known as gram, metre, second. [WBSCTE 2016]
- The body which does not deform under the action of applied forces is known as Rigid body. [WBSCTE 2016]

18. If the resultant of two forces P and Q (acting at an angle θ) makes an angle α with P , then $\tan \alpha = \frac{Q \sin \theta}{(P + Q \cos \theta)}$ [WBSCTE 2016]
19. The process of finding out the resultant force of two or more given forces is called composition of forces. [WBSCTE 2017]
20. The turning or rotational tendency of a force is called torque. [WBSCTE 2017]
21. The diagram which shows all forces including reactions at the contact surface after removing supports is known as free body diagram. [WBSCTE 2017]
22. If a number of forces are acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces. This is known as Parallelogram law. [WBSCTE 2018]
23. If the two forces P and Q are acting at an angle θ and their resultant acting at an angle α with P . Then $\tan \alpha = \frac{Q \sin \theta}{(P + Q \cos \theta)}$ [WBSCTE 2018]
24. Example of vector quality is velocity. [WBSCTE 2022]

C. State whether the following statements are True or False:

25. The radius of gyration can be given by $K_1 = \sqrt{\frac{I}{A}}$ [WBSCTE 2010]
Answer: True
26. There is no difference in component and resolved parts of a force. [WBSCTE 2012]
Answer: True
27. Algebraic sum of moment of all the forces in a plane about any point in that plane is equal to moment of their resultant force about the same point. [WBSCTE 2012]
Answer: True
28. If two forces 7 N and 4 N are acted their resultant is not always equal to 11 N. [WBSCTE 2013]
Answer: True
29. Under equilibrium condition the vector diagram is a closed polygon. [WBSCTE 2013]
Answer: True

30. Component of a force is the resolved parts of the force in two mutually perpendicular direction. [WBSCTE 2013, 2014]
Answer: True
31. In a simply supported beam carrying triangular load, the reactions cannot be vertical. [WBSCTE 2014]
Answer: False
32. Concurrent forces always are coplanar. [WBSCTE 2015]
Answer: True
33. A body isolated from all other members which are connected to it is called Free body. [WBSCTE 2016]
Answer: True
34. A roller support has reaction normal to the support plane. [WBSCTE 2016]
Answer: True
35. If two equal forces each of magnitude P act at an angle θ , then their resultant will be $P\sqrt{2(1 - \cos \theta)}$. [WBSCTE 2016]
Answer: False
36. The normal force of an object lying on an inclined plane will be lesser than the normal force acting on it, when it rests horizontally. [WBSCTE 2016]
Answer: True
37. The components of a force along two mutually perpendicular lines are called rectangular components. [WBSCTE 2017]
Answer: True

D. Pick up the correct answer:

38. A force can be completely defined by its (Magnitude and Direction/Magnitude, Direction and Sense/magnitude, Direction, sense and Line of Action) [WBSCTE 2011]
Answer: Director, sense and magnitude
39. If three forces acting on a body keep the body in equilibrium they (must meet at a point / must be parallel / must meet at a point or else they must be parallel) [WBSCTE 2012]
Answer: must meet at a point or else they must be parallel

40. The line of action of resultant of two like parallel forces lies (in between them nearer to the smaller force/ in between them nearer to the larger force/In the line of action of larger force) [WBSCTE 2012]

Answer: in between them nearer to the larger force

41. When two forces of magnitude 3 N and 4 N acts at right angles to each other their resultant is (14 N/10 N/2 N/5 N). [WBSCTE 2013, 2014]

Answer: 5 N

42. For simple problem of statics (statically determinate) the analysis can be done (graphically/analytically/both graphically and analytically). [WBSCTE 2013]

Answer: both graphically and analytically.

E. Answer the following Questions:

43. Define Scalar & Vector quantity.

[WBSCTE 2022]

Answer: Refer to Article No. 1.1 of Unit at a Glance.

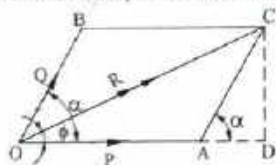
Long Answer Type Questions

1. State the parallelogram law of forces. Draw sketch to explain this.

[WBSCTE 2003, 2004, 2009, 2009, 2013]

Answer:

Parallelogram Law of Forces: If two forces are acting simultaneously on a particle and are represented in terms of magnitude and direction of two adjacent sides of a parallelogram, then the diagonal passing through the point of intersection of these forces, represents the resultant of above two forces, in terms of magnitude and direction.



Suppose two forces, P and Q acting at point O be represented by the two sides of a parallelogram, then their resultant R is represented completely by the diagonal OC of the parallelogram $OACB$.

2. The following forces act at a point –

[WBSCTE 2003, 2016, 2022]

- (i) 20N inclined at 30° to North of East, (ii) 25N towards North, (iii) 30N towards North-West and (iv) 35N inclined at 40° to South of West.

Find the magnitude and direction of the resultant of the force system.

POLY-ME

Answer:

[Hints: similar type as above]

$$\sum H = -30.7 \text{ N} \quad \sum V = 63.7 \text{ N} \quad \theta = 244.2690^\circ$$

For finding the resultant of the given forces we need to find resultant of forces in each direction.

Doing the components of the forces in x-direction we get forces as

$$20\cos 30^\circ, 25\cos 90^\circ, 30\cos 45^\circ, 35\cos 40^\circ.$$

Net force in the x-direction can be written as F_x

$$F_x = 20\cos 30^\circ + 25\cos 90^\circ - 30\cos 45^\circ - 35\cos 40^\circ$$

$$F_x = -30.7 \text{ N}$$

Doing the components of the forces in y-direction we get forces as

$$20\sin 30^\circ, 25\sin 90^\circ, 30\sin 45^\circ, 35\sin 40^\circ$$

Net force in the y- direction can be written as F_y

$$F_y = 20\sin 30^\circ + 25\sin 90^\circ + 30\sin 45^\circ - 35\sin 40^\circ$$

$$F_y = 33.7 \text{ N}$$

Resultant force will be, $F = \sqrt{F_x^2 + F_y^2} = 45.6 \text{ N}$

Direction of resultant force will be

$$\alpha = \tan^{-1}(33.7/-3.7)$$

$$\alpha = (-47.7^\circ)$$

Therefore, the magnitude and direction of the resultant force system is 45.6N and -47.7° respectively.

3. (a) The following coplanar and concurrent forces act at a point –

- (i) 30N at 30° North of West (ii) 40N towards South (iii) 20N towards North East (iv) 35N towards 60° East of South (v) 50N towards 30° South of West.

Find the magnitude and direction of the resultant of the force system.

[WBSCTE 2004]

- (b) The following forces act at a point –

- (i) 30N inclined at 40° towards North of East,

- (ii) 20N due North,

- (iii) 40N inclined at 60° South of West. Find the resultant force in magnitude and direction.

[WBSCTE 2004, 2009]

Answer:

- a) Magnitude of the resultant force. Resolving all the forces horizontally i.e., along east-West line.

POLY-ME

$$\sum H = (20 \cos 45^\circ + 30 \cos 150^\circ + 50 \cos 210^\circ + 40 \cos 270^\circ + 35 \cos 330^\circ) \text{ N}$$

$$= -24.829 \text{ N}$$

And now resolving all the forces vertically i.e., along North-South Line.

$$\sum V = (20 \sin 45^\circ + 30 \sin 150^\circ + 50 \sin 210^\circ + 40 \sin 270^\circ + 35 \sin 330^\circ) \text{ N}$$

$$= -53.358 \text{ N}$$

We know the magnitude of resultant force R

$$= R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$= \sqrt{(-24.829)^2 + (-53.358)^2} \text{ N}$$

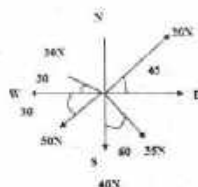
$$= 58.852 \text{ N}$$

Direction of the resultant force.

Let angle is θ which the resultant makes with the force

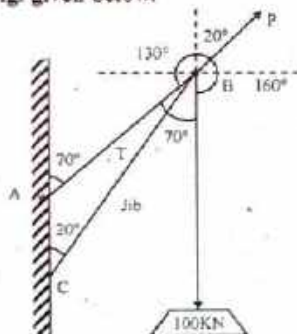
Since $\sum V$ is negative and $\sum H$ is negative, therefore θ lies between 180° and 270°

Actual $\theta = 180^\circ + 65^\circ 046' = 255^\circ 046'$



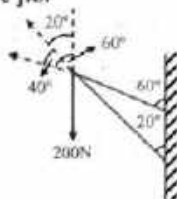
b) Similar type problem as (a).

4. Find the tension in the tie and job when a load of 100 kN is suspended from the crane as shown in Fig. given below. [WBSCTE 2006]



OR,

A jib crane shown in figure, lift a load of 200N. Determine the tension in the tie and compression force in the jib. [WBSCTE 2008]



Answer:

Let, T = required tension in the tie AB, and P = required compressive force in BC.

The point B is in equilibrium under the action of three forces which are T , P and 100 kN

$$\text{or, } \frac{P}{\sin 70^\circ} = \frac{T}{\sin 160^\circ} = \frac{100}{\sin 130^\circ}$$

$$\text{or, } P = \frac{100 \times \sin 70^\circ}{\sin 130^\circ} = 122.67 \text{ kN} \quad (\text{Ans.})$$

$$T = \frac{\sin 160^\circ}{\sin 130^\circ} \times 100 = 44.65 \text{ kN} \quad (\text{Ans.})$$

[Note: Second problem similar type]

5. Explain bow's notation.

[WBSCTE 2008]

OR,

Explain Bow's notation with proper diagram.

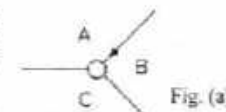
[WBSCTE 2009]

Answer:

Bow's Notation

When several members are pinned are together and the joint in total equilibrium (not moving), the resultant force must be zero. This means that if we add up all the forces as vectors, they must form a closed polygon. If one or even two of these forces is unknown, then it must be the vector, which closes the polygon.

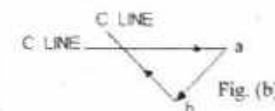
Consider three members joined by a pin as shown in figure (a). Only one of these forces is known. Bow's notation helps us to identify and label each member and draw the polygon or (in this case) the triangle of forces. The process is as follows.



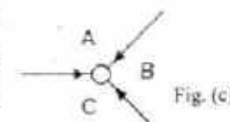
1. Label the space between each number. This is why the diagram is called a SPACE DIAGRAM.

2. Starting at any space, say A identify each member by moving clockwise around the joint so the first becomes a-b and the next b-c (in this case only).

3. Draw the known vector a-b. We known that the next vector b-c starts at b but we do not know its length. Draw a 'c' line from 'b' in the direction of member b-c. We know that when all the vectors are added, they must form a closed triangle so c-a must end at 'a'. Draw a 'c' line through 'a' in the direction of member c-a. Where the two 'c' lines cross must be point 'c'.



4. Finally, transfer the arrows back to the space diagram in the same direction as on the triangle of force (figure c). If they push onto the pin joint, the member must be in compression and so is a strut. If the arrow pulls on the joint, the member must be in tension and so is tie.



6. State the triangle law of force.

[WBSCTE 2009]

Answer:

Triangle law of force: If two forces are simultaneously acting on a particle and are represented in terms of magnitude and direction by two sides of a triangle, taken in an order, then the third side or the closing side of the triangle, drawn in opposite order, represents the resultant of above two forces in terms of magnitude and direction.

Let F_1 and F_2 forces are acting along two sides AB & BC of $\triangle ABC$

Then the resultant force R will be represented both in direction and magnitude by the side AC.

7. State the polygon law of force & transmissibility of forces. [WBSCTE 2010]

Answer: Refer to Article No. 1.3 of Unit at a Glance.

8. ABCDEF is a regular hexagon. Forces of magnitude 2, $4\sqrt{3}$, 8, $2\sqrt{3}$ & 4N act at A in direction of AB, AC, AD, AE & AF respectively determine the resultant completely? [WBSCTE 2010]

Answer:

$$\triangle ABC, \vec{AB} + \vec{BC} = \vec{AC}$$

Using the law of triangle of vector

$$2 + a = 4\sqrt{3}$$

$$a = (4\sqrt{3} - 2)N$$

$$|\vec{BC}| = |\vec{CD}| = |\vec{DE}| = |\vec{EF}| = |\vec{a}|$$

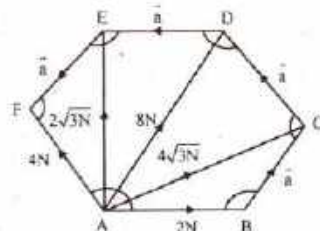
using the law of hexagon.

$$\therefore \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} = \vec{AF}$$

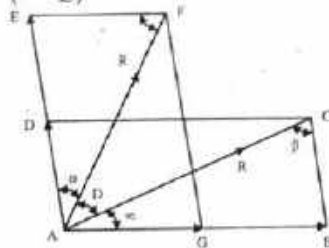
$$2 + 4(4\sqrt{3} - 2) = \vec{AF} = 2 + 16\sqrt{3} - 8$$

$$\Rightarrow 16\sqrt{3} - 6 = \vec{AF}$$

$\therefore$ Hence the resultant $\vec{AF} = (16\sqrt{3} - 6)N$ (Ans.).



9. The angle of inclination between two forces P and Q is θ . If P and Q be interchanged in position, shows that the resultant will be turned through an angle ϕ , where $\tan\left(\frac{\phi}{2}\right) = \frac{(P-Q)}{(P+Q)} \tan\left(\frac{\theta}{2}\right)$. [WBSCTE 2011]



Answer:

Let,

$$\angle BAC = \alpha = \angle EAF$$

$$\angle ACD = \beta$$

$$\text{Given, } \phi = \angle CAF, \text{ and } \theta = \angle DAB = \angle FAG$$

$$\therefore \phi = \theta - 2\alpha \quad \text{or, } \alpha = \frac{\theta - \phi}{2}$$

$$\text{Again, } \beta = \alpha + \theta = \frac{\theta - \phi}{2} + \phi = \frac{\theta + \phi}{2}$$

$$\text{From, } \triangle ABC, \text{ we get, } \frac{BC}{\sin \alpha} = \frac{AB}{\sin \beta}$$

$$\text{or, } \frac{\theta}{\sin \frac{\theta - \phi}{2}} = \frac{P}{\sin \frac{\theta + \phi}{2}}$$

$$\text{or, } \frac{P}{Q} = \frac{\sin \frac{\theta + \phi}{2}}{\sin \frac{\theta - \phi}{2}}$$

$$\text{or, } \frac{P - Q}{P + Q} = \frac{\sin \frac{\theta + \phi}{2} - \sin \frac{\theta - \phi}{2}}{\sin \frac{\theta + \phi}{2} + \sin \frac{\theta - \phi}{2}}$$

$$\begin{aligned} &= \frac{2 \cos \frac{\theta + \phi}{2} \times \sin \frac{\theta - \phi}{2}}{2 \sin \frac{\theta + \phi}{2} \times \cos \frac{\theta - \phi}{2}} = \frac{\cos \frac{\theta}{2} \sin \frac{\phi}{2}}{\sin \frac{\theta}{2} \cos \frac{\phi}{2}} = \frac{\tan \frac{\phi}{2}}{\tan \frac{\theta}{2}} = \frac{P - Q}{P + Q} \tan \frac{\theta}{2} \\ &\therefore \tan \frac{\phi}{2} = \frac{(P - Q)}{(P + Q)} \tan \frac{\theta}{2} \end{aligned}$$

10. One end of a stout rope of length 20 metre is fixed to a vertical telegraph post standing on the ground, and a man pulls at the other end with a given force. Find the point of the post at which the rope to be fixed in order that the man will have the best chances of overturning the post. [WBSCTE 2012]

Answer:

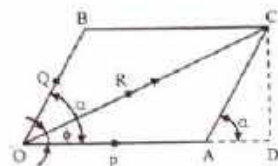
The man will have the maximum chances to over turn the post if he fix the rope at the top of the post.

If the rope is connected at point A then torque generated at point A, having a centre of rotation B, will be maximum hence the post will over turn.



11. If one of the two equal forces acting simultaneously at a point and if their resultant is equal to any one of them, then find the angle between the forces. [WBSCTE 2013]

Answer:



From the parallelogram law, we know that Resultant

$$R = (P^2 + Q^2 + 2PQ \cos \alpha)^{1/2} \quad \dots (i)$$

$$\text{and direction } \phi = \tan^{-1}[(Q \sin \alpha) / (P + Q \cos \alpha)] \quad \dots (ii)$$

Now from equation (i) we can write

$$P^2 = (P^2 + P^2 + 2P \cdot P \cos \alpha)$$

$$\therefore \cos \alpha = -1/2 = \cos 120^\circ$$

$$\therefore \alpha = 120^\circ$$

And from equation (ii) we can write $\phi = \tan^{-1}[(P \sin 120^\circ) / (P + P \cos 120^\circ)] = 60^\circ$

12. Two forces act an angle of 120° . The bigger force is of 40N and the resultant is perpendicular to the smaller one, find the smaller force. [WBSCTE 2014]

Answer:

Bigger force $P = 40\text{N}$

Angle between the resultant and the smaller force $= 90^\circ$

Angle between the two forces $\theta = 120^\circ$

$\therefore$ Angle between the bigger force and the resultant $(R) = 120^\circ - 90^\circ = 30^\circ$

Let $Q =$ smaller force,

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos 120^\circ} = \sqrt{40^2 + Q^2 - 40Q} \quad \dots (i)$$

$$\text{Now, } P^2 = Q^2 + R^2 - 2QR \cos 90^\circ \Rightarrow P^2 = Q^2 + R^2 \quad \dots (ii)$$

$\therefore$ From equation (i) & (ii) we get

$$P\theta = 2\theta^2$$

$$\therefore Q = \frac{P}{2} = \frac{40}{2} \text{ N} = 20\text{N}$$

13. State and explain triangle law of forces. [WBSCTE 2015]

Answer: Refer to Question No. 6 of Long Answer Type Questions.

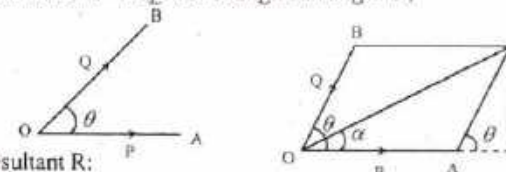
14. a) Two concurrent forces P & Q are acting at an angle θ , derive the expression of their resultant force.

- b) The resultant of two forces P and Q is R . If Q is doubled, R is doubled and if Q is reversed, R is again doubled. Show that $P:Q:R = \sqrt{2}:\sqrt{3}:\sqrt{2}$.

[WBSCTE 2016]

Answer:

- a) Two concurrent forces P & Q are acting at an angle θ ,



Derivation of Resultant R:

$\angle CAD = \theta$ [because $OB \parallel CA$ and OA is common base]

$$\text{In } \triangle ACD, \cos \theta = \frac{AD}{AC} \Rightarrow AD = AC \cos \theta$$

But, $AC = Q$; i.e. $AD = Q \cos \theta$

$$\text{And, } \sin \theta = \frac{CD}{AC} \Rightarrow CD = AC \sin \theta$$

$$\text{Now in } \triangle OCD \Rightarrow OC^2 = OD^2 + CD^2$$

$$R^2 = (OA + AD)^2 + CD^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$= P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Let, the angle made by the resultant R with the direction of force P is given by α .

$$\therefore \text{In } \triangle OCD, \tan \alpha = \frac{CD}{OD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \left[\frac{Q \sin \theta}{P + Q \cos \theta} \right]$$

α is the direction of the resultant R

- b) The resultant of two forces P and Q is R .

$\therefore$ by using the formula of vectors/forces,

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad \dots (i)$$

The given condition is, if Q is doubled, then R is doubled.

$$\therefore (2R)^2 = P^2 + (2Q)^2 + 2|P| \times |2Q| \cos \theta$$

$$\Rightarrow 4R^2 = P^2 + 4Q^2 + 4PQ \cos \theta \quad \dots (ii)$$

Again, if Q is reversed, R is again doubled

$$\therefore (2R)^2 = P^2 + (-Q)^2 + 2|P| \times |-Q| \cos(180^\circ - \theta)$$

[$\therefore$ reversing a vector means changing its direction by 180°].

On solving we get,

$$4R^2 = P^2 + Q^2 - 2PQ \cos \theta \quad \dots (iii)$$

Subtracting eqn (iii) from eqn (ii) we get,

$$0 = 3Q^2 + 6|P||Q| \cos \theta$$

Assuming Q as non zero vector/forces

$$-3|Q| = 6|P| \cos \theta$$

$$\Rightarrow Q = -2P \cos \theta$$

$$\Rightarrow Q^2 = 4P^2 \cos^2 \theta \quad \dots (iv)$$

Now, subtracting eqn (i) from eqn (ii) we get,

$$3R^2 = 3Q^2 + 2PQ \cos \theta$$

Substituting the value of Q in the above eqn.

$$3R^2 = 3(4P^2 \cos^2 \theta) + 2P(-2P \cos \theta) \cos \theta$$

$$\Rightarrow 3R^2 = 12P^2 \cos^2 \theta - 4P^2 \cos^2 \theta$$

$$\Rightarrow 3R^2 = 8P^2 \cos^2 \theta \Rightarrow R^2 = \frac{8P^2 \cos^2 \theta}{3} \quad \dots (v)$$

Now, Substituting the value of Q & R in eqn (i) we get,

$$\frac{8P^2 \cos^2 \theta}{3} = P^2 + (4P^2 \cos^2 \theta) + 2P(-2P \cos \theta) \cos \theta$$

Assuming P as non zero vector/forces

$$\Rightarrow P^2 = \frac{8P^2 \cos^2 \theta}{3} \quad \dots (vi)$$

$$\therefore P^2 : Q^2 : R^2 = \frac{8P^2 \cos^2 \theta}{3} : 4P^2 \cos^2 \theta : \frac{8P^2 \cos^2 \theta}{3}$$

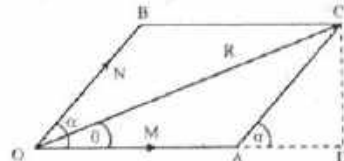
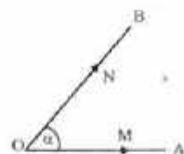
$$= \frac{8}{3} : 4 : \frac{8}{3} = 8 : 12 : 8 = 2 : 3 : 2$$

$$\therefore P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2} \quad (\text{proved})$$

15. a) Two concurrent forces M & N are acting at an angle α , derive the expression of their resultant force. [WBSCTE 2018]

Answer:

Two concurrent forces M and N are acting at an angle α .



POLY-Me

Derivation of resultant R ,

$$\angle CAD = \alpha \quad [\because OB \parallel CA \text{ \& } OA \text{ is common base}]$$

$$\text{In } \triangle ACD, \cos \alpha = \frac{AD}{AC} \Rightarrow AD = AC \cos \alpha$$

$$\text{But, } AC = N; \text{ i.e. } AD = N \cos \alpha$$

$$\text{and, } \sin \alpha = \frac{CD}{AC} \Rightarrow CD = AC \sin \alpha$$

$$\text{Now in } \triangle OCD \Rightarrow OC^2 = OD^2 + CD^2$$

$$R^2 = (OA + AD)^2 + CD^2 = (M + N \cos \alpha)^2 + (N \sin \alpha)^2$$

$$= M^2 + N^2 \cos^2 \alpha + 2MN \cos \alpha + N^2 \sin^2 \alpha$$

$$R = \sqrt{(M^2 + N^2 + 2MN \cos \alpha)}$$

Let, the angle made by the resultant R with the direction of force M is given by θ .

$$\therefore \text{In } \triangle OCD, \tan \theta = \frac{CD}{OD} = \frac{N \sin \alpha}{(M + N \cos \alpha)}$$

$$\therefore \theta = \tan^{-1} \left[\frac{N \sin \alpha}{(M + N \cos \alpha)} \right]$$

θ is the direction of the resultant R .

15. b) The resultant of two forces M & N is R . If N is doubled, R is doubled and if M is reversed again doubled. Show that $M : N : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$.

[WBSCTE 2018]

Answer:

The resultant of two forces M and N is R .

$$\therefore R^2 = M^2 + N^2 + 2MN \cos \theta \quad \dots (1)$$

Now, if N is doubled, R is doubled.

$$\Rightarrow 4R^2 = M^2 + 4N^2 + 4MN \cos \theta \quad \dots (2)$$

Again, if M is reversed, R is again doubled.

$$\therefore (2R)^2 = (-M)^2 + N^2 + 2[-M][N] \cos(180^\circ - \theta)$$

[$\therefore$ Reversing a vector means changing its direction by 180°]

$$\Rightarrow 4R^2 = M^2 + N^2 - 2MN \cos \theta \quad \dots (3)$$

Now, (2) - (3),

$$0 = 3N^2 + 6|N||M| \cos \theta$$

Assuming N as non-zero vector / forces

$$\Rightarrow -3|N| = 6|M| \cos \theta$$

$$\Rightarrow N = -2M \cos \theta$$

$$\Rightarrow N^2 = 4M^2 \cos^2 \theta \quad \dots (4)$$

POLY-Me

Now, (1) - (2),

$$3R^2 = 3(4M^2 \cos^2 \theta) + 2M(-2M \cos \theta) \cos \theta$$

$$\Rightarrow 3R^2 = 12M^2 \cos^2 \theta - 4M^2 \cos^2 \theta = 8M^2 \cos^2 \theta$$

$$\Rightarrow R^2 = \frac{8M^2 \cos^2 \theta}{3} \quad \dots (5)$$

Now, substituting the value of M and N in Eqn. (1), we get,

$$\frac{8M^2 \cos^2 \theta}{3} = M^2 + (4M^2 \cos^2 \theta) + 2M(-2M \cos \theta) \cos \theta$$

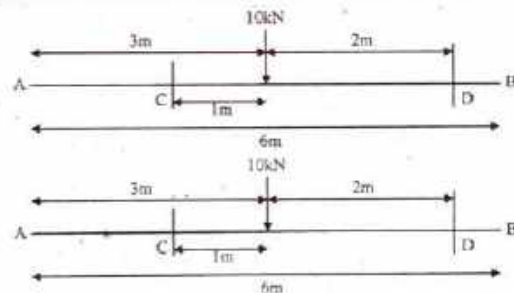
Assuming M as non-zero vector / forces

$$M^2 = \frac{8M^2 \cos^2 \theta}{3} \quad \dots (6)$$

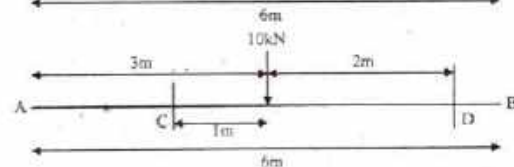
$$\therefore M^2 : N^2 : R^2 = \frac{8M^2 \cos^2 \theta}{3} : 4M^2 \cos^2 \theta : \frac{8M^2 \cos^2 \theta}{3} = \frac{8}{3} : 4 : \frac{8}{3} = 8 : 12 : 8 = 2 : 3 : 2$$

$$\Rightarrow M : N : R = \sqrt{2} : \sqrt{3} : \sqrt{2} \quad [\text{Proved}]$$

16. Replace the 10kN force by two downward force acting at C and D respectively, which will disturb the overall external effect on the beam. [WBSCTE 2018]



Answer:



Let us consider the forces applied at C and D be P_1 and P_2 respectively

The slope at A in case of load W at C is:

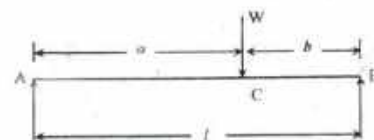
$$\theta_A = \frac{Wb(l^2 - b^2)}{6EI}$$

Slope at A due to P_1 at C

$$\theta_{AP_1} = \frac{P_1 \times 4 \times (6^2 - 4^2)}{36EI} = \frac{80P_1}{36EI}$$

Slope at B due to P_1 at C

$$\theta_{BP_1} = \frac{P_1 \times 2 \times 4 \times (2 \times 6 - 4)}{36EI} = \frac{64P_1}{36EI}$$



Again slope at A due to P_2 at D

$$\theta_{AP_2} = \frac{P_2 \times 1 \times (6^2 - 4^2)}{36EI} = \frac{35P_2}{36EI}$$

Slope at B due to P_2 at D

$$\theta_{BP_2} = \frac{P_2 \times 5 \times 1 \times (2 \times 6 - 1)}{36EI} = \frac{55P_2}{36EI}$$

When 10kN force was applied at mid of the beam, slopes at both A and B same. Applying the same logic we can write,

Slope at A due to P_1 and P_2 = Slope at B due to

$$\therefore \theta_{AP_1} + \theta_{AP_2} = \theta_{BP_1} + \theta_{BP_2}$$

$\therefore$ We can write

$$\frac{80P_1}{36EI} + \frac{35P_2}{36EI} = \frac{64P_1}{36EI} + \frac{55P_2}{36EI}$$

$$\Rightarrow 80P_1 + 35P_2 = 64P_1 + 55P_2$$

$$\Rightarrow 16P_1 = 20P_2$$

$$\Rightarrow P_1 = \frac{5}{4}P_2$$

.... (1)

Slope θ when 10kN force was applied the midpoint as given by

We can therefore say

$$\frac{80P_1}{36EI} + \frac{35P_2}{36EI} = \frac{360 \times 10^3}{16EI}$$

$$\Rightarrow 80P_1 + 35P_2 = \frac{360 \times 10^3 \times 36}{16}$$

$$\Rightarrow 80P_1 + 35P_2 = 910 \times 10^3$$

$$\Rightarrow 80 \times \frac{5}{4}P_2 + 35P_2 = 910 \times 10^3$$

[From (1)]

$$\Rightarrow 100P_2 + 35P_2 = 910 \times 10^3$$

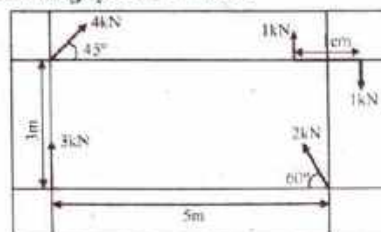
$$\Rightarrow 135P_2 = 910 \times 10^3$$

$$\Rightarrow P_2 = \frac{910}{135} \times 10^3 = 6 \times 10^3 \text{ N} = 6 \text{ kN}$$

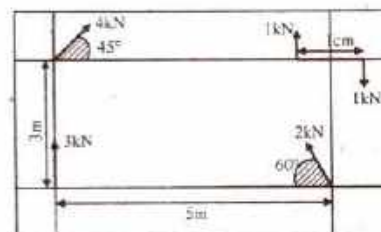
$$\text{Now } P_1 = \frac{5}{4} \times P_2 = \frac{5}{4} \times 6 = 7.5 \text{ kN}$$

$$\therefore P_1 = 7.5 \text{ kN} \quad P_2 = 6 \text{ kN}$$

17. Find the magnitude of resultant force and also the angle through which the resultant acts for following system of forces. [WBSCTE 2018]



Answer:



Net force acting along x direction

$$F_x = 4 \cos 45^\circ + (-2 \cos 60^\circ) = 4 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{2} = 2\sqrt{2} - 1 = 1.828$$

$$F_y = 4 \sin 45^\circ + 1 - 1 + 2 \sin 60^\circ + 3 = 2\sqrt{2} + \sqrt{3} + 3 = 2.828 + 1.732 + 3 = 7.56$$

Resultant force,

$$R = \sqrt{(1.828)^2 + (7.56)^2} = \sqrt{3.34 + 57.15} = \sqrt{60.49} = 7.777 \text{ kN}$$

We know

$$\tan \theta = \frac{F_y}{F_x} = \frac{7.56}{1.83}$$

$$\theta = \tan^{-1} \left(\frac{7.56}{1.83} \right) = 76.39^\circ$$

18. a) Describe briefly the types of loading and types of support of the beam. [WBSCTE 2018]

Answer:

Types of Loading:

The following are the important types of loading:

- Concentrated or point load
- Uniformly distributed load
- Uniformly varying load.

Concentrated or point load: Fig. 1 shows a beam AB, which is simply supported at the ends A and B. A load W is acting at the point C. This load is known as point load (or concentrated load). Hence any load acting at a point on a beam, is known as point load.

In actual practice, it is not possible to apply a load at a point (i.e., at a mathematical point) as it must have some contact area. But this area in comparison to the length of the beam is too small (or area is negligible).

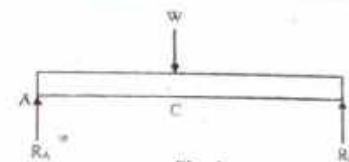


Fig. 1

Uniformly distributed load: If a beam is loaded in such a way, that each unit length of the beam carries same intensity of the load, then that type of load is known as uniformly distributed load (which is written as U.D.L.).

Fig. 2 shows a beam AB, which carries a uniformly distributed load.

For finding the reactions the total uniformly distributed load is assumed to act at the centre of gravity of the load.

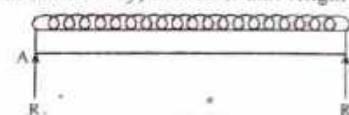


Fig. 2

Uniformly varying load: Fig. 3 shows a beam AB, which carries load in such a way that the rate of loading on each unit length of the beam varies uniformly. This type of load is known as uniformly varying load. The total load on the beam is equal to the area of the load diagram. The total load acts at the centre of gravity of the load diagram.

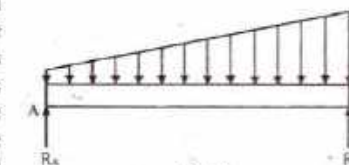


Fig. 3

Types of Supports:

Though there are many types of supports, yet the following are important from the subject point of view:

- Simple supports or knife edge supports
- Roller support
- Pin-joint (or hinged) support
- Smooth surface support
- Fixed or built-in support

Simple supports or knife edge supports: A beam supported on the knife edges A and B is shown in Fig. 4(a). The reactions at A and B in case of knife edge support will be normal to the surface of the beam. The reactions R_A and R_B with free body diagram of the beam is shown in Fig. 4(b).

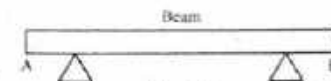


Fig. 4(a)



Fig. 4(b)

Roller support: A beam supported on the rollers at points A and B is shown in Fig. 5(a). The reactions in case of roller supports will be normal to the surface as shown in Fig. 5(b).



Fig. 5(a)

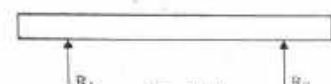


Fig. 5(b)

Pin-joint (or hinged) support: A beam, which is hinged (or pin-joint) at point A, is shown in Fig. 6. The reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If the load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined.



Fig. 6

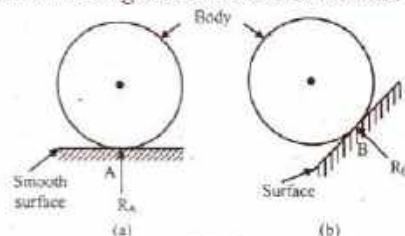


Fig. 7

Smooth surface support: Fig. 7 shows a body in contact with a smooth surface. The reaction will always act normal to the support as shown in Fig. 7(a) and 7(b).

Fig. 8 shows a rod AB resting inside a sphere, whose surface are smooth. Here the rod becomes body and sphere becomes surface. The reactions on the ends of the rod (i.e., at point A and B) will be normal to the sphere surface at A and B. The normal at any point on the surface of the sphere will always pass through the centre of the sphere. Hence reactions R_A and R_B will have directions AO and BO respectively as shown in Fig. 8.

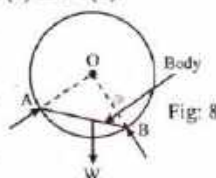


Fig. 8

Fixed or built-in support: Fig. 9 shows the end A of a beam, which is fixed. Hence the support at A is known as a fixed support. In case of fixed support, the reaction will be inclined. Also the fixed support will provide a couple.

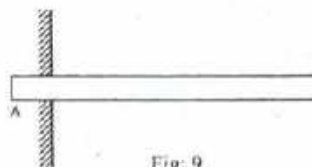
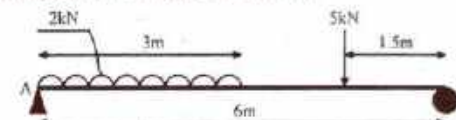
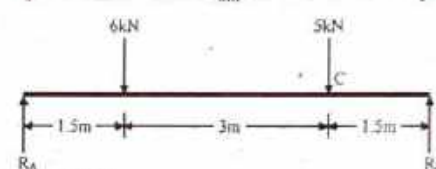
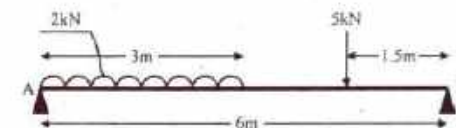


Fig. 9

18. b) A simply supported beam AB of 6m span is subjected to loading as shown in Fig. Find the support Reaction at A and B. [WBSCTE 2018]



Answer:



$$R_A + R_B = (6 + 5) = 11 \text{ kN}$$

Taking moment w.r.t. pt. A,

$$R_B \times 6 = (6 \times 1.5) + (5 \times 4.5) = 31.5$$

$$\Rightarrow R_B = 5.25 \text{ kN}$$

$$\therefore R_A = (11 - 5.25) \text{ kN} = 5.75 \text{ kN}$$

19. a) The resultant of two forces is 10N and it is inclined at 60° to one of the forces whose magnitude is 5N. Find the magnitude and direction of the other force.

b) State Triangle Law and Parallelogram Law of forces.

[WBSCTE 2022]

Answer:

a) The resultant of two forces, $P = 10 \text{ N}$ and $Q = 5 \text{ N}$

Angle between the two forces, $\alpha = 60^\circ$

The magnitude of the resultant force (R) =

$$\begin{aligned} \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} &= \sqrt{(10)^2 + (5)^2 + 2 \cdot 10 \cdot 5 \cdot \cos 60^\circ} \\ &= \sqrt{100 + 25 + \left(100 \times \frac{1}{2}\right)} = \sqrt{175} = 13.22 \text{ N} \end{aligned}$$

b) Refer to Article No. 1.3 of Unit at a Glance.

Unit 2: Moments and Couples

Unit at a glance:

2.1 Moment of a force

1. Moment of a Force about a Point

Moment of force- (often just *moment*) is the tendency of a force to twist or rotate an object.

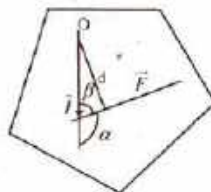
The moment of a force about a point is defined as the product of the magnitude of the force by the perpendicular distance from the point to the line of action of the force.

Moment of force F about O is Fd .

Direction M is perpendicular to the plane of the point and the force with a sense determined by right hand corkscrew rule.

$$\alpha = 180^\circ - \beta$$

$$\vec{M} = \vec{r} \times \vec{F}$$



2. Moment of a Couple

A couple is a pair of forces, equal in magnitude, oppositely directed, and displaced by perpendicular distance.

Let us consider two equal, opposite and non-collinear forces

$\vec{F}$ and $-\vec{F}$ (known as a couple) as shown in Fig.

Now, moment of the above system of forces about any arbitrary point O ,

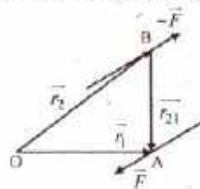
$$\vec{M} = \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\vec{F}), \text{ where } \vec{r}_1 \text{ and } \vec{r}_2 \text{ refer to position vectors of}$$

two arbitrary points A and B, located on the lines of action of $\vec{F}$ and $-\vec{F}$ respectively.

$$\therefore \vec{M} = (\vec{r}_1 - \vec{r}_2) \times \vec{F} \quad \text{i.e., } \vec{M} = \vec{r}_{21} \times \vec{F} \quad \dots(1)$$

Observe that r_{21} is independent of choice of the moment centre O . Thus, moment of the

couple, defined by the vector $\vec{M}$ in equation (1), is independent of the point about which the moment is computed. We will see subsequently that such a vector is called a **free vector**.



Moments and Couples

Me.27

3. Varignon's Theorem

The net moment of a system of forces about a point is equal to the moment of the resultant about the same point. It can also be stated, as "the moment of a vector about a point is same as that of the sum of the moments of its components about same point."

4. Properties of Couples

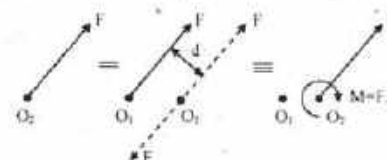
The properties which distinguish one couple from every other couple are called its characteristics. A couple whether positive or negative, has the following properties/characteristics.

- (1) The algebraic sum of the forces constituting a couple is zero.
- (2) The algebraic sum of the moments of the forces forming a couple in the same about any point in their plane.
- (3) The couple can be balanced only by another couple of the same moment but of the opposite sense.
- (4) The net effect of a number of coplanar couple is equivalent to the algebraic sum of the effects of each of the couples.

A couple is frequently indicated by a clockwise or counter-clockwise arrow when coplanar force systems are involved instead of showing two separate forces.

5. Replacement of a force by a force and couple

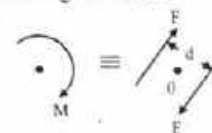
Consider a force F acting at point O_1 . Imagine two equal and opposite force F parallel to given force acting at O_2 as shown in the figure below.



These two additional forces do not alter the system. The new system is equivalent to a force F acting at O_2 , plus a couple of moment $M = F.d$.

6. Replacement of a couple by two forces

Consider a couple of moment M , where the axis of the couple is through O perpendicular to the plane of paper as shown in the figure below.



This couple is equivalent to any two parallel forces of magnitude F acting at a distance d apart such that $F.d = M$ and the directions of the forces so chosen as to give the correct direction of M .

Short Answer Type Questions

A. Fill in the blanks in the following statements:

- The unit of moment in S.I. unit is Newton – meter (Nm) [WBSCTE 2005, 2009]
- Varignon's theorem is applicable only to coplanar concurrent and non-concurrent forces. [WBSCTE 2006, 2007]
- Equivalent couple is one where two couples having the same moment. [WBSCTE 2013]
- According to Varignon's theorem algebraic sum of moment of all coplanar forces about any point in their plane is equal to Moment of their resultant force about the same point. [WBSCTE 2013, 2014]
- The resultant of two unlike unequal parallel forces is located at the point lies outside of the line and on the same side as the larger force. [WBSCTE 2013]
- The resultant of a force and a couple is a force and a couple. [WBSCTE 2015]
- If the arm of a couple is doubled, its moment will be doubled. [WBSCTE 2016]
- A resultant force is a single force which can replace two or more forces and produce the same effect as the force. [WBSCTE 2017]
- A couple has no resultant force but has resultant moment. [WBSCTE 2017]
- Principle of moment is based on Varignon's theorem. [WBSCTE 2017]
- Velocity ratio of first system of pulley is 16. [WBSCTE 2017]
- The product of either force of couple with the arm of the couple is called- moment of the couple. [WBSCTE 2022]
- The algebraic sum of moments of the forces forming couple about any point in their plane is - equal to the moment of the couple. [WBSCTE 2022]

B. State whether the following statements are True or False:

- Torque and inertia are related as $T = I\alpha$ [WBSCTE 2010]
Answer: True

Moments and Couples

- Moment of a force is a scalar quantity.
Answer: False [WBSCTE 2010]
 - Two like equal parallel forces constitute a couple.
Answer: False [WBSCTE 2012]
 - Action of a moment on a body creates tendency of rotation of the body.
Answer: True [WBSCTE 2014]
 - Two unlike unequal parallel forces constitute a couple.
Answer: False [WBSCTE 2015]
 - Rotational tendency of a force is called resultant.
Answer: False [WBSCTE 2016]
 - If the arm of a couple is doubled, it's moment will be same.
Answer: False [WBSCTE 2017]
 - Moment of a force is a scalar quantity.
Answer: False [WBSCTE 2017]
 - The resultant of two like parallel forces acts at a point between their points of application.
Answer: True [WBSCTE 2017]
 - Equivalent couple is one where moment and direction are different.
Answer: False [WBSCTE 2017]
- #### C. Pick up the correct answer:
- Numerical value of Moment of a force (vector) about any point is equal to (2 times/3 times / 0.5 times) the area of triangle constructed by that vector and the given point
Answer: 2 times [WBSCTE 2011]
 - The resultant of a force and a couple is a (force / couple / force and a couple)
Answer: force and a couple [WBSCTE 2012, 2013]
 - The resultant of two non-parallel concurrent forces may be (zero / non zero positive quantity / non zero negative quantity).
Answer: non zero positive quantity. [WBSCTE 2015]

Long Answer Type Questions

1. State and prove Varignon theorem. [WBSCTE 2004, 2009, 2016, 2018]

OR,

State and explain Varignon's theorem [WBSCTE 2010, 2013]

OR,

State Varignon's theorem. [WBSCTE 2011]

OR,

State and show by a sketch Varignon's Theorem. [WBSCTE 2022]

Answer:

Varignon's theorem of moments states: The algebraic sum of the moments of any two forces about any point in their plane is equal to the moment of their resultant about the same point.

Proof. The net moment of a system of forces about a point is equal to the moment of the resultant about the same point. It can also be stated, as "the moment of a vector about a point is same as that of the sum of the moments of its components about same point."

For concurrent forces: Let $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ are concurrent forces at a point A.

$\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = \vec{R}$ = Resultant force.

Consider moment of forces about O, which is origin of a reference system.

$$\vec{M}_O = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \dots + \vec{r} \times \vec{F}_n$$

$$= \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n) = \vec{r} \times \vec{R} = \text{moment of the resultant.}$$

For Coplanar forces (Parallel forces):

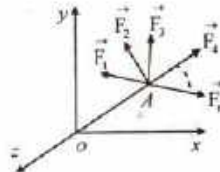
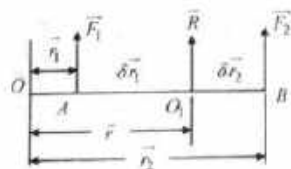
Let us assume,

$\vec{F}_1$ and $\vec{F}_2$ acting at A and B having a resultant R acting at O_1 .

Assume 'O' origin of the reference system.

$$\text{Hence } \vec{M}_O = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = (\vec{r}_1 - \delta \vec{r}_1) \times \vec{F}_1 + (\vec{r}_2 + \delta \vec{r}_2) \times \vec{F}_2$$

$$= \vec{r}_1 \times (\vec{F}_1 + \vec{F}_2) + (\delta \vec{r}_2 \times \vec{F}_2 - \delta \vec{r}_1 \times \vec{F}_1)$$



For all parallel forces in equilibrium the two moments are equal in magnitude but opposite in sense and hence,

$$\vec{\delta r}_2 \times \vec{F}_2 = \vec{\delta r}_1 \times \vec{F}_1$$

$$\vec{M}_O = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \vec{r} \times \vec{R} = \text{moment of the resultant about O.}$$

2. A horizontal bridge (AB) of 18m long weighs 18kN and rest on two support at its ends. What will be the pressure on each support when a lorry of weight 5kN starting from A is two third of the way across the bridge. [WBSCTE 2007, 2017]

Answer:

$$AP = \frac{2}{3} \times AB (\text{given}) = \frac{2}{3} \times 18\text{m} = 12\text{m}$$

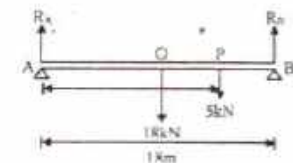
For equilibrium we can say $\sum \text{Resultant force} = 0$

Therefore, $R_A + R_B = 18 + 5 = 23\text{kN}$.

Taking a moment about a point A, it can be written

$$R_A \times 0 - 18 \times 9 - 5 \times 12 + R_B \times 18 = 0 \Rightarrow R_B = 12.33\text{kN}$$

So, R_A will be $= (23 - 12.33) = 10.66\text{ kN}$ (Ans.)



3. Two men carry a concrete block weighing 400N using a light wooden plank on their shoulders. The stronger man can carry 225N. How should the block be supported so that the stronger man gets his share if the length of the plank between the shoulders is 3.5m? [WBSCTE 2009, 2017]

Answer:

Suppose two men are located at point A and B.

Let A is the stronger man

So A can carry 225N

Again R_A & R_B are the reaction of the two men.

$$\therefore R_A + R_B = 400\text{N}$$

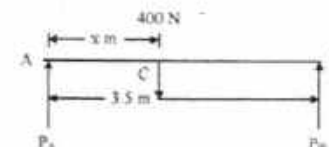
$$\therefore \text{So } R_A = 225\text{N}$$

$$\therefore R_B = 400 - 225 = 175\text{N}$$

Now taking moment about A, $R_B \times 3.5 = 400 \times x$

$$\text{or, } x = \frac{175 \times 3.5}{400} = 1.531\text{m.}$$

So the distance of the load C, is 1.531 m away from the stronger person.

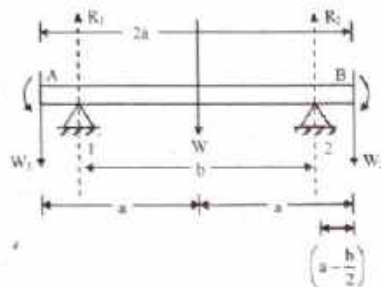


4. A uniform plank of length 2a and weight W is supported horizontally on two vertical props at a distance b apart. The greatest weight that can be placed at the two ends without upsetting the plank are

$$W_1, (W_1 + W) + W_2, (W_2 + W) = b/a.$$

[WBSCTE 2011]

Answer:


 From the figure, we have distance between the props support = b .

 Span of plank = $2a$.

 Two extreme weights support at two ends A and B respectively, are W_1 & W_2 without disturbing the equilibrium

$$W_1 + W_2 + W = R_1 + R_2$$

... (i) [As per Varignon's theorem]

Considering the moments about prop support (1)

$$+W_1 \times \left(a - \frac{b}{2}\right) - W \times \frac{b}{2} + R_2 \times b - W_2 \times \left(b + a - \frac{b}{2}\right) = 0$$

$$\therefore R_2 \times b = W_2 \times \left(a + \frac{b}{2}\right) - W_1 \times \left(a - \frac{b}{2}\right) + W \times \frac{b}{2} \quad \dots (ii)$$

Considering the moments about prop support (2)

$$-W_2 \times \left(a - \frac{b}{2}\right) + W \times \frac{b}{2} - R_1 \times b + W_1 \times \left(b + a - \frac{b}{2}\right) = 0$$

$$\therefore W_2 \times \left(a - \frac{b}{2}\right) + W \times \frac{b}{2} + W_1 \times \left(a + \frac{b}{2}\right) = R_1 \times b \quad \dots (iii)$$

$$\begin{aligned} \therefore (R_1 + R_2) \times b &= W_2 \times \left(a + \frac{b}{2}\right) - W_1 \times \left(a - \frac{b}{2}\right) + W \times \frac{b}{2} + W_2 \times \left(a - \frac{b}{2}\right) + W \times \frac{b}{2} + W_1 \times \left(a + \frac{b}{2}\right) \\ &= W \times b + W_2 \times \left(a + \frac{b}{2} + a - \frac{b}{2}\right) + W_1 \times \left(a + \frac{b}{2} - a + \frac{b}{2}\right) \end{aligned}$$

$$(W_1 + W_2 + W)b = W \times b + W_2 \times 2a + W_1 \times (b)$$

$$(W_1 + W_2 + W - W - W_1)b = W_2 \times 2a$$

$$W_2 \times b = W_2 \times 2a \quad \therefore b = 2a$$

it signifies that the greatest weight should be supported by the plank if the proper distance is equal to the length of plank.

5. Find the resultant (both magnitude and direction) of two coplanar concurrent forces P and Q acting at an angle α . [WBSCTE 2011]

Answer:

 Resultant of two forces F_1 & F_2 is given by

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

 R = resultant Force, θ = angle between two forces

Given:

$$F_1 = F_2, R = F_1$$

$$\text{Now, } R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$F_1 = \sqrt{F_1^2 + F_1^2 + 2F_1F_1 \cos \theta}$$

$$F_1^2 = 2F_1^2 + 2F_1^2 \cos \theta$$

$$2 \times \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\cos \theta = \cos 120^\circ$$

$$\theta = 120^\circ$$

6. (a) Drawing a suitable diagram show the location of the resultant of two unlike unequal parallel forces acting in a plane. What will happen if the magnitude of these two forces becomes equal.

(b) The constituent forces of a couple of moment G act at A and B, if their lines or action are turned through a right angle, they form a couple of moment H. When they both act at right angles to AB, show that they form a couple of moment $(G^2 + H^2)^{0.5}$. [WBSCTE 2012]

Answer:

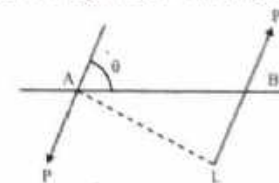
a)


 $\vec{a}$ & $\vec{b}$ are two forces which are parallel unlike and unequal.

 Their resultant will be $\vec{a} - \vec{b}$ as $a > b$ so $\vec{R} = \vec{a} - \vec{b}$ and the resultant vector will act along the direction of $\vec{a}$.

If the two forces become equal then the resultant will be zero.

- b) Let the constituent forces be of magnitude P each, acting at A and B at an angle θ with AB.


 Let $AB = d$

$$G = P \cdot AL = P \cdot d \sin \theta$$

.... (i)

When each force is turned through one right angle, the moment is H .

$$\therefore H = Pd \sin(90^\circ + \theta) = Pd \cos \theta \quad \dots (ii)$$

Squaring and adding (i) & (ii),

$$G^2 + H^2 = P^2 d^2 \quad \text{or, } Pd = \sqrt{G^2 + H^2}$$

7. (a) Forces P and Q having a resultant R act at a point. If any transversal cuts the lines of action of the forces P , Q and R at points L , M and N respectively show that $P/OL + Q/OM = R/ON$.

(b) A beam AB is simply supported over a span of 5 m. It is subjected to uniformly distributed loading 2 kN/m over a span of 3 m from support A towards B and a concentrated load of 3 kN at a distance 2 m from B . Find support reactions at A and B . [WBSCTE 2015]

Answer:

a) Let $\vec{P}$, $\vec{Q}$ and $\vec{R}$ makes angles α , β and γ with x -axis respectively.

The transversal of AB cuts the lines of action of the forces at point L , M and N respectively.

Since $\vec{R}$ is resultant of $\vec{P}$ and $\vec{Q}$.

$\therefore$ By theorem of resolved parts

$$R \cos \gamma = P \cos \alpha + Q \cos \beta \quad \dots (i)$$

From figure $\cos \alpha = \frac{OL}{OP}$

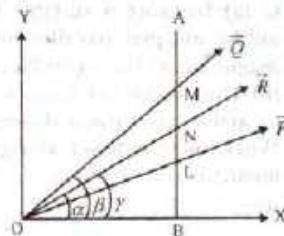
$$\cos \beta = \frac{OM}{OP}$$

and $\cos \gamma = \frac{ON}{OP}$

Using these values in Eqn. (i), we get

$$R \left(\frac{ON}{OP} \right) = P \left(\frac{OL}{OP} \right) + Q \left(\frac{OM}{OP} \right)$$

$$\therefore \frac{R}{ON} = \frac{P}{OL} + \frac{Q}{OM} \quad (\text{Proved})$$



b) Length of beam = 5 m

Rate of U.D.L. = 2 kN/m

Length of U.D.L. = 3 m

$\therefore$ Total load due to U.D.L.

$$= (\text{Length of U.D.L.}) \times \text{Rate of U.D.L.} = 3 \times 2 = 6 \text{ kN}$$

This load of 6 kN will be acting at the middle point of AC .

Let R_A = Reaction at A .

and R_B = Reaction at B .

Taking the moments of all forces about point A .

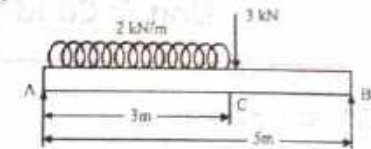
We get

$$R_B \times 5 - 6 \times 1.5 - 3 \times 3 = 0$$

$$\therefore R_B = 3.6 \text{ kN} \quad (\text{Ans.})$$

$\therefore$ Total load on the beam = $10 + 3 = 13 \text{ kN}$

$$\therefore R_A = 13 - 3.6 = 9.4 \text{ kN} \quad (\text{Ans.})$$



8. State and explain Varignon's theorem. [WBSCTE 2015]

Answer:

Refer to Question No. 1 of Long Answer Type Questions.

9. Define the properties of couple. [WBSCTE 2016]

Answer:

Refer to Question No. 11 of Long Answer Type Questions.

10. Give geometrical representation of moment of a force. [WBSCTE 2017]

Answer:

Refer to Article No. 2.1.1 of Unit at a Glance.

11. Define the properties of a couple. [WBSCTE 2018]

Answer:

Properties of a couple:

- As the two forces forming couple are equal and opposite the couple doesn't produce translational motion.
- When it acts on a body net resultant force acting on a body is zero.
- It produces pure rotational motion in the body this is because the algebraic sum of the moments of the two forces about any point in their plane is not zero.
- Moment of a couple about any point in its plane is constant, both in magnitude and direction.

12. The resultant of a force P of 40 N and an unknown force Q , is known to be perpendicular to P . If P is replaced by a force of 70 N in the same line of action, the resultant would be inclined at 45° to the force of 70 N. Find the magnitude and direction of unknown force Q . [WBSCTE 2022]

Answer:

The magnitude of unknown force,

$$Q = \sqrt{(40)^2 + (70)^2} = \sqrt{1600 + 4900} = \sqrt{6500} = 80.62 \text{ N}$$

The direction of unknown force,

$$\tan \theta = \frac{Q}{P} = \frac{80.62}{40} = 2.0155$$

$$\Rightarrow \theta = \tan^{-1} 2.0155 = 0.035$$

Unit 3: Condition of Equilibrium

Unit at a glance:

An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation.

1. Conditions of equilibrium

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0 \text{ and } \sum M_x = 0, \sum M_y = 0, \sum M_z = 0.$$

Equilibrium for two force system $\Rightarrow$ If and only if the two forces are equal in magnitude, opposite in sense and collinear in action.

Equilibrium for three force system $\Rightarrow$ If either the forces meet at one point or they are parallel. Reverse is not always true.

2. Lami's Theorem

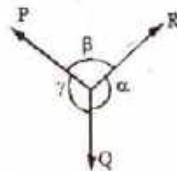
This theorem can be applied when a particle is in equilibrium under the action of three forces. It states,

"If three coplanar forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces".

Let us consider three co-planar forces P, Q & R with included angles α, β and γ as shown in Fig.

According to Lami's Theorem

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



3. Equilibrium of a Coplanar Force System

Equilibrium of a coplanar force system occurs if the resultant is neither a force R nor a couple C. The necessary and sufficient conditions that R and C be zero vectors are

$$R = \sum F = 0 \text{ and } C = \sum M = 0$$

Where $\sum F$ = vector sum of all forces of the system

$\sum M$ = vector sum of the moments (relative to any point) of all the forces of the system.

The two vector equations above may be applied directly, or the following derived scalar equations may be used for the three types of coplanar force systems.

4. Concurrent System

Any of the following sets of equations ensures equilibrium of a concurrent system i.e., the resultant is zero. Assume concurrency at origin.

Condition of Equilibrium

Me.37

Set	Equations of Equilibrium	Remarks
A	(1) $\sum F_x = 0$ (2) $\sum F_y = 0$	$\sum F_x$ = algebraic sum of the x components of the forces of the system. $\sum F_y$ = Algebraic sum of the y components of the forces of the system.
B	(1) $\sum F_x = 0$ (2) $\sum M_A = 0$	$\sum M_A$ = Algebraic sum of the moments of the forces of the system about A, which may be chosen any place in the plane except on the y axis.
C	(1) $\sum M_A = 0$ (2) $\sum M_B = 0$	$\sum M_A$ and $\sum M_B$ = algebraic sums of the moments of the forces of the system about A and B, which may be chosen any place in the plane provided A, B and the origin do not lie on the same straight line

If only three nonparallel forces act in a plane on a body in equilibrium, these three forces must be concurrent.

5. Nonconcurrent, Nonparallel System

Any of the following sets of equations ensures equilibrium of a non concurrent, non parallel system; i.e., the resultant is neither a force nor a couple.

Set	Equations	Remarks
A	(1) $\sum F_x = 0$ (2) $\sum F_y = 0$ (3) $\sum M = 0$	$\sum F_x$ = algebraic sum of x components of the forces $\sum F_y$ = algebraic sum of y components of the forces $\sum M$ = algebraic sum of the moments of the forces of the system
B	(1) $\sum F_x = 0$ (2) $\sum M = 0$ (3) $\sum M = 0$	$\sum F_x$ = algebraic sum of x components of the forces $\sum M$ & $\sum M$ = algebraic sums of the moments of the forces of the system About any two points A and B in the plane, provided that The line joining A and B is not perpendicular to the x-axis.
C	(1) $\sum M = 0$ (2) $\sum M = 0$ (3) $\sum M = 0$	$\sum M_A, \sum M_B$ & $\sum M_C$ = Algebraic sums of the moments of the forces of the system about any three points A, B and C in the plane, provided that A, B and C do not lie on the same straight line.

Short Answer Type Questions

A. Fill in the blanks in the following statements:

1. Lami's theorem is used when a particle is acted upon by three concurrent forces in equilibrium. [WBSCTE 2009, 2010]
2. The equivalent couple is one when two forces produce the same moment. [WBSCTE 2015]
3. If for a number of coplanar concurrent forces the force polygon is closed the forces are in Equilibrium. [WBSCTE 2016]
4. Equilibrant force is equal and opposite to the resultant of several forces acting on the body. [WBSCTE 2017]
5. The effect of a couple acting on a rigid body is to rotate it without moving it as a whole. [WBSCTE 2018]
6. If a body is in equilibrium. We may conclude that the resultant of all the forces acting on it is zero and the moment of the forces about any point is zero. [WBSCTE 2018]
7. The Lami's Theorem is applicable only for three coplanar and concurrent and non-collinear forces. [WBSCTE 2018]

B. State whether the following statements are True or False:

8. If a system of coplanar forces is in Equilibrium.

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0;$$
 Answer: True. [WBSCTE 2010]
9. Action of a moment on a body creates tendency of rotation of the body. [WBSCTE 2013]
 Answer: True
10. To find support reactions of a simply supported beam application of moment is not required. [WBSCTE 2013]
 Answer: False
11. Under equilibrium condition the vector diagram is a closed polygon. [WBSCTE 2014]
 Answer: True
12. When a ladder is placed leaned to a wall the wall may be frictionless in order to keep the system in stable condition. [WBSCTE 2015]

Answer: False

13. Couple causes rotation tendency of a body. [WBSCTE 2015]
 Answer: True

14. Summation of all forces in X direction is the only necessary condition of equilibrium of a system of coplanar non-concurrent forces. [WBSCTE 2015]
 Answer: False

15. A body is said to be in equilibrium, if it has no linear motion. [WBSCTE 2018]
 Answer: False

16. A couple can be balanced by a force equal to its magnitude. [WBSCTE 2018]
 Answer: False

17. A body isolated from all other supports which are in contact to it, is called a Free body. [WBSCTE 2018]
 Answer: True

18. Turning the cap of a pen is not an example of a couple. [WBSCTE 2018]
 Answer: False

C. Answer the following questions:

19. Write short notes on equivalent couple. [WBSCTE 2017]
 Answer:
 Equivalent couple is one where two couples are having the same moment.

D. Pick up the correct answer:

20. With usual notations Parallel Axis theorem states that

$$(I_{AB} = I_{xx} + Ah^2) / I_{AB} = I_{AB} = Ah^2 / I_{AB} + I_{xx} + A^2h)$$
 Answer: $I_{AB} = I_{xx} + Ah^2$ [WBSCTE 2011]
21. If three forces acting on a body in a plane keep the body in equilibrium then (they must meet at a point / they must be parallel/either they will meet at a point or else they should be parallel). [WBSCTE 2013]
 Answer: either they will meet at a point or else they should be parallel.
22. Lami's theorem is applicable for (3 forces/any number of forces/ any number of non coplanar forces). [WBSCTE 2013]
 Answer: 3 coplanar concurrent forces

23. If three forces acting on a body keep the body in equilibrium then (they must meet at a point / they must be parallel / they either must meet at a point or else they will be parallel). [WBSCTE 2015]

Answer: they either must meet at a point or else they will be parallel.

24. When a body is about to overturn about a point reaction at that point is (zero / maximum / cannot be said) [WBSCTE 2015]

Answer: maximum

25. If a number of forces keep a body in equilibrium the magnitude of their resultant is (maximum / minimum / zero). [WBSCTE 2015]

Answer: zero

26. The resultant of two equal non-collinear unlike parallel forces is (twice the forces / zero / a couple). [WBSCTE 2015]

Answer: a couple

Long Answer Type Questions

1. State Lami's theorem. [WBSCTE 2005, 2009, 2022]

OR,

State and explain Lami's theorem.

[WBSCTE 2010, 2011, 2013]

Answer: Refer to Article No. 2 of Unit at a Glance.

2. A machine weighting 1500N is supported by two chains attached to some point on the machine. One of these chain goes to the eye bolts in the wall and is inclined at 30° to the horizontal and other goes to the hook in ceiling and is inclined at 45° to that horizontal. Find the tensions in the two chains? [WBSCTE 2010]

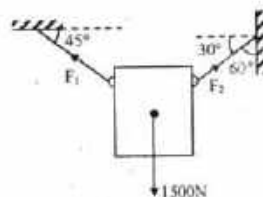
Answer:

Using Lami's theorem we get

$$\frac{F_1}{\sin 120^\circ} = \frac{F_2}{\sin 135^\circ} = \frac{1500}{\sin 105^\circ}$$

$$F_1 = 1500 \times \frac{\sin 120^\circ}{\sin 105^\circ} = 1837.11 \text{ N}$$

$$F_2 = 1500 \times \frac{\sin 135^\circ}{\sin 105^\circ} = 1098.07 \text{ N}$$



3. A man and boy carry a weight of 300N between them by means of uniform pole 2m long & weighing 100N, where must the weight be placed so that the man may bear twice as much of the weight as that of the boy? [WBSCTE 2010]

Answer:

Using the law of equilibrium we get,

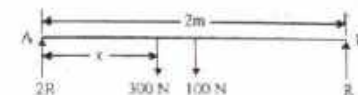
$$2R + R = 400$$

$$\Rightarrow 3R = 400$$

$$\Rightarrow R = 133.3 \text{ N}$$

Taking moment w.r.t. point A, we get,

$$R \times 2 = 100 \times 1 + 300 \times x \Rightarrow x = 0.555 \text{ m}$$



4. A string ABC has its extremities tied to two fixed points A and B in the same horizontal line; to a given point C in the string is knotted a given weight W.

Prove that the tension in the portion CA is $\frac{Wb(c^2 + a^2 - b^2)}{4c\Delta}$ where a, b, c are the

sides and Δ the area of the triangle ABC.

[WBSCTE 2011]

Answer:

By considering

$$T_1 = a \text{ and } T_2 = b$$

$$W = c$$

According to Lami's Theorem, we get

$$\frac{a}{\sin(90^\circ + \beta)} = \frac{b}{\sin(90^\circ + \alpha)} = \frac{W}{\sin(180^\circ - (\alpha + \beta))}$$

$$T_1 + T_2 = W \quad \therefore a + b = W$$

By the properties of triangles,

$$\frac{T_1}{\sin(90^\circ - \beta)} = \frac{T_2}{\sin(90^\circ - \alpha)} = \frac{W}{\sin(\alpha + \beta)}$$

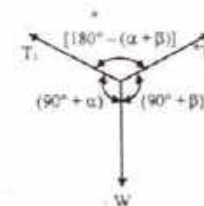
$$\frac{a}{\sin(90^\circ - \beta)} = \frac{b}{\sin(90^\circ - \alpha)} = \frac{W}{\sin(\alpha + \beta)} \quad [\text{By putting the value in the triangular figure}]$$

$$\frac{a}{\cos \beta} = \frac{b}{\cos \alpha} = \frac{W}{\sin(\alpha + \beta)}$$

$$\therefore \left[\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} \right] \text{ properties } \Delta': \Delta = \frac{1}{2} ac(\sin \beta)$$

The tension in string 'AC' is

$$\therefore \left[\frac{Wb(c^2 + a^2 - b^2)}{4c\Delta} \right] \quad (\text{Proved})$$



5. Two forces P, Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight W on the plane; prove

$$\text{that } \frac{1}{P^2} + \frac{1}{Q^2} = \frac{1}{W^2}.$$

[WBSCTE 2012]

Answer:

Let α be the inclination of plane, by Lami's theorem in the first case,

$$\frac{P}{\sin(90^\circ + 90^\circ - \alpha)} = \frac{W}{\sin 90^\circ}$$

$$\frac{P}{\sin \alpha} = W$$

$$\text{or, } \operatorname{cosec} \alpha = \frac{W}{P}$$

By Lami's theorem in the second case,

$$\frac{Q}{\sin(90^\circ + 90^\circ - \alpha)} = \frac{W}{\sin(90^\circ + \alpha)}$$

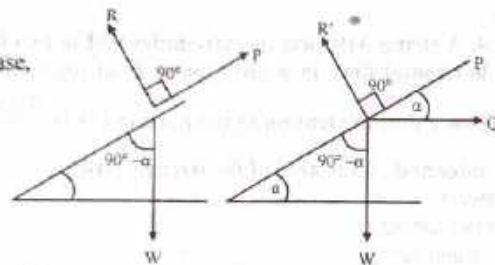
$$\therefore \frac{Q}{\sin \alpha} = \frac{W}{\cos \alpha}$$

$$\text{or, } \cot \alpha = \frac{W}{Q}$$

$$\text{Now, } \operatorname{cosec}^2 \alpha - \cot^2 \alpha = 1$$

$$\therefore \left(\frac{W}{P}\right)^2 - \left(\frac{W}{Q}\right)^2 = 1$$

$$\text{or, } \frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{W^2}$$



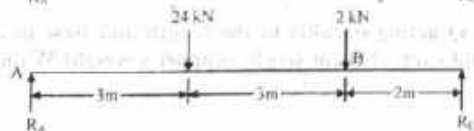
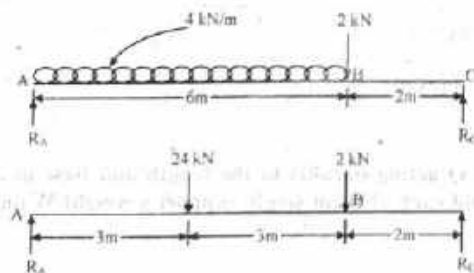
6. (a) A simply supported beam has a span of 8.0 m length. It is subjected to uniformly distributed loading of 4 kN/m from left support up to 6.0 m length. It is also subjected to a concentrated load of 6 kN at a distance 2.0 m from right support. Find the Support Reactions for this beam.

(b) A roller of radius $R = 300$ mm and weight $W = 2500$ N is to be pulled over a curb (obstacle) by a horizontal pull P applied at the centre of the roller. Find the minimum value of P such that the roller can cross the curb.

[WBSCTE 2013]

Answer:

a)



POLY-ME

$$R_d + R_c = (24 + 2) \text{ kN} = 26 \text{ kN}$$

--- (1)

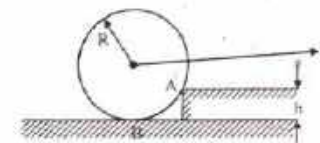
Taking moment w.r.t. point A, we can write,

$$R_c \times 8 = 24 \times 3 + 2 \times 6$$

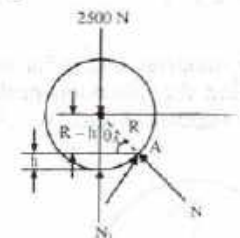
$$\therefore R_c = 10.5 \text{ kN}$$

$$\therefore R_d = 15.5 \text{ kN}$$

b)



When the roller starts rolling over the curb, contact between the roller and the ground at B is lost, which means that the ground will not exert any force on the roller under this condition. i.e., $N_B = 0$



For rolling the roller over the curb, net clockwise moment of all forces about A must be greater than zero.

$$\text{i.e., } P \times R \sin \theta - 2500 \times R \cos \theta > 0$$

$$\text{Since } R \neq 0 \therefore P > 2500 \cot \theta$$

$$\text{Now } \sin \theta = \frac{R-h}{R}$$

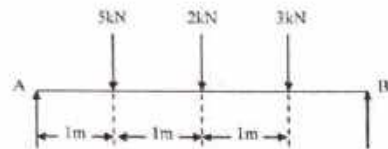
$$\therefore \theta = \sin^{-1} \left(\frac{R-h}{R} \right)$$

$$\therefore \text{Minimum value of } P \text{ is } 2500 \cot \theta, \text{ where } \theta = \sin^{-1} \left(\frac{R-h}{R} \right)$$

7. A simply supported beam AB of span 4m is carrying point loads of 5, 2 and 3 kN at 1, 2 and 3m from the support A. Calculate the reactions at the supports A & B. [WBSCTE 2014]

POLY-ME

Answer:



Using the force equilibrium and we get

$$R_A + R_B = (5 + 2 + 3) \text{ kN} = 10 \text{ kN}$$

Now, taking moment at point A we get

$$(R_A \times 0) - (5 \times 1) - (2 \times 2) - (3 \times 3) + (R_B \times 4) = 0$$

$$\therefore R_B = \frac{18}{4} = 4.5 \text{ kN}$$

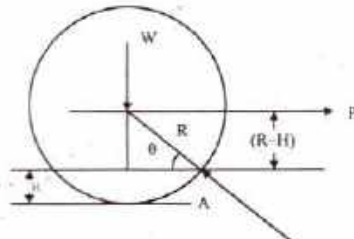
$\therefore$ The reaction at the support A

$$R_A = (10 - 4.5) = 5.5 \text{ kN and } R_y = 4.5 \text{ kN}$$

8. An oil drum of 500mm diameter and 1.5m long is to be rolled across a footstep of 100mm high. Find the minimum push required at the top of the drum. Take density of oil as 1kg/litre. Neglect weight of the drum.

[WBSCTE 2014]

Answer:



For rolling the oil drum over the foot step, net clockwise moment of all forces about A must be greater than zero.

$$P \times R \sin \theta - W \times R \cos \theta > 0$$

$$\therefore R \neq 0 \quad \therefore P > W \cot \theta$$

Now, W = weight of the oil drum

$$= mg = \rho v \times g = d \cdot \pi R^2 H \times g$$

$$= 1 \times 3.14 \times 2.5^2 \times 15 \times 9.81 \text{ N} = 2887.82 \text{ N}$$

And,

$$\sin \theta = \frac{R - H}{R}$$

$$\therefore \theta = \sin^{-1} \frac{250 - 100}{250} = 36^\circ$$

Condition of Equilibrium

Me.45

The minimum value of P is $w \cot \theta$

$$= 2887.82 \times \cot 36^\circ$$

$$= 1.376 \text{ N}$$

9. A heavy uniform rod of length 2 m rests with one end against a smooth vertical wall, the other end being tied to a point of the wall by a string of length 3m. Prove that the rod may remain in equilibrium at an angle 'A' to the wall, given by: $\cos A = (5/12)^{0.5}$. [WBSCTE 2015]

Answer:

If 3 force systems are in equilibrium, then 3 forces must meet at a point.

Here D is that point

From geometry,

$$AB = BC = 1 \text{ m}$$

$$AD = DE = 1 \text{ m}$$

$$OC = CE = x$$

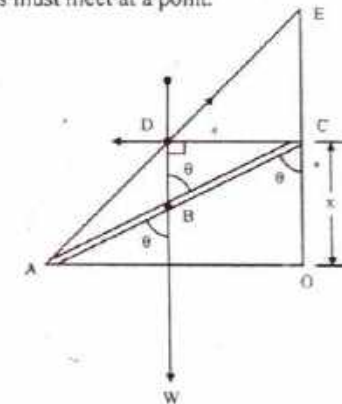
$$\text{From } \triangle ACO \Rightarrow AC^2 = AO^2 + x^2$$

$$\text{From } \triangle AEO \Rightarrow AE^2 = AO^2 + (2x)^2$$

$$\therefore AE^2 - AC^2 = (2x)^2 - x^2$$

$$\Rightarrow 3^2 - 2^2 = 3x^2 \quad \therefore x = \sqrt{\frac{5}{3}}$$

$$\therefore \cos \theta = \frac{x}{AC} = \frac{\sqrt{\frac{5}{3}}}{2} = \frac{\sqrt{5}}{4} \quad (\text{Proved})$$



10. What is equilibrant force?

[WBSCTE 2015]

Answer:

Equilibrant force: Equilibrant force is a force, which brings equilibrium state. It is considered to be the equal and opposite of the resultant force. Equilibrant force is the force, which keeps any object motionless and acts on virtually every object in the world that is not moving.

For forces $(F_1, F_2, \dots)$, equilibrium (F_e) will be opposite to the resultant force (F_R)

$$\therefore F_1 + F_2 = F_R = F_e \quad (\text{Opposite direction})$$

11. Two forces $P = 40 \text{ N}$ and $Q = 60 \text{ N}$ acting parallel to the length and base of an inclined plane respectively, would each of them separately supports a weight W on that plane. Find the value of W . [WBSCTE 2015]

Answer: Refer to Question No. 5 of Long Answer Type Questions.

12. A uniform ladder is in equilibrium with one end resting on the ground and the other end against a vertical wall; if the ground and wall be both rough, the coefficient of friction being 0.35 and 0.25 respectively, and if the ladder be on the point of slipping at both ends, find the inclination of the ladder to the horizon.

[WBSCTE 2015]

Answer:

A uniform ladder of weight W inclined to the horizon at θ and $\mu = 0.35$ and $\mu' = 0.25$

Resolved the forces about horizontally and vertically

$$\therefore R_2 + \mu R_1 = P \quad \dots (i)$$

$$\therefore R_1 = W + \mu' R_2 \quad \dots (ii)$$

$\therefore$ Taking the moment of forces about point A, we get -

$$-W \times \frac{\ell}{2} \cos \theta - 0.25 \times R_2 \times \ell \cos \theta + R_2 \times \ell \sin \theta = 0$$

$$\therefore \frac{W}{2} \cos \theta + \frac{R_2 \ell}{4} \cos \theta - R_2 \ell \sin \theta = 0$$

$$\therefore R_2 \left(\frac{\ell}{4} \cos \theta - \ell \sin \theta \right) = -\frac{W \ell}{2} \cos \theta$$

$$\therefore R_2 = \frac{-\frac{W}{2} \cos \theta}{\frac{1}{4} \cos \theta - \sin \theta} \quad \dots (iii)$$

Solving (i) and (ii)

$$R_2 = \frac{P - 0.35W}{1.0875}$$

Put the value R_2 in equation (iii)

$$\frac{P - 0.35W}{1.0875} = \frac{-\frac{W}{2} \cos \theta}{\frac{1}{4} \cos \theta - \sin \theta}$$

Now, if the ladder be on the point of slipping at both ends

$$\therefore P = 0$$

$$\therefore \frac{-0.35W}{1.0875} = \frac{-0.5W \cos \theta}{0.25 \cos \theta - \sin \theta}$$

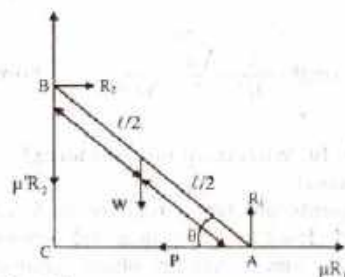
$$\therefore 0.0875 \cos \theta - 0.35 \sin \theta = 0.544 \cos \theta$$

$$\therefore -0.4565 \cos \theta = -0.35 \sin \theta$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{0.4565}{0.35}$$

$$\therefore \tan \theta = 1.304$$

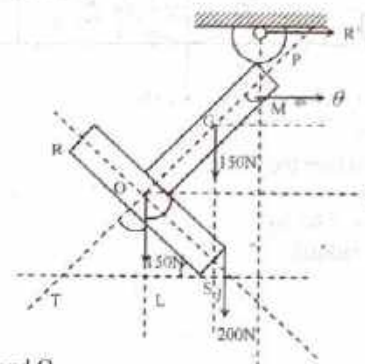
$$\therefore \theta = 52.52^\circ$$



13. Two identical prismatic bars PQ and RS, each 150 N are welded together to form a Tee and are suspended in a vertical plane at P. Calculate the value of the angle θ that the bar PQ will make with the vertical when a vertical load of 200 N is applied at S of RS.

[WBSCTE 2015]

Answer:



We need to find out R' and Q

Suspended along p. So, there acts the moment:

$\therefore$ moment of $R' = 0$. $\therefore$ only need to find out θ .

Let, $PQ = RS = \ell$, G and Q are the mid points of P Q and RS respectively.

And $\angle QPN = \theta$, then $\angle LQT = \theta$ and $\angle QSL = \theta$ (corresponding angle $\angle LQT$ and $\angle QSL$)

$$GM = PG \sin \theta = \frac{\ell}{2} \sin \theta$$

$$QN = PQ \sin \theta = \ell \sin \theta$$

$$SL = SL - LQ = SL - QN = \frac{\ell}{2} \cos \theta - \ell \sin \theta$$

For, equilibrium,

$$\sum M_{\text{torque}} = 0 \therefore 200 \times SL - 150 \times GM - 150 \times QN = 0$$

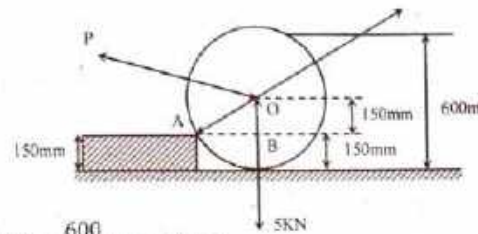
$$\Rightarrow 150 \times \frac{\ell}{2} \sin \theta + 150 \times L \sin \theta = 200 \left(\frac{\ell}{2} \cos \theta - L \sin \theta \right)$$

$$\Rightarrow 425 \sin \theta = 100 \cos \theta$$

$$\Rightarrow \tan \theta = \frac{4}{17} \Rightarrow \theta = 13^\circ 12'$$

14. A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height. Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner of the block. Also, find the reaction of the block. Consider all the surfaces to be smooth. [WBSCTE 2016]

Answer:



Radius of the wheel is $= \frac{600}{2} \text{ mm} = 300 \text{ mm}$

Let P = least pull required to turn the wheel.

So, from $\triangle OAB$, we get,

$$OB = 150 \text{ mm} [\because (300 - 150) = 150 \text{ mm}]$$

$$AO = 300 \text{ mm} [\because AO \text{ is the radius}]$$

$$\therefore \sin \theta = \frac{150}{300}, \theta = 30^\circ$$

Now,

$$AO^2 = AB^2 + OB^2$$

$$\Rightarrow AB^2 = AO^2 - OB^2$$

$$\Rightarrow AB^2 = (300)^2 - (150)^2$$

$$\Rightarrow AB = \sqrt{(300)^2 - (150)^2}$$

$$= 259.80 = 260 \text{ mm}$$

Now taking moment about point A, considering body is in equilibrium,

$$P \times 300 - 5 \times 260 = 0$$

$$\therefore P = 4.33 \text{ kN}$$

Let, R = Reaction of the block, *

Since, body is in equilibrium, resolving all the force in horizontal direction and equate to zero,

$$R \cos 30^\circ - P \sin 30^\circ = 0$$

$$\therefore R = 2.5 \text{ kN}$$

15. a) State, explain and prove Lami's theorem.

b) A string AC of length 24 cm is attached to a point on a smooth vertical wall and to a point on the surface of a sphere of radius CO is 12 cm. The sphere whose weight is 100 kN hangs in equilibrium against the wall. Find the tension in the string and the reaction of the wall. (Fig. 1) [WBSCTE 2017]

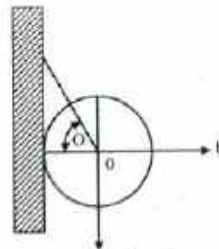


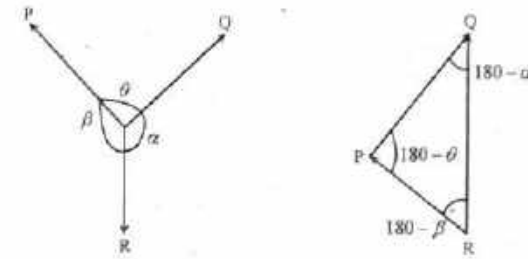
Fig: 1

Answer:

a) Lami's theorem states that, if three concurrent forces act on a body keeping it in Equilibrium, then each force is proportional to the sine of the angle between the other two forces.

Let P, Q, R be the 3 concurrent forces in equilibrium as shown in fig.

Since the forces are vectors, we can move them to form a triangle



Applying sine rule we get

$$P/(\sin(180-\alpha)) = Q/(\sin(180-\beta)) = R/(\sin(180-\gamma)) \Rightarrow P/(\sin \alpha) = Q/(\sin \beta) = R/(\sin \gamma)$$

..... Hence Proved.

b) $CO = 12 \text{ cm}, OB = 12 \text{ cm}, AC = 24 \text{ cm}$

$$OA = OC + AC = 12 + 24 = 36 \text{ cm}$$

Consider, $\angle OAB = \theta$

$$\therefore \sin \theta = \frac{OB}{OA} = \frac{12}{36} \Rightarrow \theta = 19.47^\circ$$

$$\angle ABO = 90^\circ$$

$$\therefore \angle AOB = 180^\circ - (\angle OAB + \angle ABO) = 180^\circ - (90^\circ + 19.47^\circ) = 70.53^\circ$$

$\therefore$ Applying Lami's theorem we get

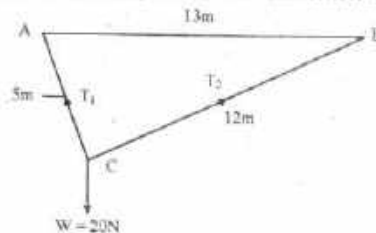
$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 70.53^\circ}$$

$$\Rightarrow T = \frac{W \sin 90^\circ}{\sin 70.53^\circ} = \frac{W}{0.942} \text{ kN} = \frac{100}{0.942} = 106.16 \text{ kN}$$

$$\therefore \frac{R}{\sin 210^\circ} = \frac{W}{\sin 70.53^\circ}$$

$$\Rightarrow R = W \times \frac{\sin 210^\circ}{\sin 70.53^\circ} = W \times 0.53 = 53 \text{ kN}$$

16. a) State and explain triangular law of equilibrium.
 b) A body of weight 20 N is suspended by two strings 5 meter and 12 meter long and other ends being fastened to the extremities of a rod of length 13 meter. If the rod is to remain horizontal, determine the tensions in the strings. (Figure below) [WBSCTE 2017]



Answer:

a) Three forces in equilibrium can be represented in magnitude and direction by the three sides of a triangle taken in order.

As an example of an object in equilibrium, consider an object held stationary by two ropes in the arrangement below:

Let us draw a force diagram for the object. In the force diagram the object is drawn as a dot and all forces acting on the object are drawn in the correct directions starting from that dot. In this case, three forces are acting on the object.

Each rope exerts a force on the object in the direction of the rope away from the object.

Since the object has mass, it is attracted towards the centre of the earth.

Since the object is stationary, the resultant force acting on the object is zero. In other words the three force vectors drawn tail-to-head form a closed triangle.

In general, when drawn tail-to-head the forces acting on an object in equilibrium form a closed figure with the head of the last vector joining up with the tail of the first vector. When only three forces act on an object this closed figure is a triangle. This leads to the **triangle law for three forces in equilibrium**.

b) From the figure, we get

$$\angle ACB = 90^\circ$$

$$\text{Let } \angle CAB = \alpha, \text{ then } \sin \alpha = \frac{12}{13}$$

$$\text{Let } \angle ACD = \beta \text{ then } \beta = (90 - \alpha) \quad \dots (1)$$

For the rod to remain horizontal, all forces in the x direction and y direction must be in equilibrium

$$\sum F_x = 0 \Rightarrow T_1 \sin \beta - T_2 \sin \alpha = 0 \quad \dots (2)$$

$$\sum F_y = 0 \Rightarrow T_1 \cos \beta + T_2 \cos \alpha = 20 \quad \dots (3)$$

From Eqns. (1) and (2) we get,

$$T_1 \sin(90 - \alpha) - T_2 \sin \alpha = 0$$

$$\Rightarrow T_1 \cos \alpha - T_2 \sin \alpha = 0$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

Again from Eqns. (1) and (3)

$$T_1 \cos \beta + T_2 \cos \alpha = 20$$

$$\Rightarrow T_1 \cos(90 - \alpha) + T_2 \cos \alpha = 20$$

$$\Rightarrow T_1 \sin \alpha + T_2 \cos \alpha = 20$$

$$\Rightarrow T_1 \sin \alpha + T_1 \cos \alpha \cdot \cot \alpha = 20$$

$$\Rightarrow T_1 \sin \alpha + T_1 \cos \alpha \cdot \frac{\cos \alpha}{\sin \alpha} = 20$$

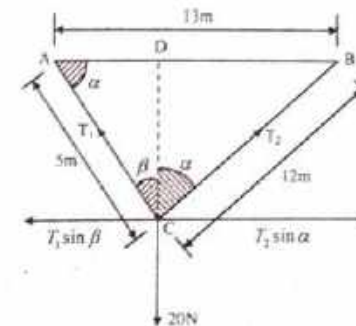
$$\Rightarrow T_1 \sin \alpha + T_1 \frac{\cos^2 \alpha}{\sin \alpha} = 20$$

$$\Rightarrow T_1 \left[\frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha} \right] = 20$$

$$\Rightarrow T_1 \times \frac{1}{\sin \alpha} = 20$$

$$\Rightarrow T_1 = 20 \sin \alpha = 20 \times \frac{12}{13} = 18.46 \text{ N}$$

$$T_2 = T_1 \cot \alpha = 18.46 \times \frac{5}{12} = 7.69 \text{ N}$$



17. a) State the conditions of equilibrium.

b) Forces each of 4N in magnitude act along AB and CD of a square ABCD and forces each of 6N in magnitude act along the sides AD and CB respectively in the same square. Find the moment of the couple which will produce equilibrium. Length of each sides of the square is 80mm. [WBSCTE 2022]

Answer:

a) Refer to Article No.1 of Unit at a Glance.

b) Let us, consider x-y coordination system.

The resultant of all the forces is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

Length of each side of the square

$$AB = BC = CD = DA = 80 \text{ mm} = 0.08 \text{ m}$$

Force at B = 6N

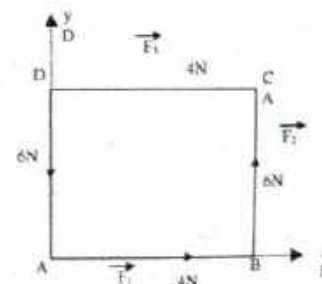
Force at C = 6N

Force at D = 4N

Force at A = 4N

a) The moment of couple C about point A

$$\therefore \text{Force at C} \times \text{Length AB} = 6 \times 0.08 = 0.48 \text{ Nm.}$$



b) The moment of couple at D about point A

$$\therefore \text{Force at D} \times \text{Length AD} = 4 \times 0.08 = 0.32 \text{ Nm.}$$

$$\therefore \text{Resultant moment of all forces about A} = 0.48 + 0.32 \text{ Nm} = 0.8 \text{ Nm}$$

18. ABCDEF is a rectangular hexagon, forces of 3, P, Q, 5 and 4N are acting along AB, CA, AD, AE and FA respectively and are in equilibrium. Determine the value of P & Q [WBSCTE 2022]

Answer:

ABCD is a rectangle in which AB = 4 m & BC = 3 m.

$$\text{Then, } \tan \theta = \frac{BC}{AB} = \frac{3}{4}$$

The forces 3, P, 5, 4 and Q N have the resultant R as shown in figure:

$$R \cos \theta + 5 - 3$$

$$\Rightarrow Q \cos \theta + 2 \quad \dots (i)$$

$$R \sin \theta = P + 4 - Q \sin \theta \quad \dots (ii)$$

$$\Rightarrow QAB \sin \theta + 5BC = 4AB$$

$$\Rightarrow Q \cdot 4 \left(\frac{3}{5} \right) + 5 \times 3 = 4 \times 4 \quad \left(\because \sin \theta = \frac{3}{5} \text{ and } \cos \theta = \frac{4}{5} \right)$$

$$\Rightarrow \frac{12}{5} Q = 16 - 15$$

$$\therefore Q = \frac{5}{12} N$$

Find R from (i) and P from (ii)

$$\frac{4R}{5} = \frac{5}{12} \times \frac{4}{5} + 2$$

$$\Rightarrow \frac{4R}{5} = \left(\frac{1}{3} + 2 \right)$$

$$\Rightarrow 4R = 5 \times \frac{7}{3}$$

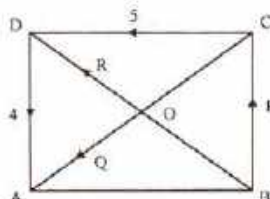
$$\therefore R = \frac{35}{12} \text{ and } \frac{35}{12} \times \frac{3}{5} = P + 4 - \frac{5}{12} \times \frac{3}{5}$$

$$\Rightarrow \frac{7}{4} = P + \left(4 - \frac{1}{4} \right)$$

$$\Rightarrow \frac{7}{4} = P + \frac{15}{4}$$

$$\Rightarrow \frac{15}{4} - \frac{7}{4} = P$$

$$\therefore P = \frac{8}{4} = 2$$



Unit 4: Friction

Unit at a glance:

1. Definition

- When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces.

Terminology	Definition
Sliding friction	Encountered commonly in the case of two solid surfaces having a tendency of relative motion.
Rolling Friction	Resistance between two surfaces is experienced when they roll over each other.
Fluid friction	Resistive force is offered by a fluid to body which moves over or through it.
Limiting friction	Maximum value of frictional force, which comes into play, when the motion is impending.
Static friction	Applied force is less than the limiting friction, the body remains at rest.
Dynamic friction	Applied force exceeds the limiting friction, the body starts moving over the other body.

2. Laws of Friction

The results of experiments conducted by Coulomb are known as Laws of dry friction or Laws of Coulomb's friction. They are as follows:

- The direction of friction force always acts in a direction opposite to which the body tends to move.
- The frictional force developed is independent of the area of contact.
- The total frictional force depends on the roughness of contact surfaces.
- The frictional force is equal to the force applied to the body, so long as the body is at rest.
- The coefficient of Kinetic friction is less than the coefficient of Static friction over a wide range.
- The total frictional force is proportional to the normal Reaction force i.e., ($F \propto N$).

3. Coefficient of Friction

The total frictional force is proportional to the normal reaction force

$$\therefore F = \mu N \dots (1)$$

i.e., $F \propto N$

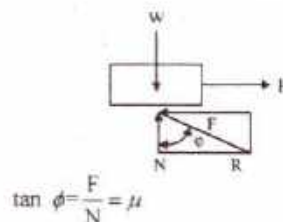
Where, F = Frictional force developed in the contact surface, N = Normal force and μ = Coefficient of friction.

Thus, the coefficient of friction is defined as the ratio between the limiting friction and the normal reaction.

$$\text{Hence, } \mu = \frac{F}{N}$$

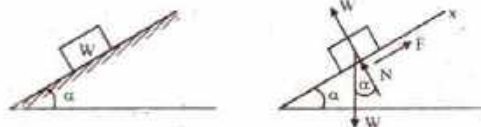
4. Angle of Friction

When the force of friction is limiting, the force of friction F and the normal reaction N combined gives a resultant force R as shown in Fig. Here the angle ϕ is called the angle of friction.



5. Angle of Repose

The maximum inclination of the plane on which a body, free from external forces, can slip.



$$\sum F_x = 0 \text{ gives, } F - W \sin \alpha = 0, F = W \sin \alpha$$

$$\sum F_y = 0 \text{ gives, } F - W \cos \alpha = 0, F = W \cos \alpha$$

α = angle of repose

$$\text{from the above equation we get, } \frac{F}{N} = \tan \alpha$$

Comparing equation, we get,

$$\mu = \tan \phi = \tan \alpha$$

$$\therefore \phi = \alpha$$

Thus, Angle of repose, (α) = Angle of friction, (ϕ)

Limiting friction

It is the maximum value up to which static friction can rise and balance a given externally applied force and beyond which it cannot rise, is called limiting friction.

Short Answer Type Questions

A. Choose the correct answer from the given alternative:

1. The angle which an inclined surface makes with the horizontal when a body placed on it is on the point of moving down is known as – [WBSCTE 2006]
(a) angle of friction (b) angle of limiting friction
(c) angle of repose (d) angle of inclination

Answer: (c)

2. When a body is about to move upwards in an inclined plane by application of a force the frictional force will act – [WBSCTE 2013]
(a) upward the inclined plane
(b) downward the inclined plane
(c) in the direction of applied force

Answer: (b)

3. The force of friction always acts in a direction opposite to that [WBSCTE 2014]
(a) in which the body tends to move (b) in which the body is moving
(c) both of the two (d) none of the two

Answer: (a)

4. The magnitude of the force of friction between two bodies are lying above the other, depends upon the roughness of the [WBSCTE 2014]
(a) upper body (b) lower body (c) both the bodies

Answer: (c)

B. Fill in the blanks in the following statements:

5. Angle of friction is equal to angle of repose. [WBSCTE 2010]
6. The ratio of limiting friction and normal reaction is known as co-efficient of static friction. [WBSCTE 2011]
7. For a perfect Elastic collision coefficient of restitution is 1. [WBSCTE 2011]
8. Frictional force depends upon nature of the Surface. [WBSCTE 2013]
9. By application of a force when a body is about to rise along a rough inclined plane the frictional force acts along down the inclined plane. [WBSCTE 2015]
10. Coefficient of friction is the ratio of the frictional force divided by the normal force. [WBSCTE 2015]
11. When frictional force (F) increases, the ratio between F and normal reaction (N) Increase. [WBSCTE 2016]

12. The angle which an inclined surface makes with the horizontal when a body placed on it is on the point of moving down is known as angle of repose. [WBSCTE 2016]
13. Maximum value of static friction is known as limiting friction. [WBSCTE 2016]
14. Dynamic or kinetic friction is always less than the static friction. [WBSCTE 2017]
15. Frictional force is directly proportional to normal reaction. [WBSCTE 2017]
16. Coefficient of friction depends upon nature of the surfaces. [WBSCTE 2022]
17. A heavy string attached at two ends at same horizontal level and when central dip is very small approaches the following curve Parabola. [WBSCTE 2022]
18. Frictional force encountered after commencement of motion is called dynamic friction. [WBSCTE 2022]
19. The skidding away of the vehicle on a level circular path can be avoided if the force of friction between the wheels and ground is- less than the centrifugal force. [WBSCTE 2022]
20. The slope on the road surface generally provided on the curves is known as- angle of banking. [WBSCTE 2022]
21. On a ladder resting on smooth ground and leaning against vertical wall, the force of friction will be upward at its upper end. [WBSCTE 2022]
22. The velocity of a body on reaching the ground from a height h, is $\sqrt{2Hg}$. [WBSCTE 2022]
23. The ratio of static friction to dynamic friction is always greater than one. [WBSCTE 2022]
24. A flywheel on a motor goes from rest to 1000 rpm in 6 sec. The number of revolutions made is nearly equal to -50. [WBSCTE 2022]
25. The angle which an inclined plane makes with the horizontal when a body placed on it is about to move down is known as angle of reposes. [WBSCTE 2022]

C. State whether the following statements are True or False:

26. If θ = angle of friction, the coefficient of friction is equal to 'tan θ '. [WBSCTE 2009, 2010]

Answer: True

27. Friction is not always harmful. [WBSCTE 2010]
Answer: True
28. For a ladder resting on floor and a wall the floor may be smooth (friction less). [WBSCTE 2011]
Answer: False
29. Frictional force always acts in the direction of tendency of motion. [WBSCTE 2012]
Answer: False
30. A ladder will be in a stable condition if the resting wall is smooth. [WBSCTE 2013]
Answer: False
31. Angle of repose is equal to angle of friction. [WBSCTE 2013]
Answer: True
32. Angle of repose is equal to angle of friction. [WBSCTE 2014]
Answer: True
33. If a ladder is not in equilibrium against a smooth vertical wall, then it can be made equilibrium by decreasing the angle of inclination. [WBSCTE 2014]
Answer: True
34. Angle of Repose is equal to angle of friction. [WBSCTE 2015]
Answer: False
35. In SI, the unit of co-efficient of friction is Nm-1. [WBSCTE 2016]
Answer: False
36. Semi-central angle in cone of friction is equal to angle of friction. [WBSCTE 2017]
Answer: False
37. The force of friction between two bodies in contact is always normal to the surface of their contact. [WBSCTE 2018]
Answer: False
38. Angle of repose is equal to angle of friction. [WBSCTE 2018]
Answer: True

D. Pick up the correct answer:

39. Frictional force is (greater than applied force/less than applied force/self adjusting force) [WBSCTE 2011]
Answer: Self adjusting force

40. With the change in surface area with constant weight the frictional force (increases/decreases/remains unchanged) [WBSCTE 2012]

Answer: remains unchanged

41. The Law of Limiting Friction is given by ($F = \mu R$ / $\mu F = R$ / $F = R$). [WBSCTE 2013]

Answer: $F = \mu R$

42. The angle of friction is the angle between (frictional force and normal reaction / Resultant reaction and normal reaction / frictional force and resultant reaction). [WBSCTE 2015]

Answer: frictional force and normal reaction

43. In an inclined plane frictional force always acts (down the inclined plane / up the inclined plane / against the tendency of motion). [WBSCTE 2015]

Answer: against the tendency of motion

44. An ideal machine is one where frictional loss is (maximum / minimum / nil). [WBSCTE 2015]

Answer: nil

Long Answer Type Questions

1. Define the term friction. How does it come into play? [WBSCTE 2003, 2009]

Answer:

When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces.

2. Explain the terms – (i) co-efficient of friction (ii) angle of friction (iii) limiting friction (iv) angle of repose. [WBSCTE 2003, 2008]

Answer: Refer to Article No. 3, 4, 5 of Unit at a Glance.

3. Explain the useful and harmful effects of friction. [WBSCTE 2004]

OR,

Write a short note on Useful and harmful effects of frictions. [WBSCTE 2016]

Answer:

Useful effect of friction:

- Screws or nails would not stick to wood.
- Walking would have been impossible without friction as the force exerted on the feet would cause them to stop in absence of friction. That is why it is difficult to walk on a pond of ice.
- Brakes would not stop cycles or motor cars as there would be no force, necessary to stop the motion of the wheels.

Harmful effect of friction:

- It causes much of the wear and tear of the machinery and heating of its parts. The energy spent in overcoming friction is dissipated in heat and is not recovered.
- It reduces the efficiency of the machines and often causes much of the work done by effort to be wasted.
- Friction may sometimes cause fire.

4. Define angle of friction and angle of repose and show that they are equal. [WBSCTE 2009]

OR,

Prove that angle of repose equal to the angle of friction [WBSCTE 2008]

OR,

Define angle of friction & angle of repose is the relationship between them. [WBSCTE 2010]

Answer:

Refer to Article No. 4 of Unit at a Glance.

5. State laws of static friction. [WBSCTE 2009, 2016, 2018]

Answer:

Refer to Article No. 2 of Unit at a Glance.

6. A force of 200N is required just to move a certain body up an inclined plane of angle 20° , the force is acting parallel to the plane. If the angle of inclination of the plane is made 25° , the force required parallel to the plane is 230N. Find the coefficient of friction and weight of the body. [WBSCTE 2009, 2016]

Answer:

$$R = W \cos \theta$$

Let, coefficient of friction is μ

Let F_f is the frictional force

F is the applied force.

$$\text{So, } F_f + W \sin \theta = F$$

When, $\theta = 20^\circ$, Then $F = 200\text{N}$

$$\therefore \text{ again, } F_f = \mu R$$

$$\therefore \mu W \cos 20^\circ + W \sin 20^\circ = 200$$

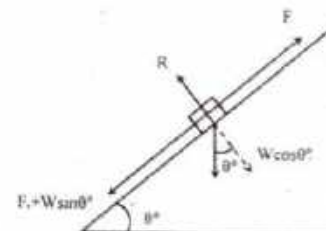
$$\text{or, } W = \frac{200}{\mu \cos 20^\circ + \sin 20^\circ} \dots (i)$$

When $\theta = 25^\circ$, then $F = 230\text{N}$

$$\mu W \cos 25^\circ + W \sin 25^\circ = 230$$

$$\text{or, } W = \frac{230}{\mu \cos 25^\circ + \sin 25^\circ} \dots (ii)$$

Equating (i) & (ii) we get –



$$\frac{200}{\mu \cos 20^\circ + \sin 20^\circ} = \frac{230}{\mu \cos 25^\circ + \sin 25^\circ}$$

$$\text{or, } 200(\mu \times 0.906 + 0.422) = 230(\mu \times 0.939 + 0.342)$$

$$\text{or, } \mu \times 34.77 = 05.74$$

$$\therefore \mu = 0.165$$

$$\text{Again we get, } W = \frac{200}{\mu \cos 20^\circ + \sin 20^\circ} = \frac{200}{0.165 \times 0.939 + 0.342} = 402.36 \text{ N}$$

7. The force required to pull a body of weight 60N on a rough horizontal plane is 15N. Determine the coefficient of friction if the force is applied at an angle of 15° with the horizontal. [WBSCTE 2010]

Answer:

In equilibrium $\sum$ vertical component = 0 &

$\sum$ Horizontal component = 0

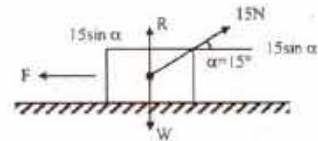
$$P = 15 \text{ N, } F = P \cos \theta$$

$$= 15 \times \cos 15^\circ = 14.49 \text{ N}$$

$$\text{again, } F = \mu R = 0$$

$$\Rightarrow 14.49 = \mu \times 60$$

$$\Rightarrow \mu = 0.24.$$



8. A uniform ladder of weight 1000 N, inclined to the horizon at 45° , rests with its upper extremity against a rough vertical wall and its lower extremity on the ground. Find the least horizontal force which will move the lower end towards the wall. Given 0.25 and 0.20 are the co-efficient of friction at the lower and upper ends respectively. [WBSCTE 2011, 2017]

Answer:

l = length of the ladder

$$\sum M_A = -1000 \times \frac{l}{2} \cos 45^\circ - F_B \times l \cos 45^\circ + R_B \times l \sin 45^\circ = 0$$

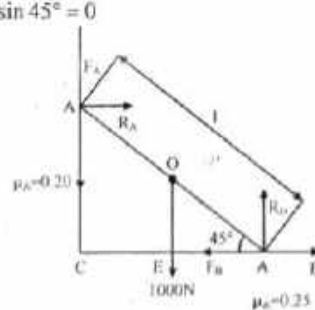
$$\sum M_o = 0 \text{ (by considering equilibrium)}$$

$$\text{or, } -500 \times l \times \frac{1}{\sqrt{2}} - 0.25 \times R_B \times l \times \frac{l}{\sqrt{2}} + R_B \times l \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow R_B \times \frac{1}{\sqrt{2}} = \frac{500 \times l}{\sqrt{2}} + \frac{0.25 \times R_B \times l}{\sqrt{2}}$$

$$\Rightarrow \frac{R_B}{\sqrt{2}} = \frac{500}{\sqrt{2}} + \frac{0.25 R_B}{\sqrt{2}}$$

$$\Rightarrow R_B = 500 + 0.25 R_B$$



$$\Rightarrow R_B(1 - 0.25) = 500$$

$$\Rightarrow R_B = \frac{500}{0.75} = 666.66 \text{ N}$$

Components along y axis:

$$F_A + R_B = 1000 \quad F_A = 1000 - R_B = 333.33 \text{ N}$$

$$R_A = \frac{F_A}{\mu_A} = \frac{333.33}{0.20} \quad R_A = 1666.6 \text{ N}$$

Least horizontal force that will be actual to move the ladder towards the wall is equal to frictional force $F_B = 166.66 \text{ N}$. (Ans.)

9. A metallic chain of length 100 cm is partially resting on a rough horizontal table and part of the chain is suspending from the end of the table. If the coefficient of friction between the chain and the table is 0.4 find the maximum length of the chain that can be suspended under equilibrium condition. [WBSCTE 2012]

Answer:

Let mass/cm of the chain is $m \text{ gm/cm}$.

$\therefore$ Weight of x length = $x \times m \times g \text{ gm}$

Weight of $(100 - x)$ length = $(100 - x) \times m \times g$.

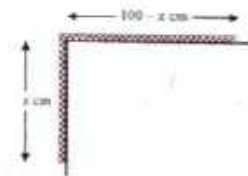
Now for equilibrium

$$x \times m \times g = (100 - x) \times m \times g \times 0.4$$

$$x = 0.4 \times (100 - x) = 40 - 0.4x$$

$$1.4x = 40$$

$$x = \frac{40}{1.4} = 28.57 \text{ cm.}$$



That is the length that can be suspended from the table.

10. Determine the horizontal force P to be applied to a block of weight 500 N to hold it in position on a smooth inclined plane which makes an angle 30° with the horizontal. [WBSCTE 2013]

Answer:

$$\sum F_y = 0 \text{ gives,}$$

$$P \cos 30^\circ = 500 \sin 30^\circ$$

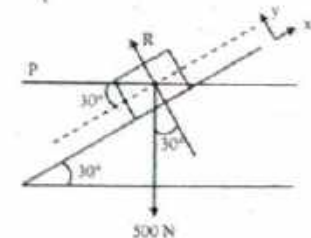
$$\therefore P = 500 \times \frac{\sin 30^\circ}{\cos 30^\circ} = 288.675 \text{ N}$$

$$\sum F_x = 0 \text{ gives,}$$

$$R = (500 \cos 30^\circ + 288.675 \sin 30^\circ) \text{ N}$$

$$= (453 + 144.3) \text{ N} = 597.33 \text{ N}$$

$\therefore$ The horizontal force applied to a block = 288.675 N.



11. A weight 300N just starts moving down a rough inclined plane supported by a force of 100N acting parallel to the plane and it is at the point of moving up the plane when pulled by a force 200N parallel to the plane. Find the inclination of the plane with the horizontal and the coefficient of friction between the inclined plane and the weight. [WBSCTE 2013]

Answer:

Case - I

$$\sum F_x = 0 \text{ gives,}$$

$$F_1 + 100 = 300 \sin \alpha$$

$$\text{or, } \mu R + 100 = 300 \sin \alpha \quad \dots (1)$$

$$\text{and } R = 300 \cos \alpha \quad \dots (2)$$

Case - II

$$\sum F_x = 0 \text{ gives,}$$

$$200 = F_2 + 300 \sin \alpha$$

$$= \mu R + 300 \sin \alpha \quad \dots (3)$$

From (1) & (3), we can write,

$$\mu R + 100 = 200 - \mu R$$

$$\text{or, } 2\mu R = 100$$

$$\therefore \mu R = 50$$

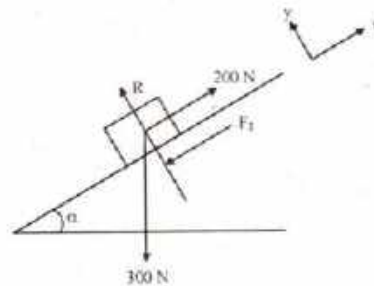
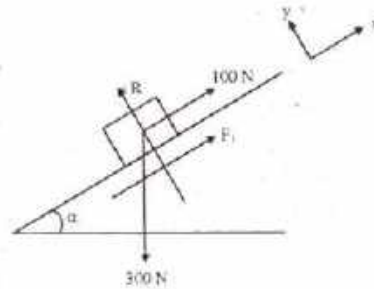
$$\therefore 50 + 100 = 300 \sin \alpha$$

$$\therefore \sin \alpha = \frac{150}{300} = \frac{1}{2}$$

$$\therefore \alpha = 30^\circ$$

$$\therefore R = 300 \cos 30^\circ \text{ N} = 259.8 \text{ N}$$

$$\therefore \mu = \frac{300 \times \frac{1}{2} - 100}{259.8} = 0.192$$



12. A uniform ladder of length 3 m weighing 150 N is placed against a vertical wall with angle of inclination with the horizontal 60° . The coefficient of friction between the ladder and the wall is 0.25 and between the floor and the ladder is 0.35. The ladder in addition to its self weight has to support a man weighing 500 N at a distance 2 m from bottom end of the ladder. Calculate the minimum horizontal force to be applied at the bottom end of the ladder towards the wall to prevent slipping. [WBSCTE 2013]

Answer:

$$\frac{CB}{AC} = \sin 60^\circ$$

$$\therefore CB = 3 \sin 60^\circ$$

$$\frac{AB}{AC} = \cos 60^\circ$$

$$\therefore AB = 3 \cos 60^\circ$$

The weight $w = 150 \text{ N}$ of the ladder AC acts vertically downward through its midpoint G

Let the minimum horizontal force F to be applied at the bottom end of the ladder 'A' towards the wall to prevent slipping.

In addition the weight of man $W_m = 500 \text{ N}$ on the ladder AF acts vertically downward through its point F.

Besides, the following forces are acting on the ladder

R_1 , the normal reaction of the wall acting at C and R_2 , the normal reaction of the ground acting at A.

At C, the frictional force $\mu_1 R_1$ towards CB and at A, the frictional force $\mu_2 R_2$ towards AB.

At equilibrium, the force acting in the horizontal direction, we get

$$R_1 + \mu_2 R_2 - F = 0$$

$$\text{or, } F = R_1 + \mu_2 R_2$$

Now, considering the forces acting in the vertical direction we get,

$$R_2 - 150 - 500 - \mu_1 R_1 = 0$$

$$\therefore R_2 = \mu_1 R_1 + 650$$

$$\therefore F = R_1 + \mu_2 (\mu_1 R_1 + 650)$$

Again, the algebraic sum of the moments of the forces acting on the ladder w.r.t. the point A should be zero.

$$R_1 \cdot CB - R_2 \mu_1 AB - 150 AD - 500 AE = 0$$

$$\therefore R_1 \cdot 3 \cdot \sin 60^\circ - \mu_1 R_1 \cdot 3 \cos 60^\circ - 150(1.5 \cos 60^\circ) - 500 \cdot 2 \cdot \cos 60^\circ = 0$$

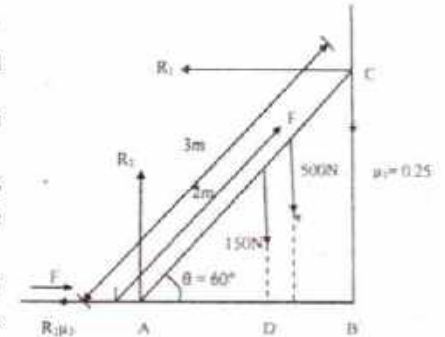
$$\therefore R_1 \cdot 3 \cdot \frac{\sqrt{3}}{2} - \mu_1 R_1 \cdot 3 \cdot \frac{1}{2} - 225 \cdot \frac{1}{2} - 1000 \cdot \frac{1}{2} = 0$$

$$\therefore R_1 \left(\frac{3\sqrt{3}}{2} - 0.25 \cdot \frac{3}{2} \right) - 112.5 - 500 = 0$$

$$\therefore R_1 = \frac{612.5}{2.22} = 275.52$$

$$\therefore F = 275.52 + 0.35(0.25 \times 275.52 + 650) = 527.13 \text{ N}$$

$\therefore$ The minimum horizontal force to be applied at the bottom end of the ladder towards the wall to prevent slipping is 527.13 N.



13. A body resting on a rough horizontal plane, required a pull of 180N inclined at 30° to the plane just to move it. It was found that a push of 200N inclined at 30° to the plane just removed the body. Determine the weight of the body and the coefficient of friction. [WBSCTE 2014]

Answer:

From equilibrium condition

$$R_1 + 180 \sin 30^\circ = w \quad \dots (i)$$

$$\text{and } F_1 = 180 \cos 30^\circ \quad \dots (ii)$$

where, R_1 = Normal Reaction

w = Weight of the body

F_1 = Frictional force

In figure we get

Again from equilibrium condition,

$$R_2 = w + 200 \sin 30^\circ \quad \dots (iii)$$

$$\text{and } F_2 = 200 \cos 30^\circ \quad \dots (iv)$$

Dividing eq. (i) and (iii) we get,

$$\frac{R_1}{R_2} = \frac{w - 180 \sin 30^\circ}{w + 200 \sin 30^\circ}$$

$$\text{or, } \frac{180}{200} = \frac{w - 180 \times \frac{1}{2}}{w + 200 \times \frac{1}{2}} = \frac{2w - 180}{2w + 200}$$

$$\text{or, } 18w + 1800 = 20w - 1800$$

$$\text{or, } 2w = 3600$$

$$\therefore w = 1800 \text{ N}$$

$$\therefore \text{The weight of the body} = 183.48 \text{ kg (Ans.)}$$

From equation (i) we get,

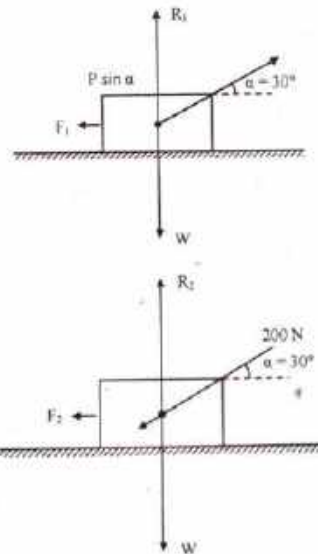
$$R_1 = w - 180 \sin 30^\circ = 1800 - \left(180 \times \frac{1}{2}\right) = 1710 \text{ N}$$

From equation (ii) we get

$$\mu R_1 = 180 \cos 30^\circ$$

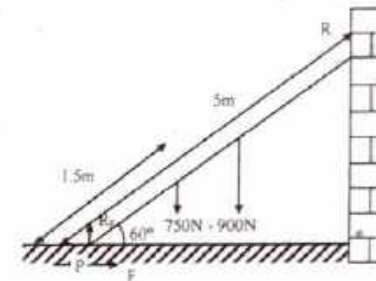
$$\therefore \mu = \frac{180 \times \frac{\sqrt{3}}{2}}{1710} = 0.091$$

$$\therefore \text{The co-efficient of friction } \mu = 0.091 \text{ (Ans.)}$$



14. An uniform ladder 5m long rests on a horizontal ground floor and leans against a smooth vertical wall at angle of 60° with the horizontal. The weight of the ladder is 900N acts at the middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5m from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor. [WBSCTE 2014]

Answer:



Given data:

$$l = 5 \text{ m}$$

$$\alpha = 60^\circ$$

$$\therefore \text{Weight of the ladder } (w_1) = 900 \text{ N}$$

$$\therefore \text{Weight of the man } (w_2) = 750 \text{ N}$$

Distance between the man and bottom of ladder = 1.5m

Force acting on the ladder are shown in given figure.

Let, μ_f = The co-efficient of friction between ladder and floor

and R_f = Normal reaction on the floor

$\therefore$ Resolving the forces vertically,

$$R_f = (900 + 750) \text{ N} = 1650 \text{ N}$$

$\therefore$ Force at P is

$$F = \mu_f \times R_f = \mu_f \times 1650$$

Now taking moments about R and equating the same, we get,

$$\therefore R_f \times 5 \sin 60^\circ = (F \times 5 \cos 60^\circ) + (900 \times 2.5 \sin 60^\circ) + (750 \times 3.5 \sin 60^\circ)$$

$$\therefore R_f \frac{5\sqrt{3}}{2} = \left(\mu_f \times 1650 \times \frac{5}{2}\right) + \left(900 \times 2.5 \times \frac{\sqrt{3}}{2}\right) + \left(750 \times 3.5 \times \frac{\sqrt{3}}{2}\right)$$

$$\therefore 1650 \times \frac{5\sqrt{3}}{2} = 4125 \mu_f + 1948.5 + 2273.25$$

$$\therefore \mu_f = \frac{2911.75}{4125} = 0.70$$

15. Determine the horizontal force P to be applied to a block of weight 450N to hold it in position on a smooth inclined plane which makes an angle 30° with horizontal. [WBSCTE 2014]

Answer:

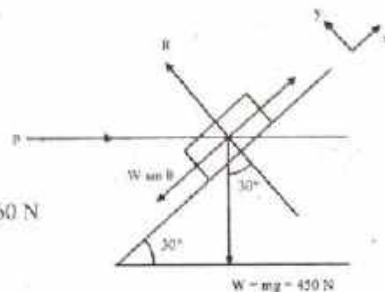
At Equilibrium:

$$\therefore \sum F_x = 0 \text{ gives}$$

$$\therefore P \cos 30^\circ = w \sin 30^\circ$$

$$\therefore P = 450 \times \frac{1}{2} \times \frac{2}{\sqrt{3}} = 260 \text{ N}$$

$\therefore$ The horizontal force applied to a block = 260 N



16. A body of weight 70 N can just be sustained on a rough inclined plane by a force $P = 40$ N and just dragged up the plane by a force $Q = 60$ N, P and Q both acting up the line of the greatest slope. Find the coefficient of friction between the body and the inclined plane. [WBSCTE 2015]

Answer:

Case-I:

$$\sum F_x = 0 \text{ gives}$$

$$\therefore F_1 + 40 = 70 \sin \theta$$

$$\text{or, } \mu R + 40 = 70 \sin \theta \quad \dots (1)$$

$$\text{and } R = 70 \cos \theta \quad \dots (2)$$

Case-II:

$$\sum F_x = 0 \text{ gives}$$

$$60 = F_2 + 70 \sin \theta$$

$$\text{or, } 60 = \mu R + 70 \sin \theta \quad \dots (3)$$

From (1) and (3) we can write

$$\mu R + 40 = 60 - \mu R$$

$$\therefore 2\mu R = 20$$

$$\therefore \mu R = 10$$

$$\therefore 70 \sin \theta = 10 + 40 = 50$$

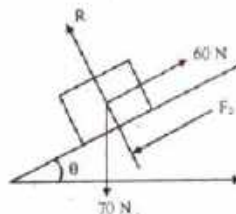
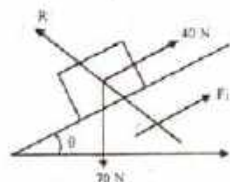
$$\therefore \sin \theta = \frac{50}{70}$$

$$\therefore \theta = \sin^{-1} \frac{5}{7} = 45.58^\circ$$

$$\therefore R = 70 \times 0.6998 = 48.98 = 49 \text{ N}$$

$$\therefore \mu = \frac{10}{49} = 0.204$$

$\therefore$ The coefficient of friction between the body and the inclined plane (μ) = 0.204.



17. A chain of length 1.0 m is partly suspended from a horizontal rough table and is partly on the table. If the coefficient of friction between the chain and the table be 0.3 find the maximum length of the chain that can be suspended from the table to keep the system in equilibrium. [WBSCTE 2015]

Answer:

Let mass/cm of the chain is m gm/cm.

$$\therefore \text{Weight of } x \text{ cm length} = x \times m \times g \text{ N}$$

$$\therefore \text{Weight of } (100 - x) \text{ cm length} = m \times g \text{ N}$$

Now for equilibrium

$$x \times m \times g = (100 - x) \times m \times g \times 0.3$$

$$\therefore xm = 30 - 0.3xm$$

$$\therefore 1.3xm = 30 \text{ m}$$

$$\therefore x = \frac{30}{1.3} = 23 \text{ cm.}$$

$\therefore$ The maximum length of the chain that can be suspended from the table to keep the system in equilibrium = 23 cm.

18. A metal block of weight 100 N is resting on a horizontal surface. The coefficient of friction between the surfaces being 0.35. A pull is exerted on the block by means of a string inclined at 45° to the horizontal which is sufficient to cause motion of the block. Calculate the magnitude of force exerted by the string. [WBSCTE 2016]

Answer:

$$F_{\text{net}} = mg - F_N - T_V$$

No acceleration in vertical direction, $F_{\text{net}} = 0$

$$F_N = mg - T_V$$

$$= mg - T \sin \theta$$

$$= 100 - T \sin 45^\circ$$

$$= 100 - T(0.7071)$$

$F_{\text{net}} = T_H - F_f$ No acc. In horizontal axis, $F_{\text{net}} = 0$

$$\therefore T_H - F_f \Rightarrow T_H = T \cos 45^\circ = T(0.7071)$$

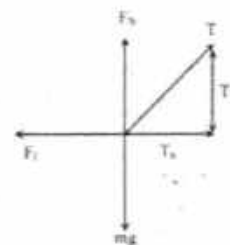
$$F_f = \mu F_N = (0.35)(100 - T \sin 45^\circ) = 35 - T(0.7071)$$

$$\therefore T_H = F_f$$

$$\Rightarrow T(0.7071) = 35 - T(0.7071)$$

$$2T(0.7071) = 35$$

$$T = \frac{35}{2 \times 0.7071} = \frac{35}{1.4142} = \boxed{24.75 \text{ N}}$$



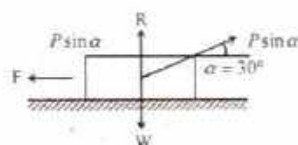
19. (a) Define limiting friction and normal reaction.
 (b) A body resting on a horizontal plane required a pull of 80 N inclined to 30° to horizontal just to move it. It was also found that a push of 100 N inclined at 30° to the horizontal just moved the body. Find the weight of the body and coefficient of friction. [WBSCTE 2017]

Answer:

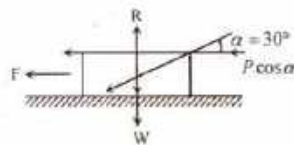
a) **Limiting friction:** Maximum value of frictional force, which comes into play, when the motion is impending.

Normal Reaction: The force exerted by a surface on an object in contact with it which prevents the object from passing through the surface; the force is perpendicular to the surface, and is the only force that the surface exerts on the object in the absence of frictional forces.

b)



When the block is pulled



When the block is pushed

where, W = Weight of the body

R = Normal reaction

μ = Coefficient of friction

Resolving the forces when the block is pulled we get,

$$F = P \cos \alpha = 80 \cos 30^\circ = 80 \times 0.866 = 69.28 \text{ N}$$

$$R = W - P \sin \alpha = W - 80 \sin 30^\circ = (W - 40) \text{ N}$$

$$\therefore 69.28 = \mu R = \mu (W - 40) \quad \dots (i)$$

Resolving the forces when the block is pushed,

$$F = P \cos \alpha = 100 \cos 30^\circ = 86.6 \text{ N}$$

$$R = W + P \sin \alpha = W + (100 \times 0.5) = (W + 50) \text{ N}$$

$$86.6 = \mu (W + 50) \quad \dots (ii)$$

Dividing (i) by (ii) we get,

$$\frac{69.28}{86.6} = \frac{\mu (W - 40)}{\mu (W + 50)}$$

$$\Rightarrow 69.28W + 3464 = 86.6W - 3464$$

$$\Rightarrow 17.32W = 6928$$

$$\Rightarrow W = 400 \text{ N}$$

Now substituting the value of W in equation (i)

$$69.28 = \mu (400 - 40)$$

$$\Rightarrow \mu = 0.192$$

20. a) Prove that angle of repose is equal to the angle of friction.
 b) A load W acts on an inclined plane of inclination θ to the horizontal. If the co-efficient of friction is μ , show that the horizontal force necessary to move the

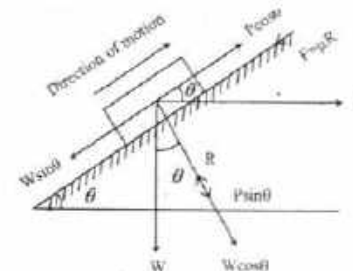
load up the plane is $\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \times W$.

[WBSCTE 2017]

Answer:

a) Refer to Article No. 5 of Unit at a Glance.

b)



The body is in equilibrium under the action of the forces shown in above figure.

Resolving the forces along the plane

$$P \cos \theta = W \sin \theta + \mu R \quad \dots (i)$$

Resolving the forces normal to the plane

$$R = P \sin \theta + W \cos \theta \quad \dots (ii)$$

Substituting the value of R in equation (i)

$$P \cos \theta = W \sin \theta + \mu (P \sin \theta + W \cos \theta)$$

$$= W \sin \theta + \mu \sin \theta + \mu W \cos \theta$$

$$\text{or, } P \cos \theta - \mu P \sin \theta = W \sin \theta + \mu W \cos \theta$$

$$\text{or, } P (\cos \theta - \mu \sin \theta) = W (\sin \theta + \mu \cos \theta)$$

$$\text{or, } P = \frac{W (\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)} = W \times \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)$$

21. Define the terms 'Coefficient of friction' and 'Angle of friction'.

[WBSCTE 2017]

Answer:

Co-efficient of friction (μ): It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies. It is denoted by the symbol μ . Thus

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$

$$\therefore F = \mu R$$

Angle of friction (ϕ): It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction (R). It is denoted by ϕ . Figure below shows a solid body resting on a rough horizontal plane.

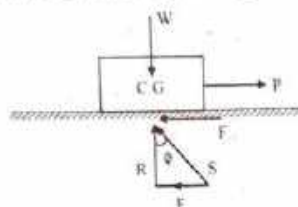


Fig: Angle of friction

Let S = resultant of the normal reaction (R) and limiting force of friction (F)
Then angle of friction = ϕ = Angle between S and R

From figure above, we have

$$\tan \phi = \frac{F}{R} = \frac{\phi R}{R} \quad [\because F = \mu R]$$

$$= \mu = \text{Co-efficient of friction}$$

22. A metal block of weight 200 N is resting on a horizontal surface. The coefficient of friction between the surfaces being 0.3. A pull is exerted on the block by means of a string inclined at 30° to the horizontal which is sufficient to cause motion of the block. Calculate the magnitude of force exerted by the string. [WBSCTE 2018]

Answer:

$$F_{\text{net}} = mg - F_N - T_v$$

$\therefore$ No acceleration in vertical direction, $F_{\text{net}} = 0$.

$$F_N = mg - T_v = mg - T \sin \theta$$

$$= 200 - T \sin 30^\circ = 200 - T(0.5)$$

$F_{\text{net}} = T_h - F_f$, No acceleration in horizontal axis,

$$F_{\text{net}} = 0$$

$$\therefore T_h - F_f \Rightarrow T_h = T \cos 30^\circ = T(0.866)$$

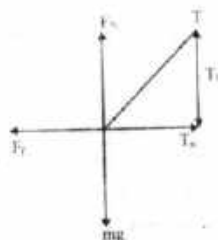
$$\therefore F_f = \mu F_N = (0.3)(200) - T \sin 30^\circ = 60 - T(0.5)$$

$$\therefore T_h = F_f$$

$$\Rightarrow T(0.866) = 60 - T(0.5)$$

$$\Rightarrow 1.366T = 60$$

$$\Rightarrow T = \frac{60}{1.366} = 43.94 \text{ N}$$



Unit 5: Centroid and Centre of Gravity

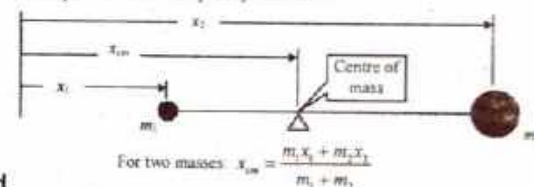
Unit at a glance:

1. Centre of gravity

The point through which resultant gravitational force (weight) acts for any orientation of the body.

2. Centre of Mass

The concept of the centre of mass is that of an average of the masses factored by their distances from a reference point. In one plane, that is like the balancing of a seesaw about a pivot point with respect to the torques produced.



3. Centroid

A point in a plane area such that the moment of area about any axis through that point is zero.

4. Centre of Gravity

Methods of finding centroid of a plane figure.

- By geometrical Consideration
- By methods of moments

5. Centroid of a plane figure by geometrical Consideration

The position of the centroid of certain simple plane figure can be seen by visual inspection.

- If figure has two axis at their intersection rectangular section
- If area possesses an axis of symmetry
- Centroid is located on that axis
- T like section

If area possesses of centre of symmetry

- Centroid with its centre of
- Z like section

Examples

1) Centroid of a straight line or uniform rod.

Centroid lies on the middle for a straight line or a uniform rod.

If straight line/ uniform rod having length L , then (centroid located at $L/2$ from either end)

2) Centroid of a rectangle or Parallelogram

Centroid located at that point where diagonals meet each other.

6. Triangle

Centroid is located at that point where medians are intercept each other.

7. By methods of moments

This is calculated with reference to some axis called axis of reference. This axis usually taken at the lowest edge of the figure to calculate y and left edge to calculate X , where x , y are the co ordinates of its centroid with reference to axis.

Short Answer Type Questions

A. Fill in the blanks in the following statements:

1. The distance of CG of a solid hemisphere having radius ' r ' from the base is $\frac{3}{8}r$
[WBSCTE 2009, 2012]

2. Moment of Inertia of a rectangle of width b and height h about its base is $I = \frac{1}{12}bh^3$
[WBSCTE 2012]

3. The centroid of a semi circular lamina of radius 10 cm is at a distance $\frac{40}{3\pi}$ cm from its diameter.
[WBSCTE 2013]

4. The centroid of an unsymmetrical lamina can be determined by composite method.
[WBSCTE 2013]

5. The centre of gravity of an equilateral triangle with each side (a) is $\frac{a}{2\sqrt{3}}$
[WBSCTE 2014]

6. The centre of gravity of a right circular cone of diameter (d) and higher (h) lies at a distance of $\frac{h}{4}$ from the base measured along the vertical radius. [WBSCTE 2014]

7. The centre of gravity of an isosceles triangle with base (p) and sides (q) is $\frac{1}{3}\sqrt{q^2 - \frac{p^2}{4}}$ from its base.
[WBSCTE 2014]

8. For a hemisphere of circular base of radius R , the location of centre of gravity is at a height $\frac{3}{5}R$ from the plane surface.
[WBSCTE 2015]

9. The C.G. of an equilateral triangle with each side ' a ' is at a distance $\frac{a}{2\sqrt{3}}$ from any of the three sides.
[WBSCTE 2016]

10. The C.G. of a solid circular cone divides the height in the ratio 1:4. [WBSCTE 2017]

11. The centre of gravity of a right circular cone of diameter (d) and height (h) lies at a distance of $\frac{h}{4}$ from the base measured along the vertical radius. [WBSCTE 2018]

12. C.G. is the point through which the whole weight of a body may be assumed to act.
[WBSCTE 2018]

13. The centre of gravity a T-section 100mm×150mm×50mm from its bottom is 87.5.
[WBSCTE 2022]

14. From a circular plate of diameter 6cm is cut out a circle whose diameter is a radius of the plate. Find the C.G. of the remainder from the centre of circular plate 0.5 cm.
[WBSCTE 2022]

B. State whether the following statements are True or False:

15. The centroid of a circle is its centre
Answer: True [WBSCTE 2009]

16. There may be more than one centre of gravity in a hollow section
Answer: False [WBSCTE 2011]

17. Centre of gravity is a unique point.
Answer: False [WBSCTE 2012]

18. Unit of Moment of Inertia is mm^4 .
Answer: True [WBSCTE 2012]

19. Position of Centroid may change if Reference Axis is changed.
Answer: True [WBSCTE 2012]

20. There may be two numbers of centre of gravity in an unsymmetrical object.

[WBSCTE 2013]

Answer: False

21. Centroid corresponds to solid whereas centre of gravity corresponds to plane lamina.

[WBSCTE 2013]

Answer: False

22. The centre of gravity of a triangle lies at a point where any two medians meet each other.

[WBSCTE 2014]

Answer: False

23. The centre of gravity of a semicircle lies at a distance of $3r/4\pi$ from its base measured along the vertical radius (r).

[WBSCTE 2014]

Answer: False

24. Centroid of a lamina of remaining portion is shifted from its original position away from the cut out hole on the lamina.

[WBSCTE 2015]

Answer: True

25. In locating centroid of a body, we apply principle of moments.

[WBSCTE 2016]

Answer: True

26. The C.G. of a semicircle lies at a distance from its base measured along the vertical radius is $4r/3\pi$.

[WBSCTE 2016]

Answer: True

27. Centroid of a triangular lamina lies at a point where its three medians meet.

[WBSCTE 2016]

Answer: True

28. The distance of C.G. from its base of a right circular cone of height ' h ' is $\frac{h}{4}$.

[WBSCTE 2017]

Answer: True

29. Centroid of a triangle lies at $\frac{2h}{3}$ from the base, where h is the height of a triangle.

[WBSCTE 2018]

Answer: False

C. Pick up the correct answer:

30. In a half filled circular cylindrical vessel the Centre of Gravity will be at (higher/lower/same height) compared to the same vessel when it was empty.

[WBSCTE 2012]

Answer: lower

31. Moment of Inertia of any lamina about its base is (less than/more than/equal to) the Moment of Inertia about its Centroidal axis.

[WBSCTE 2012]

Answer: more than

32. The centroid of a lamina will be located (on the X axis/ on the axis of symmetry / at equal distance from X and Y axis).

[WBSCTE 2013, 2014]

Answer: on the axis of symmetry

33. From a symmetrical lamina if a part is taken out which is not symmetrical about one axis the centroid of the remaining portion will be shifted (towards the centre of cut out part / away from centre of cut out part / will remain unchanged).

[WBSCTE 2015]

Answer: away from centre of cut out part.

D. Answer the following Questions:

34. Define CG of any section.

[WBSCTE 2022]

Answer:

The center of gravity (CG) of an object is the point at which weight is evenly dispersed and all sides are in balance.

Long Answer Type Questions

1. Calculate the centroid of uniform rectangular lamina by method of integration.

[WBSCTE 2004]

OR,

Derive the formula for the moment of inertia of a rectangular lamina about centroidal axis xx parallel to the width.

[WBSCTE 2010]

Answer:

Let, b = Width of the rectangular area

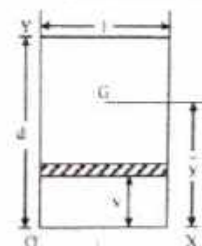
d = depth of the rectangular area and

w = weight per unit area.

Then, total weight on the area is given by $W = b \times d \times w$

This weight acts through G , which is the C.G. of the area i.e. at $(\bar{x}, \bar{y})$

Let $\bar{x}$ = distance of C.G. of the area from y axis, and y = distance of the C.G. of the area from x = axis.



At any distance 'y' from x-axis, let us consider an elementary strip of very small thickness dy, the strip being taken parallel to width b of the rectangle. Weight on this elementary strip = $b \times dy \times w$

Moment of this elementary strip about x-axis = $b \times dy \times w \times y$

∴ The algebraic sum of the moments of all such elementary weights of the rectangular area about x-axis,

$$= \int_0^d b \times dy \times w \times y = \int_0^d bwy \, dy$$

Now, moment of w about x-axis = $w \times y = b \times d \times w \times \bar{y}$

According to the principle of moments, $b \times d \times w \times \bar{y}$

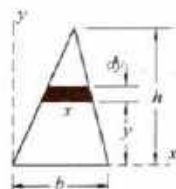
$$= \int_0^d b \times dy \times w \times y, \text{ or } b \times d \times w \times \bar{y} = b \times w \times \int_0^d y \, dy \text{ (as } b \text{ \& } w \text{ are constants)}$$

$$\text{or, } d \times \bar{y} = \left[\frac{y^2}{2} \right]_0^d = \frac{d^2}{2}$$

$$\text{or, } \bar{y} = \frac{d}{2}$$

Similarly, taking strips parallel to d, it can be shown that, $\bar{x} = \frac{b}{2}$

2. Find out the centroid of a uniform triangular lamina by the method of integration. [WBSCTE 2005, 2009, 2017]



Answer:

Step-I: First draw the figure clearly and mark all necessary dimensions.

The x-axis is taken to coincide with the base. A differential strip of area $dA = x \, dy$ is chosen.

$$\text{By similar triangles, } \frac{x}{h-y} = \frac{b}{h} \therefore x = \frac{b}{h}(h-y)$$

Now,

$$\bar{y} = \frac{\int y \, dA}{A} = \frac{\int_0^h y \, x \, dy}{\frac{1}{2} b \times h} = \frac{\int_0^h y \left(\frac{b}{h} \right) (h-y) \, dy}{\frac{1}{2} b \times h}$$

$$\therefore \bar{y} = \frac{h}{3} \quad \therefore \bar{x} = \frac{h}{3}$$

3. Determine the centroid of semi-circular area.

[WBSCTE 2006]

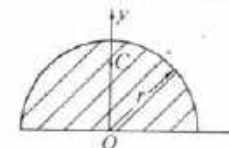
OR,

Determine the centroid of a semi-circle.

[WBSCTE 2008]

Answer:

It is straight forward to realize that when we rotate a semicircular area about one of its diameter (say, x-axis) by an angle 2π , a spherical volume is generated.



From your prior knowledge of elementary mensuration, you know that in this case,

$$V = \frac{4}{3} \pi r^3 \quad \text{and} \quad \int dA = \frac{\pi r^2}{2}$$

From Pappus theorem,

$$V = 2\pi \bar{y} A$$

$$\therefore \frac{4}{3} \pi r^3 = 2\pi \bar{y} \left(\frac{\pi r^2}{2} \right) \quad \text{or, } \bar{y} = \frac{4r}{3\pi}$$

4. Define the centre of gravity.

[WBSCTE 2009, 2011, 2013]

Answer: Refer to Article No. 4 of Unit at a Glance.

5. Determine the centroid of the area remaining after a circle of diameter, 'r' is removed from a circle of radius 'r' as shown in Fig. [WBSCTE 2009]

OR,

From a uniform circular disc of radius 'a' is cut off a circular portion of a radius $\frac{a}{2}$. Find the distance between the centers of disc and hole if the "Centre of Mass" of the remainder is on the circumference of the hole.

[WBSCTE 2010]

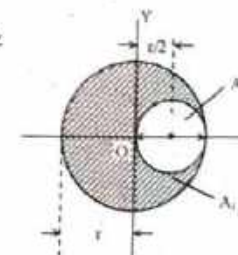
Answer:

Let A_1 and A_2 be areas of (1) & (2) & x_1, x_2 be C.G of (1) & (2) respectively.

Let $\bar{x}$ is the C.G of resultant area.

$$\therefore A_1 = \pi r^2 \quad A_2 = \frac{\pi r^2}{4}$$

We know that the C.G of a circle is of the centre of the circle.



$$\text{So, } x_1 = 0, x_2 = \frac{r}{2}$$

$$\text{So, } \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{\pi r^2 \cdot 0 - \frac{\pi r^2}{4} \cdot \frac{r}{2}}{\pi r^2 - \frac{\pi r^2}{4}} = -\frac{\pi r^3}{8} \times \frac{4}{3\pi r^2} = -\frac{r}{6}$$

$\therefore$ The centroid will be at a distance of $\frac{r}{6}$ left of the center O.

6. Define the terms centroid.

[WBSCTE 2009, 2013]

Answer: Refer to Article No. 3 of Unit at a Glance.

7. From a uniform circular disc of radius 'a' is cut off a circular portion of a radius $\frac{a}{2}$. Find the distance between the centres of disc and hole if the "Centre of Mass" of the remainder is on the circumference of the hole. [WBSCTE 2010]

Answer:

$$A_1 = \pi a^2 \quad \text{(using the basic formula of a circle)}$$

$$x_1 = a$$

$$A_2 = \pi \left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$$

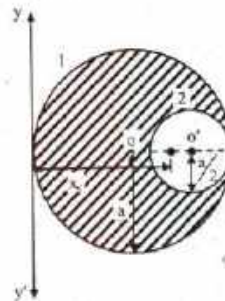
$$x_2 = a + \frac{a}{2} = \frac{3a}{2}$$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{\pi a^2 \times a - \frac{\pi a^2}{4} \times \frac{3a}{2}}{\pi a^2 - \frac{\pi a^2}{4}}$$

$$\bar{x} = \frac{\pi a^2 \left[a - \frac{3a}{8} \right]}{\pi a^2 \left[1 - \frac{1}{4} \right]} = \frac{\frac{5a}{8}}{\frac{3}{4}} = \frac{5a}{8} \times \frac{4}{3} = \frac{5a}{6}$$

The C.G. of the remaining position after the cut off circular Disc of radius $a/2$ is

$$\bar{x}, \bar{y} = \left(\frac{5a}{6}, 0 \right)$$



8. AB and AC are two uniform rods of lengths 12 m and 8 m respectively. If angle $BAC = 60^\circ$, find the distance from A to the C.G. of the two rods.

[WBSCTE 2011]

POLY-Me

Answer:

Wt. mass per unit length be λ

Then mass of AB = $M_{AB} = 12\lambda$

Mass of AC = $M_{AC} = 8\lambda$

Let centre of gravity for the rod AB is $C_1 = (x_1, y_1)$

then $AC_1 = \frac{12}{2} = 6\text{ m}$ and $x_1 = 6, y_1 = 0$

Similarly, centre of gravity for AC is $C_2 = (x_2, y_2)$

then $AC_2 = \frac{8}{2} = 4\text{ m}$ and

$$x_2 = 4 \cos 60^\circ = 2$$

$$y_2 = 4 \sin 60^\circ = 2\sqrt{3}$$

Now, by C.M. formula

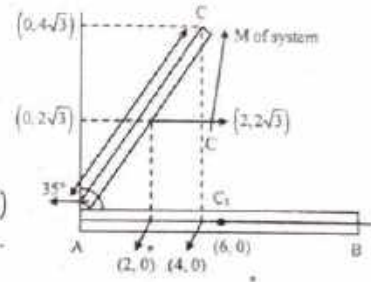
$$X_{CM} = \frac{M_{AB}x_1 + M_{AC}x_2}{M_{AB} + M_{AC}} = \frac{12\lambda \times 6 + 8\lambda \times 2}{8\lambda + 12\lambda} = \frac{72\lambda + 16\lambda}{20\lambda} = \frac{88\lambda}{20\lambda} = 4.4$$

$$Y_{CM} = \frac{M_{AB}y_1 + M_{AC}y_2}{M_{AB} + M_{AC}} = \frac{12\lambda \times 0 + 8\lambda \times 2\sqrt{3}}{12\lambda + 8\lambda} = \frac{16\sqrt{3}\lambda}{20\lambda} = 1.38$$

Hence, coordinates of CM of system C = (4.4, 1.38)

$$A = (0, 0)$$

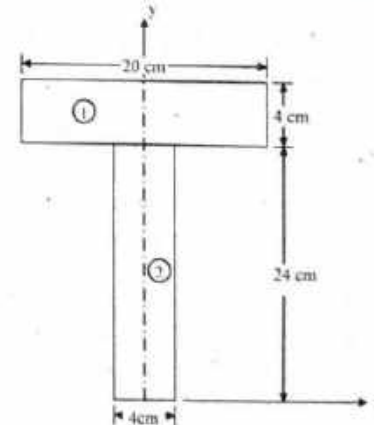
$$\therefore \text{Distance AC} = \sqrt{(4.4)^2 + (1.38)^2} = 4.6 \text{ units.}$$



9. A T section has a top flange (horizontal leg) of 20 cm $\times$ 4 cm; the web (vertical leg) has a dimension of 24 cm $\times$ 4 cm. Find out the centroid of the section.

[WBSCTE 2013]

Answer:



POLY-Me

From the symmetry of the figure, $\bar{x} = 0$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} = \frac{\left(24 + \frac{4}{2}\right) \times (20 \times 4) + 12 \times (24 \times 4)}{20 \times 4 + 24 \times 4} \text{ cm} = 18.36 \text{ cm}$$

$\therefore$ Centroid (0, 18.36), dimensions are in cm.

10. An I-section has the following dimensions in mm units: [WBSCTE 2014]

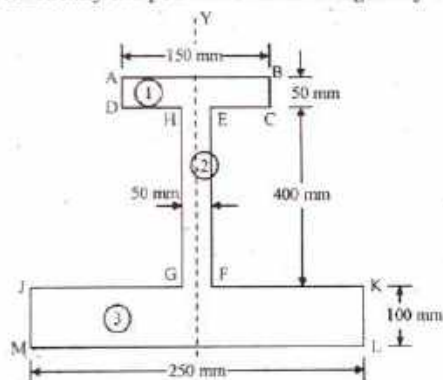
Bottom flange = 250×100

Top flange = 150×50

Web = 400×50

Determine mathematically the position of centre of gravity of the section.

Answer:



The I-section is split up into three rectangles ABCD, EFGH and JKLM as shown in Fig. above. The given I-section is symmetrical about Y-axis. Hence the C.G. of the section will lie on this axis. The bottom line of the figure is ML. Hence the moment of areas are taken about this line, which is the axis of reference.

Let $\bar{y}$ = Distance of the C.G. of the I-section from the bottom line ML.

$$a_1 = \text{Area of rectangle ABCD} = (150 \times 50) \text{ mm}^2 = 7500 \text{ mm}^2$$

$$y_1 = \text{Distance of C.G. of rectangle ABCD from bottom line ML} = (100 + 40 + 25) \text{ mm} = 165 \text{ mm.}$$

$$a_2 = \text{Area of rectangle EFGH} = (50 \times 400) \text{ mm}^2 = 20000 \text{ mm}^2$$

$$y_2 = \text{Distance of C.G. of rectangle EFGH from bottom line ML} = 100 + 200 = 300 \text{ mm.}$$

$$a_3 = \text{Area of rectangle JKLM} = (250 \times 100) \text{ mm}^2 = 25000 \text{ mm}^2$$

$$y_3 = \text{Distance of C.G. of rectangle JKLM from bottom line ML} = 50 \text{ mm.}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{8487500}{52500} = 161.67 \text{ mm (Ans.)}$$

11. Find the location of centroid of a T section with flange $120 \text{ mm} \times 20 \text{ mm}$ and the web $200 \text{ mm} \times 20 \text{ mm}$. [WBSCTE 2015]

Answer:

From the symmetry of the T section

$$\bar{x} = 0$$

$$\bar{y} = \frac{\left(200 + \frac{20}{2}\right) \times (120 \times 20) + (100 \times 200 \times 20)}{(120 \times 20) + (200 \times 20)} \text{ mm} = 141.25 \text{ mm}$$

$\therefore$ The location of centroid of a T section = (0, 141.25) dimensions are in mm.

12. AB and AC are two uniform thin weightless rods of lengths $2a$ and $2b$ respectively. If $\angle BAC = \theta$, prove that the distance from A of the centroid of the two rods is $(a^4 + 2a^2b^2\cos\theta + b^4)^{0.5}/(a+b)$. [WBSCTE 2015]

Answer:

Let weight/unit length = λ [as rod is weightless, So $\lambda = 1$]

Let C.G. of AB (x_1, y_1)

$$x_1 = \frac{2a}{2} = a; \quad y_1 = \frac{0}{2} = 0$$

$$\therefore (x_1, y_1) = (a, 0)$$

Let C.G. of AC (x_2, y_2)

$$x_2 = \frac{2b\cos\theta}{2} = b\cos\theta$$

$$y_2 = \frac{2b\sin\theta}{2} = b\sin\theta$$

$$M_{AB} = 2a\lambda = 2a$$

$$M_{AC} = 2b\lambda = 2b$$

C.G. of the composite body

$$x_{cm} = \frac{M_{AB}x_1 + M_{AC}x_2}{M_{AB} + M_{AC}} = \frac{2a \cdot a + 2b \cdot b\cos\theta}{2a + 2b} = \frac{a^2 + b^2\cos\theta}{a + b}$$

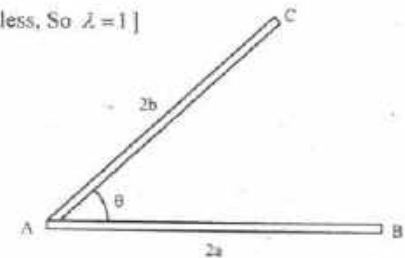
$$\text{Similarly, } y_{cm} = \frac{2a \cdot 0 + 2b \cdot b\sin\theta}{2a + 2b} = \frac{b^2\sin\theta}{a + b}$$

Distance between centroid and point A (0, 0)

$$L = \sqrt{\left(\frac{a^2 + b^2\cos\theta}{a + b}\right)^2 + \left(\frac{b^2\sin\theta}{a + b}\right)^2}$$

$$= \frac{1}{a + b} \sqrt{(a^2 + b^2\cos\theta)^2 + (b^2\sin\theta)^2} = \frac{1}{a + b} \sqrt{a^4 + b^4\cos^2\theta + 2a^2b^2\cos\theta + b^4\sin^2\theta}$$

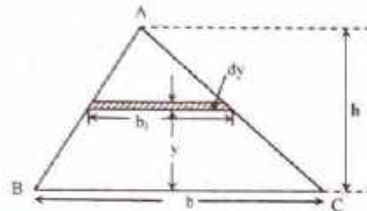
$$= \frac{1}{a + b} \sqrt{a^4 + 2a^2b^2\cos\theta + b^4} = \frac{1}{a + b} (a^4 + 2a^2b^2\cos\theta + b^4)^{0.5}$$



13. a) Determine the position of centroid of a triangle of base 'b' and height 'h' with respect to its base.
b) Find the C.G. of an unequal angle section 12cm × 8cm × 1cm. [WBSCTE 2016]

Answer:

a)



Consider the triangle ABC base width b and height h. Let us locate the distance of centroids from the base.

Let b_1 be the width of elemental strip of thickness dy at a distance y from its base.

Since, $\triangle AEF$ and $\triangle ABC$ are similar triangles, we can write:

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(\frac{h-y}{h} \right) b = \left(1 - \frac{y}{h} \right) b$$

$$\therefore \text{Area of the element} = dA = b_1 dy \Rightarrow dA = \left(1 - \frac{y}{h} \right) b dy$$

$$\text{Area of triangle } A = \frac{1}{2} bh$$

$$\text{Now, } \bar{y} = \frac{\text{Movement of area}}{\text{total area}} = \frac{\int y dA}{A}$$

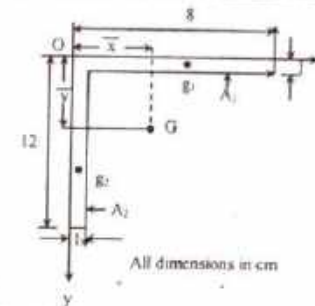
$$\therefore \int y dA = \int_0^h y \left(1 - \frac{y}{h} \right) b dy = \frac{b}{h} \int_0^h \left(y - \frac{y^2}{h} \right) dy = \frac{b}{h} \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h = \frac{bh^2}{6}$$

$$\therefore \bar{y} = \frac{\int y dA}{A} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2}bh} \therefore \bar{y} = \frac{h}{3}$$

Thus, the centroid of a triangle is at a distance $\frac{h}{3}$ from the base of the triangle where h is the height of the triangle.

Centroid and Centre of Gravity

b)



This composite figure can be divided into two rectangles:

$$A_1 = 8 \times 1 = 8 \text{ cm}^2$$

$$A_2 = (12-1) \times 1 = 11 \text{ cm}^2$$

$$\text{Total area, } A = A_1 + A_2 = (8 + 11) \text{ cm}^2 = 19 \text{ cm}^2$$

Selecting the reference axis x and y as shown in figure, the centroid of A_1 is $g_1 \left(4, \frac{1}{2} \right)$

and that of A_2 is,

$$g_2 \left[\frac{1}{2}, 1 + \frac{1}{2}(12-1) \right]$$

$$\text{i.e. } g_2 \left(\frac{1}{2}, \frac{13}{2} \right)$$

$$\therefore \bar{x} = \frac{\text{Movement about y axis}}{\text{Total area}}$$

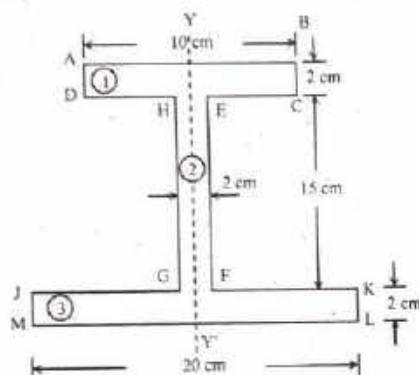
$$= \frac{A_1 x_1 + A_2 x_2}{A} = \frac{(8 \times 4) + \left(11 \times \frac{1}{2} \right)}{19} = \frac{32 + \frac{11}{2}}{19} = \frac{37.5}{19} = 1.97 \text{ cm}$$

$$\therefore \bar{y} = \frac{\text{Movement about x axis}}{\text{Total area}}$$

$$= \frac{A_1 y_1 + A_2 y_2}{A} = \frac{\left(8 \times \frac{1}{2} \right) + \left(11 \times \frac{13}{2} \right)}{19} = \frac{57.5}{19} = 3.03 \text{ cm}$$

Thus, the centroid is at $\bar{x} = 1.97 \text{ cm}$ and $\bar{y} = 3.03 \text{ cm}$

14. Find out the centroid of an I-section has bottom flange 20cm × 2cm, top flange 10cm × 2cm and web 15cm × 2cm. [WBSCTE 2017]



The I-section is split up into three rectangles ABCD, EFGH and JKLM. The given I-section is symmetrical about Y-axis. Hence, the C.G. of the section will lie on this axis. The bottom line of the figure is ML. Hence, the moment of areas are taken about this line, which is the axis of reference.

Let, $\bar{y}$ = Distance of the C.G. of the I-section from the bottom line ML.

$$a_1 = \text{Area of rectangle ABCD} = (10 \times 2) \text{ cm}^2 = 20 \text{ cm}^2$$

$$y_1 = \text{Distance of C.G. of rectangle ABCD from bottom line ML} = (2 + 1.5 + 1) = 4.5 \text{ cm}$$

$$a_2 = \text{Area of rectangle EFGH} = (15 \times 2) \text{ cm}^2 = 30 \text{ cm}^2$$

$$y_2 = \text{Distance of C.G. of rectangle EFGH from bottom line ML} = (2 + 7.5) \text{ cm} = 9.5 \text{ cm}$$

$$a_3 = \text{Area of rectangle JKLM} = (20 \times 2) \text{ cm}^2 = 40 \text{ cm}^2$$

$$y_3 = \text{Distance of C.G. of rectangle JKLM from bottom line ML} = 1 \text{ cm}$$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(20 \times 4.5) + (30 \times 9.5) + (40 \times 1)}{20 + 30 + 40} = 4.61 \text{ cm}$$

15. a) What is Centroid and Centre of Gravity?

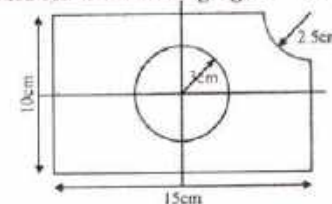
[WBSCTE 2018]

Answer:

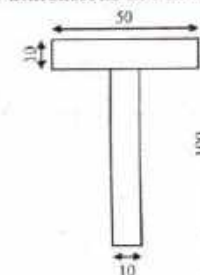
Centroid: The point at which the total area of a plane figure (like rectangle, square, triangle, quadrilateral, circle etc.) is assumed to be concentrated, is known as the centroid of that area. The centroid is also represented by C.G. or simply G. The centroid and centre of gravity are at the same point.

Centre of Gravity: Centre of gravity of a body is the point through which the whole weight of the body acts. A body is having only one centre of gravity for all positions of the body. It is represented by C.G. or simply G.

15. b) Find the Centroid for the following figure having dimensions as shown.

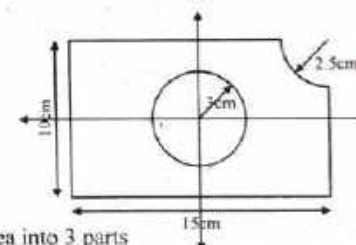


Find the centre of Gravity of a 'T' section with equal width throughout having dimensions as shown in fig. All dimensions are in mm. [WBSCTE 2018]



Answer:

1st Part:



First let us divide the area into 3 parts

- Area 1 $\rightarrow$ Rectangle with length 15 cm and breadth 10 cm
- Area 2 $\rightarrow$ Circle with radius 3 cm
- Area 3 $\rightarrow$ Quarter circle with radius 2.5 cm

Now let us consider the centre of the centre of Area 1 and Area 2 as the origin.

For Area 1:

$$\text{Length} = 15 \text{ cm, breadth} = 10 \text{ cm, Area } A_1 = 15 \times 10 = 150 \text{ cm}^2$$

Coordinates of centroid:

$$X_1 = 0 \quad Y_1 = 0$$

For Area 2:

$$\text{Radius} = 3 \text{ cm, Area } A_2 = \pi \times (3)^2 = 9\pi = 28.26 \text{ cm}^2$$

Coordinates of centroid:

$$X_2 = 0 \quad Y_2 = 0$$

For Area 3:

$$\text{Radius} = 2.5 \text{ cm, Area } A_3 = \frac{\pi \times (2.5)^2}{4} = 4.90 \text{ cm}^2$$

Coordinates of centroid:

$$X_3 = 7.5 - \frac{4 \times (2.5)}{3\pi} = 6.439 \quad Y_3 = 5 - \frac{4 \times (2.5)}{3\pi} = 3.939$$

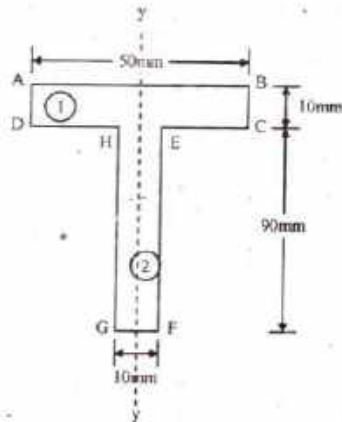
 $\therefore$ x coordinate of the centroid of the whole area,

$$X = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = \frac{150 \times 0 + (-28.26) \times 0 + (-4.90) \times 6.439}{150 - 28.26 - 4.90} = \frac{-31.5511}{116.84} = -0.27 \text{ cm}$$

 Secondly y coordinate of the centroid of the whole area,

$$Y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{150 \times 0 + (-28.26) \times 0 + (-4.90) \times 3.939}{150 - 28.26 - 4.90} = \frac{-19.3011}{116.84} = -0.165 \text{ cm}$$

 $\therefore$ The coordinate of centroid of the Area as $(-0.27 \text{ cm}, -0.165 \text{ cm})$

 2nd Part:

 Let, $\bar{y}$ = Distance of the C.G. of the T-section from the bottom line GF (axis of reference)

$$A_1 = \text{Area of rectangle } ABCD = (50 \times 10) = 500 \text{ mm}^2$$

$$y_1 = \text{Distance of C.G. of area } A_1 \text{ from bottom line } GF = 90 + \frac{10}{2} = 95 \text{ mm}$$

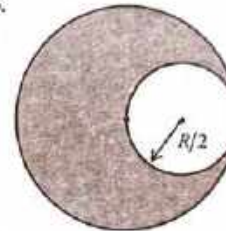
$$A_2 = \text{Area of rectangle } EFGH = (90 \times 10) = 900 \text{ mm}^2$$

$$y_2 = \text{Distance of C.G. of area } A_2 \text{ from bottom line } GF = \frac{90}{2} = 45 \text{ mm}$$

Using equation,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(500 \times 95) + (900 \times 45)}{(500 + 900)} = 62.86 \text{ mm}$$

16.



Determine the CG of the section above.

Answer:

$$A_1 = \pi R^2 \text{ (Using the basic formula of a circle)}$$

$$x_1 = R$$

$$A_2 = \pi \left(\frac{R}{2} \right)^2 = \frac{\pi R^2}{4}$$

$$x_2 = R + \frac{R}{2} = \frac{3R}{2}$$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{\pi R^2 \times R - \frac{\pi R^2}{4} \times \frac{3R}{2}}{\pi R^2 - \frac{\pi R^2}{4}} = \frac{\pi R^2 \left[R - \frac{3R}{8} \right]}{\pi R^2 \left[1 - \frac{1}{4} \right]} = \frac{\frac{5R}{8}}{\frac{3}{4}} = \frac{5R}{8} \times \frac{4}{3} = \frac{5R}{6}$$

 The centre of gravity (C.G.) of the portion is $(\bar{x}, \bar{y}) = \left(\frac{5R}{6}, 0 \right)$

[WBSCTE 2022]

Unit 6: Simple Lifting Machines

Unit at a glance:

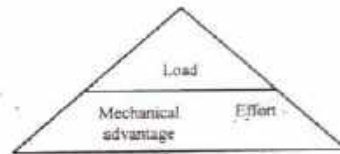
1. Definition

A simple machine is a device that simply transforms the direction or magnitude of a force, but a large number of more complex machines exist.

A lever consists of a rigid rod free to rotate about fulcrum having load and effort at two other points.

2. Mechanical Advantage

Mechanical advantage is defined as the ratio of load to effort. Pulley systems rely on this important relationship between load and effort. The formula seen below is best understood by writing it within a triangle. This helps when it is necessary to change the formula to find either; mechanical advantage or the load or the effort. In this way three formulas can be generated from the single formula inside the triangle.



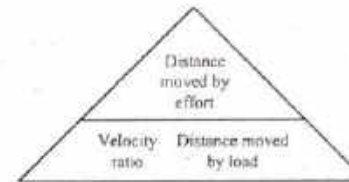
$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}}$$

$$\text{Load} = \text{Mechanical advantage} \times \text{Effort}$$

$$\text{Effort} = \frac{\text{Load}}{\text{Mechanical advantage}}$$

3. Velocity Ratio

Velocity Ratio (sometimes called movement ratio) - is defined as the ratio of the distance moved by the effort to the distance moved by the load. The formula seen below is best understood by writing it within a triangle. This helps when it is necessary to change the formula to find either; velocity ratio or the distance moved by the load or the distance moved by effort. In this way three formulas can be generated from the single formula inside the triangle.



$$\frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \text{Velocity ratio}$$

$$\text{Velocity ratio} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\text{Distance moved by effort} = \text{Distance moved by load} \times \text{Velocity ratio}$$

In the ideal frictionless/weightless machine

Velocity Ratio = Mechanical Advantage

The efficiency of a simple machine

Efficiency = Work done by the machine / Work supplied to machine

Relationship between MA, VR and efficiency.

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Work Output}}{\text{Work Input}} = \frac{\text{Load} \times \text{Distance Moved by Load}}{\text{Effort} \times \text{Distance Moved by Effort}} \\ &= \frac{\text{Load/Effort}}{\text{Distance Moved by Effort/Distance Moved by Load}} \\ &= \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \end{aligned}$$

Results:

- (i) Efficiency of an ideal machine is 1, hence mechanical advantage = velocity ratio.
- (ii) Efficiency of practical machine is less than 1, hence mechanical advantage is less than velocity ratio.

4. Reversible and Irreversible Machine

Let P is the actual effort applied to a machine to lift a load W. If the actual effort P is removed, the load W may or may not fall down to the initial position. If the load W falls down, the machine is said to be reversible. In this case work is done by the machine in reverse direction. And if the load W does not fall down, the machine is said to be irreversible or self locking. In this case work is not done by the machine in reverse direction.

5. Condition for Irreversibility of a Machine

$$\therefore \eta < \frac{1}{2} \text{ or } 50\%$$

Thus the condition for irreversibility or self-locking of a machine is that its efficiency should be less than 50%.

6. Law of a Machine

The law of a machine is defined by an equation which gives the relationship between the effort required to raise the corresponding load. This is obtained by drawing a graph between the efforts and the corresponding loads lifted by the efforts.

Lifting machines are the machines used for lifting heavy loads or weights by applying comparatively small force.

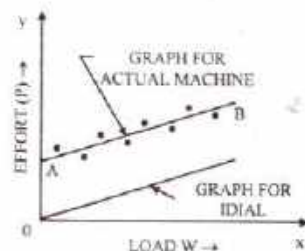


Fig: Law of a machine

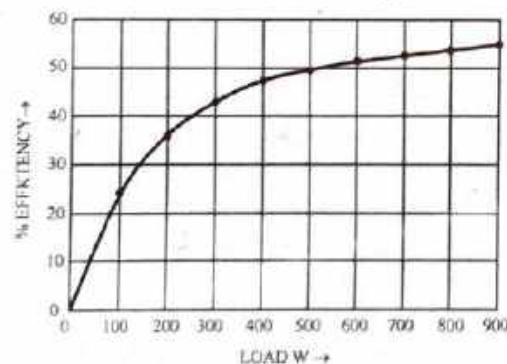
Expression for maximum mechanical advantage

$$\therefore \text{Max. M.A.} = \frac{1}{m}$$

Expression for Maximum Efficiency

$$\eta_{\text{max}} = \frac{1}{m \times (V.R.)}$$

7. Efficiency Vs Load Curve



Short Answer Type Questions

A. Choose the correct answer from the given alternative:

1. In actual machine –
(a) $MA = VR$ (b) $MA > VR$ (c) $MA < VR$ (d) MA is unity [WBSCTE 2006]

Answer: (c)

B. Fill in the blanks in the following statements:

2. Mechanical advantage is the ratio of load & Effort. [WBSCTE 2010]

3. With usual notations Velocity Ratio of Differential Screw Jack is $\frac{2AR}{P_A - P_B}$. [WBSCTE 2011]

4. A self locking machine is one where energy is not conserved. [WBSCTE 2012]

5. With usual notations the Velocity ratio of differential wheel and axle is given by $\frac{2D}{D_1 - D_2}$. [WBSCTE 2013]

6. The Law of Simple Lifting Machine is given by $P = mW + C$. [WBSCTE 2013, 2014]

7. Maximum efficiency of a lifting machine is greater than 50%. [WBSCTE 2015]

8. Velocity Ratio of first system of pulleys is 2. [WBSCTE 2015]

9. In a simple lifting machine effective load can be lifted by application of comparatively smaller effort. [WBSCTE 2015]

10. The efficiency of an actual machine is always high. [WBSCTE 2015]

11. If efficiency of a lifting machine is kept constant, its velocity ratio is directly proportional to its mechanical advantage (MA). [WBSCTE 2016]

12. Instead of simple wheel and axle, differential wheel and axle gives more V.R. [WBSCTE 2016]

13. In the equation $P = mW + C$ of a lifting machine, 'C' indicates the effort required to overcome friction under no load condition. [WBSCTE 2016]

14. The maximum mechanical advantage of a lifting machine is 1/m. [WBSCTE 2017]

15. The efficiency of a screw jack may be increased by increasing its pitch.
[WBSCTE 2018]
16. A simple machine has only one point for the application of effort and load.
[WBSCTE 2018]
17. Instead of simple wheel and axle, differential wheel and axle gives $\frac{2D}{d_1 - d_2}$ V.R.
[WBSCTE 2018]
18. The amount of effort required to overcome the machine friction is given by Actual effort-ideal effort.
[WBSCTE 2018]
19. The mechanical advantage of a lifting machine is the ratio of - load lifted to the effort applied.
[WBSCTE 2022]
20. In ideal machines, mechanical advantage is numerically equal velocity ratio.
[WBSCTE 2022]
21. The angular velocity (in rad/s) of a body rotating at N revolutions per minute is $\frac{N}{60} \times 2\pi$ RAD/S.
[WBSCTE 2022]
22. A machine having an efficiency greater than 50%, is known as reversible machine.
[WBSCTE 2022]

C. State whether the following statements are True or False:

23. In a differential machine velocity ratio is decreased than a simple machine.
[WBSCTE 2011]
Answer: True
24. In a simple lifting machine a higher load is lifted by application of a relatively small effort.
[WBSCTE 2011]
Answer: True
25. For a reversible machine efficiency is greater than 50%.
[WBSCTE 2012]
Answer: True
26. Normal acceleration signifies change in direction of a rotating body.
[WBSCTE 2012]
Answer: False
27. If more time is required to complete a work the power is more.
[WBSCTE 2012]
Answer: False
28. For self locking machine the efficiency is lower than 50%.
[WBSCTE 2013]
Answer: True

29. The effort lost due to friction is given by $\left(P - \frac{W}{VR}\right)$ in simple lifting machine.
Symbols used having their usual significance.
[WBSCTE 2014]
Answer: True
30. The maximum mechanical advantage of a lifting machine is $(1 + m)$.
[WBSCTE 2014]
Answer: False
31. Velocity Ratio of a simple Lifting Machine is a constant.
[WBSCTE 2015]
Answer: False
32. If efficiency of a machine is 40% the machine will be reversible.
[WBSCTE 2015]
Answer: False
33. The velocity Ratio of a Simple Screw Jack is $2 \times \pi \times L / (p_1 - p_2)$.
[WBSCTE 2015]
Answer: False
34. In case of a lifting machine load vs. effort graph is a curve.
[WBSCTE 2016]
Answer: False
35. If the law of lifting machine is $P = 0.04W + 20$, the maximum mechanical advantage is 25.
[WBSCTE 2016]
Answer: True
36. Mechanical advantage is the ability of a machine to multiply force.
[WBSCTE 2016]
Answer: True
37. At the point of reversing the efficiency of a simple lifting machine is 60%.
[WBSCTE 2017]
Answer: True
38. Velocity ratio of a simple machine is constant.
[WBSCTE 2017]
Answer: True
39. In actual machine $MA > VR$.
[WBSCTE 2017]
Answer: False
40. Load vs Effort graph is a curve.
[WBSCTE 2017]
Answer: True
41. A machine is said to be self-locking when its efficiency is less than 50%.
[WBSCTE 2017]
Answer: True

42. The efficiency of a screw jack is maximum when the helix angle is equal to $\left(45^\circ - \frac{\theta}{2}\right)$. [WBSCTE 2018]

Answer: True

43. In case of a lifting machine load vs. Effort graph is a curve. [WBSCTE 2018]

Answer: False

44. Mechanical advantage is the ability of a machine to multiply force. [WBSCTE 2018]

Answer: True

45. The velocity ratio of a single purchase crab which can be increased by increasing number of teeth on the pinion. [WBSCTE 2018]

Answer: False

D. Pick up the correct answer:

46. A machine is said to be at the point of reversing of its efficiency is (equal to 50%, more than 50%, less than 50%) [WBSCTE 2011]

Answer: more than 50%

47. Law of Machine is ($W = P + mc/W = mP + c/P = mW + c$) [WBSCTE 2011]

Answer: $P = mW + c$

48. The Velocity Ratio of Single Purchase Crab Winch is (LN/nr ; $LT1/rT2$; $2D/D-d$). [WBSCTE 2012]

Answer: $LT1/rT2$

49. For a Simple Machine with law given by $P = mW + c$, the maximum Mechanical Advantage is ($1/m$; m ; mW). [WBSCTE 2012]

Answer: m

50. For a freely falling body the Potential Energy is (maximum at the highest point/minimum at the highest point/constant in all the height). [WBSCTE 2012]

Answer: maximum at the highest point

51. The effort lost due to friction is given by ($P - W/V.R.$; $P \times V.R. - W$; $W - P/V.R.$) in simple lifting machine. [WBSCTE 2013]

Answer: $P - \frac{W}{V.R.}$

52. At the point of reversing the efficiency of a simple lifting machine is (40%/50%/60%). [WBSCTE 2013, 2014]

Answer: 50%

53. A lifting machine having an efficiency less than 50% is known as (reversible machine, non-reversible machine, ideal machine, none). [WBSCTE 2014]

Answer: non-reversible machine

54. In an ideal machine, the mechanical advantage is (equal to, less than, greater than velocity ratio). [WBSCTE 2014]

Answer: equal to

55. The frictional load is given by ($P - W/V.R.$; $P \times V.R. - W$; $W - P/V.R.$) [WBSCTE 2015]

Answer: $W - P/V.R.$

E. Answer the following questions:

56. If the law of lifting machine is $P = 0.04W + 20$, the maximum mechanical advantage [WBSCTE 2018]

Answer: 25

Long Answer Type Questions

1. Derive the expression to obtain maximum efficiency of a machine. [WBSCTE 2003]

Answer:

To define, maximum efficiency it is needed to familiar with the term mechanical advantage (MA) and velocity ratio (V.R).

$$M.A = \frac{\text{Load lifted by the machine}}{\text{Effort required to lift the load}} = \frac{W}{P}$$

$$\text{Velocity ratio (V.R.)} = \frac{\text{distance moved by the effort in lifting the load}}{\text{distance moved by the load}} = \frac{Y}{X}$$

$$\text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Input}} = \frac{\text{load lifted} \times \text{distance through which the load has been lifted}}{\text{effort applied} \times \text{distance through which the effort has moved}}$$

$$= \frac{W \cdot X}{P \cdot Y} = \frac{P}{X} \cdot \frac{M.A}{V.R.} = \frac{W}{P \cdot V.R.}$$

It is known that $P = mW + C$ (where m = coefficient of friction

C = another constant which represented the machine friction)

For maximum efficiency, put $P = m.W + C$

$$\text{Max } \eta' = \frac{W}{(mW + C) \cdot V.R.}$$

$$= \frac{1}{\left(m + \frac{C}{W}\right) \cdot V.R.} = \frac{1}{m \cdot V.R.} \left[\text{where } \frac{C}{W} \text{ neglected} \right]$$

2. In a lifting machine, the efforts required to lift loads of 250N and 375N are 60N and 70N respectively. If the V.R. of the machine be 20, determine: (i) the law of the machine (ii) efficiency of the machine corresponding to the load of 250N and (iii) effort lost in friction in the first case. [WBSCTE 2003]

Answer:

We have, $P = mW + C$

The loads are 250N & 375 N respectively, the force required to lift the loads is 60N & 70N. Using the above formula we get,

$$60 = m \times 250 + C \dots (1)$$

$$\& \quad 70 = m \times 375 + C \dots (2)$$

(2) - (1) we get,

$$125m = 10$$

$$\Rightarrow m = \frac{10}{125} = 0.08$$

Putting that in equation (1) we get $60 = 0.08 \times 250 + C$

$$\Rightarrow C = 60 - 20 = 40$$

The required law of the machine in

i) $P = 0.08W + 40$ (Ans.)

ii) Efficiency = $\frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}}$

When the load 250N, effort is 60N

$$\text{Mechanical Advantage (MA)} = \frac{250}{60} = 4.17$$

$$\text{Efficiency at this load} = \frac{4.17}{20} = 0.208 = 20.8\% \text{ (Ans.)} \quad [V.R. = 20 \text{ (Given)}]$$

iii) Effort lost in friction = $P - \frac{W}{V.R.}$

When $P = 60\text{N}$ and $W = 250\text{N}$, the required effort lost in

$$\text{friction} = 60 - \frac{250}{20} = (60 - 12.5)\text{N} = 47.5\text{N} \text{ (Ans.)}$$

3. Define mechanical advantage, velocity ratio and efficiency of a lifting machine. [WBSCTE 2004]

Answer: Refer to Article No. 2 and 3 of Unit at a Glance.

4. State and explain the law of machine.

[WBSCTE 2005, 2006]

OR,

In a lifting machine find the relationship between effort (P), Load (W) with usual other notations [WBSCTE 2011]

OR,

What is Law of machine?

[WBSCTE 2022]

Answer:

Law of a machine:

The term 'law of a machine' defined as the relationship between the load lifted and the effort applied. Thus for any machine, if we record the various efforts required to raise the corresponding loads and plot a graph between effort and load, we get a straight line AB as shown in Fig. We know that the intercept OA represented the amount of friction (frictional force) offered by the machine.

Or in other words this is the effort required by the machine to overcome the friction, before it can lift any load. Mathematically, the law of lifting machine is given by the relation.

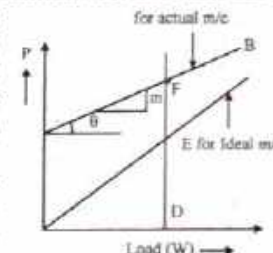
$$P = mw + c$$

Where P = Effort applied to lift.

m = A constant (called coefficient of friction) which equals to the line AB.

W = load lifted and

c = another constant, which represented the machine friction (i.e., OA).



5. What do you understand by the term 'Reversibility' Determine the condition of reversibility of a lifting machine. [WBSCTE 2009, 2016]

Answer:

If the removal of effort while lifting results in lowering of load the machine is said to be reversible.

Suppose, Load = W ; Effort = P

Velocity Ratio = $V.R.$

Distance moved by effort = D ; Distance moved by the load = d

Input work = $P \times D$

Output energy = $W \times d$

Work lost in the form of friction = $PD - Wd$

The machine will be reversible if the load can start moving down after removal of effort overcoming frictional resistance = Wd .

So condition for reversibility -

$$Wd > (PD - Wd)$$

$$\text{or, } 2Wd > PD$$

$$\text{or, } \frac{W/P}{D/d} > \frac{1}{2}$$

$$\text{or, } \frac{M.A.}{V.R.} > \frac{1}{2} \quad \text{or, } \eta > \frac{1}{2} \text{ or } 50\%$$

So a machine is reversible if $\eta > 50\%$

6. Explain the working principle of screw jack?

Answer:

This device is commonly used to lift the load (heavy). Screw jack works on the principle same as the inclined plane. The axial distance moved by the nut when it makes one complete revolution --- known as 'lead of the screw head'.

R = length of the lever & diameter of the screw.

W = weight (load) lifted using an effort ' P '. Effort ' P ' applied at lever end it is equivalent to an effort P_1 at the screw & P_1 is given by the condition:

$$\therefore P \times R = P_1 \times \frac{d}{2} \quad P_1 = \frac{2PR}{d}$$

P = load of screw

d = mean diameter of screw

Now, consider one complete revolution of the lever, the load ' W ' is lifted up by a distance ' P ' equal to lead of the screw.

Applying effort ' P ' at the end of the lever is as good as applying an effort P_1 (at screw) on the inclined plane. Resolving horizontally, the forces parallel to P_1

$$P_1 = R \sin(\theta + \phi)$$

R = Resultant Reaction

Resolving vertically: $W = R \cos(\theta + \phi) \quad \frac{P}{W} = \tan(\theta + \phi)$

$$P_1 = W \tan(\theta + \phi)$$

$$\text{But, } P_1 = \frac{2PR}{d} \therefore \frac{2PR}{d} = W \tan(\theta + \phi)$$

$$P = \frac{d}{2R} W \tan(\theta + \phi)$$

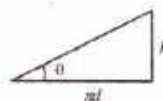
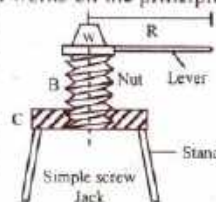
$$\therefore P = \frac{d}{2R} W \frac{(\tan \theta + \tan \phi)}{(1 - \tan \theta \tan \phi)} = \frac{d}{2R} W \frac{(\mu + \tan \theta)}{(1 - \mu \tan \theta)}$$

$$\text{When } \tan \theta = \left(\frac{P}{\mu d} \right)$$

If the load is descending, then the friction will be acting in reverse direction. So that the resultant reaction ' R ' shifts as per figure

$$P = \frac{d}{2R} W \tan(\theta - \phi)$$

[WBSCTE 2010]



$$\text{Torque required } T = P \cdot R = \frac{d}{2} W \tan(\theta + \phi)$$

$$\text{Hence, torque required while ascending: } T = \frac{d}{2} W \tan(\theta + \phi)$$

$$\text{Torque required while descending: } T = \frac{d}{2} W \tan(\theta - \phi)$$

$$\therefore V.R. = \frac{\text{Distance moved effort}}{\text{Distance moved by load}} = \frac{2\pi R}{P}$$

7. A machine raised a load of 360-N through a distance of 200 mm. The effort a force of 60N moved 1.8m during the process. Calculate - (i) Mechanical advantage, (ii) velocity ratio, (iii) Efficiency of this load, (iv) Effect of friction?

[WBSCTE 2010]

Answer:

Given the data are L = Load = 360N

E = effort = 60N

d_E = distance travelled by effort = 1.8m and d_L = distance travelled by load = 0.2 m

$$\text{i) } M.A. = \frac{L}{E} = \frac{360}{60} = 6$$

$$\text{ii) } V.R. = \frac{d_E}{d_L} = \frac{1.8}{0.2} = 9$$

$$\text{iii) } \text{Efficiency} = \frac{M.A.}{V.R.} = \frac{6}{9}$$

$$\% \text{ efficiency} = \frac{6}{9} \times 100 = 66.66\%$$

$$\text{iv) Friction, } P_f = \frac{W}{V.R.} = \frac{360}{9} = 40 \text{ (effort due to friction)}$$

$$P - P_f = 60 - 40 = 20 \text{ N (effort lost due to friction)}$$

8. In a simple machine an effort of 10N is required to raise a load of 400N through a distance of 50 mm & it is noted that the effort for moved 2.5m. Is the machine Reversible? If so what effort should be applied if the machine is on the point of reversal?

[WBSCTE 2010]

Answer:

P = Effort = 10N W = load = 400N

D = Distance moved by effort = 2.5m d = Distance moved by load = 50mm = 0.050m

$$M.A. = \frac{400}{10} = 40 \quad V.R. = \frac{2.5}{0.050} = \frac{2500}{50} = 50$$

$$\eta = \text{Efficiency} = \frac{M.A.}{V.R.} \times 100 = \frac{40}{50} \times 100 = 80\%$$

So, it is Reversible machine because $\eta > 50\%$

Ideal load $W_i = P \times V.R. = 10 \times 50 = 500 \text{ N}$

Frictional Resistance = $w_i - w = 500 - 400 = 100 \text{ N}$ (Ans.)

9. In a lifting machine an effort of 150 N raised a load of 7700 N. What is the Mechanical Advantage? Find the Velocity Ratio if the efficiency at this load is 60%. If by the same machine, a load of 13200 N is raised by an effort of 250 N, what is the efficiency? Calculate the maximum mechanical advantage and the maximum Efficiency. [WBSCTE 2011]

Answer:

Given data: Load (L) = 7700 N

Effort (E) = 150 N and Efficiency = 60%

$$\therefore \text{The Mechanical Advantage (M.A.)} = \frac{L}{E} = \frac{7700}{150} = 51.33 \text{ (Ans.)}$$

$$\therefore \text{The Velocity Ratio (V.R.)} = \frac{\text{M.A.}}{\text{Efficiency}} = \frac{51.33}{60} \times 100 = 85.56 \text{ (Ans.)}$$

Again, Load (L) = 13200 N and Effort (E) = 250 N

$$\therefore \text{Efficiency} = \frac{M.R.}{V.R.} = \frac{L}{E} = \frac{13200}{250} \times 100\% = \frac{52.8}{85.56} \times 100 = 61.71\% \text{ (Ans.)}$$

To calculate the maximum Mechanical Advantage and maximum Efficiency:

We know, the law of machine, $P = mW + C$

So, from Case 1:

$$150 = 7700m + C \quad \dots(i)$$

and from Case 2:

$$250 = 13200m + C \quad \dots(ii)$$

By solving equation (i) and (ii) we get,

$$\therefore m = 0.018$$

$$\therefore \text{Maximum Mechanical Advantage} = \frac{1}{m} = \frac{1}{0.018} = 55.56$$

$$\therefore \text{Maximum efficiency } (\eta) = \frac{1}{m \times V.R.} \times 100\% = \frac{1}{0.018 \times 85.56} \times 100\% = 64.93\%$$

10. The law of machine, with a Velocity Ratio 160, is given by $P = 0.014W + 15$. Determine: (a) The effort required to lift a load of 50 kN, (b) Mechanical Advantage under condition (a) : (c) Efficiency under condition (a); (d) Maximum efficiency of the machine. [WBSCTE 2012]

Answer:

$$(a) \text{ Required effort } P = 0.014 \times 50 \times 10^3 + 15 = 715 \text{ N}$$

$$(b) \text{ Mechanical Advantage} = \frac{50 \times 10^3}{715} = 69.93$$

$$(c) \text{ Efficiency under this condition} = \frac{69.93}{160} = 43.7\%$$

$$(d) \text{ Maximum efficiency} = \frac{1}{m \times V.R.} = \frac{1}{0.014 \times 160} = 44.64\%$$

11. (a) Define Power. What is the practical unit of Power?

(b) A bullet of weight 0.3 N is fired horizontally in to a body of weight 100 N which is suspended by a string 0.8 m long. Due to this impact the body swings through an angle 30° . Find the velocity of the bullet. [WBSCTE 2012]

Answer:

a) Power is the time rate at which work is done or energy is transferred. Power is the derivative of work with respect to time.

The power P developed by a force F that does an amount of work U is

$$P = \frac{dU}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$\text{i.e., } P = \vec{F} \cdot \vec{v}$$

Unit of power in SI system is $\text{Nm/s} = \text{J/s}$ which is known as Watt.

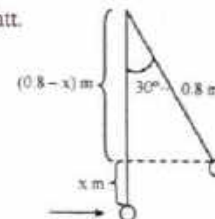
$$b) \frac{0.8 - x}{0.8} = \cos 30^\circ$$

$$\therefore x = 0.1071 \text{ m.}$$

Kinetic energy of bullet = Energy transferred to the body

$$\frac{1}{2} \times \frac{0.3}{9.8} \times V^2 = 100 \times 0.1071 \text{ [formula used: } \frac{1}{2}mv^2 = Mgh]$$

$$V = 26.43 \text{ m/s.}$$



12. (a) Define Mechanical Advantage, Velocity Ratio, Input, Output and Efficiency of a Simple Lifting Machine and establish the relationship between them.

(b) A load of 40 kN is to be lifted by a differential wheel and axle. It consists of differential axle of 250 mm and 300 mm diameter and the wheel diameter is 800 mm. Find the effort required if the efficiency of this machine is 75%. [WBSCTE 2013]

Answer:

a) Definition of Mechanical Advantage, Velocity Ratio, Efficiency:

Refer to Question No. 3 of Long Answer Type Questions.

Input: Input of a simple lifting machine is defined as the product of effort applied for lifting load and the distance through which the effort has moved.

$\therefore$ Input = Effort applied $\times$ Distance through which the effort has moved.

Output: Output of a simple lifting machine is defined as the product of weight or load lifted and the distance through which the load has been lifted.

∴ Output = Load lifted × Distance through which the load has been lifted.

Relation among them:

By definition,

$$M.A. = \frac{\text{Load lifted by the machine}}{\text{Effort required to lift the load}} = \frac{W}{P}$$

$$\text{and Velocity Ratio (V.R.)} = \frac{\text{Distance moved by the effort in lifting the load}}{\text{Distance moved by the load}} = \frac{Y}{X}$$

$$\therefore \text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Input}}$$

$$= \frac{\text{Load lifted} \times \text{distance through which the load has been lifted}}{\text{Effort applied} \times \text{distance through which the effort has moved.}}$$

$$= \frac{W \cdot X}{P \cdot Y} = \frac{P}{Y} = \frac{M.A.}{V.R.} = \frac{W}{P \cdot (V.R.)}$$

We know the law of machine

$$P = mW + C$$

∴ For maximum efficiency put

$$P = mW + C$$

$$\therefore \eta_{\max} = \frac{W}{(mW + C) (V.R.)} = \frac{1}{\left(m + \frac{C}{W}\right) (V.R.)}$$

$$\therefore \eta_{\max} = \frac{1}{m \cdot (V.R.)} \quad \left[\text{neglecting } \frac{C}{W} \right]$$

b) For a differential wheel and axle

$$\text{Velocity Ratio (V.R.)} = \frac{2D}{D_1 - D_2}$$

where, D = Diameter of the handle = 300 mm

D_1 = Diameter of the wheel = 800 mm

D_2 = Diameter of the axle = 250 mm

$$\therefore V.R. = \frac{2 \cdot 300}{800 - 250} = \frac{2 \cdot 300}{550} = \frac{12}{11}$$

$$\therefore \text{Efficiency} = \frac{\text{Load}}{\text{Effort} \cdot V.R.}$$

$$\text{or, Effort} = \frac{40 \times 1000 \times 11}{75 \times 12} = 488.88 = 489 \text{ N}$$

∴ If the efficiency of this machine is 75%, the required effort is 489 N

13. In a simple machine, whose velocity ratio is 40, a load of 2000 N is lifted by an effort of 120 N and a load of 3000 N is lifted by an effort of 180 N. Find the law of machine and calculate the load that could be lifted by a force of 200 N. Calculate also: (i) The amount of effort wasted in overcoming the friction; (ii) Mechanical Advantage; and (iii) The Efficiency. [WBSCTE 2013]

Answer:

We know, the law of machine is $P = mW + C$... (i)

where, P = Effort required lifting the load

m = A constant, which equals the slope of the line

W = Load Lifted.

and C = Another constant, which represent the machine friction.

∴ From equation of the law of machine (i) we get,

$$3000m + c = 180 \quad \dots (ii)$$

$$2000m + c = 120 \quad \dots (iii)$$

By Solving Equation (ii) & Equation (iii) we get,

$$\therefore m = \frac{60}{1000} = \frac{3}{50} = 0.06$$

From the equation (iii), we get, $2000 \times \frac{3}{50} + c = 120$ or $120 + c = 120$

∴ $c = 0$ i.e. No. amount of effort is wasted in overcoming the friction (Ans.)

∴ The law of machine is $P = 0.06W$

Again, When $P = 200$ N

$$\therefore W = \frac{200}{0.06} \text{ N} = \frac{200}{6} \times 100 = \frac{10000}{3} = 3333.33 \text{ N}$$

∴ The load 3333.33 N could be lifted by a force of 200 N

$$\therefore \text{Mechanical Advantage (M.A.)} = \frac{W}{P} = \frac{3333.33}{200} = 16.66$$

∴ Velocity ratio of the simple machine (V. R.) = 40.

$$\therefore \text{The efficiency} = \frac{M.A.}{V.R.} = \frac{16.66}{40} = 0.42$$

$$\therefore \eta = 42\% \text{ (Ans.)}$$

14. In a differential wheel and axle, the diameter of the effort wheel is 40 cm. The radii of the axles are 15 cm and 10 cm respectively. Find the load which can be lifted by an effort of 100 N, assuming the efficiency of the machine to be 75%. [WBSCTE 2014]

Answer:

Given Data:

$$d_1 = 30\text{cm}; d_2 = 20\text{cm}$$

$$D = 40\text{cm}$$

$$W = ?$$

$$P = 100\text{N}$$

$$\therefore \text{Velocity ratio (V.R)} = \frac{2D}{d_1 - d_2} = \frac{80}{10} = 8$$

$$\therefore \text{Mechanical Advantage (M.A.)} = \frac{W}{P}$$

$$\therefore \text{Again, } M.A. = \eta \times (V.R)$$

$$\text{or, } \frac{W}{P} = \eta \times 8$$

$$\therefore W = \eta \times 8 \times P = \frac{75}{100} \times 8 \times 100 = 600\text{ N}$$

$\therefore$ The load can be lifted by an effort of 100 N is 600 N.

15. In a simple machine, whose velocity ratio is 40, a load of 1500N is lifted by an effort of 100N and a load of 2500N is lifted by an effort of 160N. Find the law of machine. Also calculate the load that could be lifted by an effort of 200N.

[WBSCTE 2014]

Answer:

We know, the law of machine is

$$P = mw + C \quad \dots (i)$$

Where P = Effort required lifted the load

m = A constant

W = Load lifted.

and C = Another constant represent the machine friction.

$\therefore$ From equation of the land of machine, we get,

$$1500m + C = 100 \quad \dots (ii)$$

$$2500m + c = 160 \quad \dots (iii)$$

By solving equation (ii) & equation (iii) we get,

$$\therefore m = \frac{60}{1000} = 0.06$$

From the equation (ii) we get,

$$1500 \times \frac{60}{1000} + C = 100$$

$$\therefore C = 100 - 15 = 85$$

$\therefore$ The land of machine is $P = 0.06w + 85$

Again, when $P = 200\text{N}$

$$\therefore w = \frac{200 - 85}{0.06} = 1916.67\text{ N}$$

$\therefore$ The load 1916.67N could be lifted by a force of 200 N.

16. (a) What is law of machine? From equation of law of machine find maximum Mechanical Advantage and maximum efficiency of a simple lifting machine.
(b) In a differential wheel and axle the larger axle is 160 mm in diameter and the smaller one is 120 mm in diameter. The effort handle is 360 mm long. Find the Velocity Ratio. If an effort of 100N lifts a load of 2100 N, find the efficiency.

[WBSCTE 2015]

Answer:

a) Refer to Question No. 4 of Long Answer Type Questions.

$$\text{b) } \therefore \text{The velocity ratio} = \frac{2 \times 360}{160 - 120} = 18$$

$$\therefore \text{The efficiency } (\eta) = \frac{2100}{18} \times 100\% = 116.67\%$$

17. (a) What is the minimum effort P required to lift a load W by a screw jack when helix angle α and coefficient of friction is $\tan \phi$ where ϕ is the angle of friction.

(b) The pitch of a screw of a jack is 10 mm and the mean diameter of the thread is 60 mm and the length of the lever is 500 mm. Find the effort required to lift a load of 10 KN. Take coefficient of friction, $\mu = 0.08$.

[WBSCTE 2015]

Answer:

a)

Let p = Pitch of the screw

d = Mean diameter of the screw

α = Helix angle

P = Effort applied at the circumference of the screw to lift the load

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut
= $\tan \phi$, where ϕ is the friction angle

From the figure we find that

$$\tan \alpha = p / \pi d$$

Resolving the forces along the plane

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu R_N \quad \dots (i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad \dots (ii)$$

Substituting this value of R_p in equation (i), we have

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

$$\text{or, } P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

$$\text{or, } P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} = W \times \frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)} = W \tan(\alpha + \phi)$$

This is the minimum effort P required to lift a load W by a screw jack.

b) Given data:

$$p = 10 \text{ mm}$$

$$d = 60 \text{ mm}$$

$$w = 10 \text{ kN}$$

$$\mu = \tan \phi = 0.08$$

$$\therefore \phi = \tan^{-1}(0.08) = 4.57^\circ$$

Now, α = Helix angle

$$\therefore \tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 60} = 0.053$$

$$\therefore \alpha = 3.03^\circ$$

$\therefore$ The effort required to lift a load (P) = $W \tan(\alpha + \phi)$

$$\therefore P = 10 \times 10^3 \tan(7.60^\circ) = 1334 \text{ N}$$

18. In a Weston's differential pulley block, the difference in the number of teeth of the two pulley is 4. If the efficiency of the machine is 50% and an effort of 100 N just lifts a load of 2 kN, find the number of teeth of the two pulleys.

[WBSCTE 2016]

Answer:

We know that, in weston's differenced pulley,

$$VR = \frac{2 \times \text{No. of teeth of first pulley}}{\text{difference in the no. of the two pulleys}} \quad \dots (i)$$

Now, we have efficiency $\eta = 50\% = 0.5$

Effort, $p = 100\text{N}$, load $W = 2\text{kN} = 2000\text{N}$

$$\therefore \text{Mechanical Advantage } \frac{W}{P} = \frac{2000}{100} = 20$$

$$\text{Again, } \eta = \frac{MA}{VR} \therefore VR = \frac{MA}{\eta} = \frac{20}{0.5} = 40$$

Now, let us assume that the no. of teeth of the first pulley is x and the no. of teeth of the second pulley is y .

$$\therefore x - y = 4 \quad \dots (ii)$$

So, from eqn (i) we get

$$40 = \frac{2 \times x}{4} \Rightarrow x = 80$$

Now, substituting the value of x in eqn (ii) we get, $y = 76$

$\therefore$ The no. of teeth of the two pulleys are 80 and 76 respectively.

19. In a simple lifting machine, efforts of magnitude 0.4kN and 0.7kN are required to be applied to lift loads of magnitude 6kN and 12kN respectively. The velocity ratio of the machine is 32. Find: (i) The law of the machine (ii) Efficiencies of the machine at both the load options (iii) Effort lost in friction in both the cases (iv) Maximum MA and maximum efficiency. Also give your opinion on the reversibility of the machine in both load-effort combinations.

[WBSCTE 2016]

Answer:

(i) The law of Machine, $P = mW + c$

Since, efforts of magnitude 0.4kN and 0.7kN are required to be applied to lift loads of magnitude 6kN and 12kN respectively,

$$\therefore 0.4 = m \times 6 + c \quad \dots (i)$$

$$0.7 = m \times 12 + c \quad \dots (ii)$$

Now, solving these two equations, we get,

$$m = \frac{0.3}{6} = 0.05 \text{ and } c = 0.1\text{N}$$

$$\therefore \text{The law of machine is } \Rightarrow P = 0.05W + 0.1\text{N}$$

(ii) Given that, $VR = 32$

$$\therefore \text{Mechanical Advantage of 6kN load} = \frac{W}{P} = \frac{6}{0.4} = 15$$

$$\text{And MA of 12kN load} = \frac{W}{P} = \frac{12}{0.7} = 17.14$$

$\therefore$ Efficiency, η of the machine at load 6kN,

$$\frac{MA}{VR} = \frac{15}{32} = 46.87\%$$

efficiency η of the machine at load 12 KN,

$$\frac{MA}{VR} = \frac{17.14}{32} = 53.56\%$$

(iii) Effort lost in friction is given by the formula,

$$P_f = P - (W/VR)$$

$\therefore$ for 6KN load,

$$P_f = 0.4 - (6/32) = 0.2125N$$

For 12KN load,

$$P_f = 0.7 - (12/32) = 0.325N$$

(iv) Maximum $MA = \frac{1}{m} = \frac{1}{0.05} = 20$

$$\text{Maximum } \eta = \frac{1}{m \times VR} = \frac{1}{1.6} = 0.625 = 62.5\%$$

Since, the efficiency of the machine at load 6KN and effort 0.4KN is 46.87% (less than 50%), therefore the machine is irreversible.

And, the efficiency of the machine at load 12KN and effort 0.7KN is 53.56% (greater than 50%), therefore the machine is reversible.

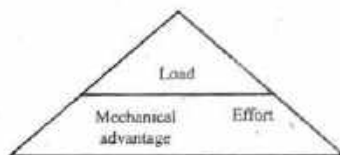
20. a) Define mechanical advantage, velocity ratio, efficiency and effort of a simple machine.
 b) For a differential wheel and axle, the diameter of the wheel is 240 mm. The larger and the smaller diameter of the differential axle are 80 mm and 70 mm respectively. An effort of 320N is applied to lift a load of 8000 N. Determine – (i) V.R. (ii) M.A. (iii) Efficiency of the machine, and (iv) Effort lost in friction.

[WBSCTE 2017]

Answer:

a) Mechanical Advantage

Mechanical advantage is defined as the ratio of load to effort. Pulley systems rely on this important relationship between load and effort. The formula seen below is best understood by writing it within a triangle. This helps when it is necessary to change the formula to find either; mechanical advantage or the load or the effort. In this way three formulas can be generated from the single formula inside the triangle.



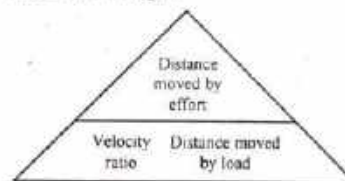
$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}}$$

$$\text{Load} = \text{Mechanical advantage} \times \text{Effort}$$

$$\text{Effort} = \frac{\text{Load}}{\text{Mechanical advantage}}$$

Velocity Ratio

Velocity Ratio (sometimes called movement ratio) - is defined as the ratio of the distance moved by the effort to the distance moved by the load. The formula seen below is best understood by writing it within a triangle. This helps when it is necessary to change the formula to find either; velocity ratio or the distance moved by the load or the distance moved by effort. In this way three formulas can be generated from the single formula inside the triangle.



$$\text{Distance moved by load} = \frac{\text{Distance moved by Effort}}{\text{Velocity ratio}}$$

$$\text{Velocity ratio} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\text{Distance moved by effort} = \text{Distance moved by load} \times \text{Velocity ratio}$$

In the ideal frictionless/weightless machine

Velocity Ratio = Mechanical Advantage

The efficiency of a simple machine

Efficiency = Work done by the machine / Work supplied to machine

Relationship between MA, VR and efficiency.

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Work Output}}{\text{Work Input}} = \frac{\text{Load} \times \text{Distance Moved by Load}}{\text{Effort} \times \text{Distance Moved by Effort}} \\ &= \frac{\text{Load/Effort}}{\text{Distance Moved by Effort/Distance Moved by Load}} \\ &= \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \end{aligned}$$

Results:

- (i) Efficiency of an ideal machine is 1, hence mechanical advantage = velocity ratio.
 (ii) Efficiency of practical machine is less than 1, hence mechanical advantage is less than velocity ratio.

Effort:

A force called the effort force is applied at one point on the lever in order to move an object, known as the resistance force, located at some other point on the lever.

b) Given, $d_1 = 80 \text{ mm}$, $d_2 = 70 \text{ mm}$

$D = 240 \text{ mm}$, $W = 8000 \text{ N}$, $P = 320 \text{ N}$

Velocity ratio is defined as $= \frac{y}{x}$ [where y = Distance moved by effort, x = Distance moved by the load]

$$V.R. = \frac{2D}{d_1 - d_2} = \frac{2 \times 240}{80 - 70} = 48 \text{ (Ans.)}$$

$$\text{Mechanical advantage, } M.A. = \frac{W}{P} = \frac{8000}{320} = 25 \text{ (Ans.)}$$

$$\text{Efficiency } \eta = \frac{M.A.}{V.R.} = \frac{25}{48} = 52.08\% \text{ (Ans.)}$$

Friction effort lost,

$$F_{\text{effort}} = P - \frac{W}{V.R.} = 320 - \frac{8000}{48} = 153.33 \text{ N}$$

21. a) Derive the conditions maximum efficiency of a lifting machine.

b) In a lifting machine, the effort required to lift a load of 250 N and 375 N are 60 N and 70 N respectively. If the velocity ratio of the machine is 20. Determine- (i) law of machine, (ii) efficiencies corresponding to load of 250 N and 375 N, (iii) effort lost in friction in both cases. [WBSCTE 2017]

Answer:

a) Refer to Question No. 1 of Long Answer Type Questions.

b) Refer to Question No. 2 of Long Answer Type Questions.

22. a) A simple screw jack has a thread of pitch of 10 mm, find the mechanical advantage and efficiency, if an effort of 20 N is applied at the end of an arm 500 mm long to lift a load 2.6 kN.

b) The number of teeth on the worm wheel of a double threaded worm and worm wheel is 25. The effort handle is 25 cm long and load drum is of 15 cm diameter. Find the efficiency of the machine, if an effort of 300 N can lift load of 3.25 kN. [WBSCTE 2017]

Answer:

a) Given, $p = 10 \text{ mm} = \text{Pitch of the screw pack.}$

$l = 500 \text{ mm} = \text{length}$

$P = 20 \text{ N} = \text{Effort required to lift the load.}$

$W = 2.6 \text{ kN} = 2600 \text{ N}$

$$V.R. = \frac{2\pi l}{p} = \frac{2\pi \times 500}{10} = 314.16$$

$$M.A. = \frac{W}{P} = \frac{2600}{20} = 130$$

$$\text{The efficiency of the screw jack} = \frac{M.A.}{V.R.} = \frac{130}{314.16} = 41.38\%$$

b) Given, No. of teeth on worm-wheel, $T = 25$

Dia. of effort wheel = 25 cm.

$$\therefore \text{Radius of effort wheel} = \frac{25}{2} = 12.5 \text{ cm} = L$$

Dia. of load drum = 15 cm.

$$\therefore \text{Radius of load drum} = \frac{15}{2} = 7.5 \text{ cm} = r$$

Effort, $P = 300 \text{ N}$

Load, $W = 3.25 \text{ kN} = 3250 \text{ N}$

$$(i) V.R. = \frac{LT}{2r} = \frac{12.5 \times 25}{2 \times 7.5} = 20.83$$

$$(ii) \text{Efficiency, } \eta = \frac{M.A.}{V.R.} = \frac{\left(\frac{W}{P}\right)}{\left(\frac{V.R.}{1}\right)} = \frac{\left(\frac{3250}{300}\right)}{20.83} = 52\%$$

23. a) What is the law of a machine. From equation of law of machine find the maximum Mechanical advantage and maximum efficiency of a simple lifting machine. [WBSCTE 2018]

Answer:

Law of a Machine: The law of a machine is defined by an equation which gives the relationship between the effort required to raise the corresponding load. This is obtained by drawing a graph between the efforts and the corresponding loads lifted by the efforts.

Figure 1 shows the graph between effort and the load. The various values of the efforts required to raise the corresponding loads are plotted. Then a straight line is drawn as shown in Fig. 1. For an ideal machine the straight line will pass through the origin. But for an actual machine, the straight line will intercept the y -axis as shown in Fig. 1. The intercept $OA = \text{Effort applied to the machine to overcome friction.}$ If the effort applied is less than OA , the load will not be lifted. The law of a machine is given mathematically as:

$$y = mx + c$$

where, $y = \text{Effort} = P$

$x = \text{Load} = W$

and $c = \text{Constant}$

$\therefore$ The law of machine becomes

$$P = mW + c \quad \dots (1)$$

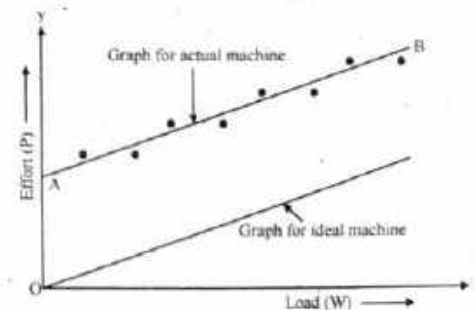


Fig: 1 Law of a machine

where,

m = Slope of the line AB and is equal to a constant which is known coefficient of friction.

c = Intercept of the line on y -axis and is equal to the effort required to overcome friction.

Expression for maximum mechanical advantage:

Eqn. (1) gives the law of a machine as

$$P = mW + c$$

where P = Effort applied

W = Load lifted

From the above equation, the value of W is obtained as

$$mW = P - c$$

$$\text{or, } W = \frac{P - c}{m} = \frac{P}{m} \left[1 - \frac{c}{P} \right]$$

$$\text{or, } \frac{W}{P} = \frac{1}{m} \left[1 - \frac{c}{P} \right] \quad \dots (2)$$

But $\frac{W}{P}$ = Mechanical advantage

From Fig. 1, it is clear that with the increase of W (load), the effort (P) increases.

With the increase of P , the value of $\frac{c}{P}$ decreases as c is constant. Hence from Eqn. (2)

it is obvious that with the decrease of $\frac{c}{P}$, the mechanical advantage increases. If the

value of P is so large, that the value $\frac{c}{P}$ becomes negligible, then mechanical advantage

will be maximum and its value will be obtained from Eqn. (2), as

$$\frac{W}{P} = \frac{1}{m} \left[\because \frac{c}{P} \text{ is negligible} \right]$$

$$\therefore \text{Max. M.A.} = \frac{1}{m}$$

Expression for Maximum Efficiency:

Let V.R. = Velocity ratio of a machine

$$\text{Then } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{W}{P}}{\text{V.R.}} = \frac{W}{P \times (\text{V.R.})}$$

We have

$$\frac{W}{P} = \frac{1}{m} \left[1 - \frac{c}{P} \right]$$

Simple Lifting Machines

$$\therefore \eta = \frac{1}{m} \left[1 - \frac{c}{P} \right] \times \frac{1}{\text{V.R.}} = \frac{1}{m \times (\text{V.R.})} \left[1 - \frac{c}{P} \right] \quad \dots (3)$$

With the increase of load, effort increases. With the increase of effort (P), the value of

$\frac{c}{P}$ decreases and hence efficiency increases. If the value of P is so large that the value

of $\frac{c}{P}$ is negligible or zero, the efficiency will be maximum. Hence maximum efficiency

will be obtained by putting the value of $\frac{c}{P} = 0$ in Eqn. (3)

$$\eta_{\max} = \frac{1}{m \times (\text{V.R.})}$$

23. b) In a differential wheel and axle the larger axle is 200 mm in diameter and the smaller one is 150 mm in diameter. The effort handle is 400 mm long. Find the Velocity Ratio. If an effort of 150 N lifts a load of 1500 N, find the efficiency. [WBSCTE 2018]

Answer:

Effort handle, $L = 400$ mm

Large diameter of axle, $d_1 = 200$ mm

Smaller diameter of axle, $d_2 = 150$ mm

Let, D = diameter of effort wheel

$W = 1500$ N

$P = 150$ N

$$\therefore \text{Velocity ratio, V.R.} = \frac{2D}{d_1 - d_2} = \frac{2 \times (2L)}{(d_1 - d_2)} = \frac{4 \times 400}{(200 - 150)} = 32$$

$$\text{Mechanical advantage, M.A.} = \frac{W}{P} = \frac{1500}{150} = 10$$

$$\therefore \text{Efficiency, } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{10}{32} = 31.25\%$$

24. a) Deduce the expression for the condition of self-locking of a lifting machine. [WBSCTE 2018]

Answer:

Condition for self-locking of a lifting machine:

When the actual effort P is removed and load W does not fall down, to the initial position, the machine is said to be self-locking. The load W will not fall down if the work done by the force of friction is more than the output of the machine.

Let, W = Load lifted by the machine

P = Effort applied to lift the load

y = Distance moved by the effort

x = Distance moved by the load

Then Input = $P \times y$

Output = $W \times x$

But Input = Output + Work done by force of friction.

$\therefore$ Work done by force of friction = Input - Output = $P \times y - W \times x$ (1)

When the effort P is removed, the load W will not fall down if the work done by the force of friction is more than the output of the machine. And if the load is not falling, the machine is said to be irreversible. Hence the condition for self-locking is

Work done by force of friction > Output of machine

$\therefore P \times y - W \times x > W \times x$ [$\because$ Output = $W \times x$]

or, $P \times y > (W \times x + W \times x)$

$> 2Wx$

or, $2Wx < P \times y$

or, $\frac{Wx}{Py} < \frac{1}{2}$ (2)

We know, $\frac{\text{Output}}{\text{Input}} = \frac{Wx}{Py} = \eta$

Substituting this value in Eqn. (2), we get

$\therefore \eta < \frac{1}{2}$ or, 50%

Thus the condition for self-locking or irreversibility of a machine is that its efficiency should be less than 50%.

24. b) In a Weston's differential pulley clock, the difference in the number of teeth of the two pulleys is 4. If the efficiency of the machine is 50% and an effort of 100 N just lifts a load of 2 kN, find the number of teeth of the two pulleys.

[WBSCTE 2018]

Answer:

Mechanical Advantage, M.A. = $\frac{W}{P} = \frac{2000}{100} = 20$

$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{20}{\text{V.R.}}$

$\Rightarrow \text{V.R.} = \frac{20}{0.5} = 40$ [$\because \eta = 50\% = 0.5$]

Now, $\text{V.R.} = \frac{2T_1}{T_1 - T_2}$

$\Rightarrow 40 = \frac{2T_1}{4}$ [$\because (T_1 - T_2) = 4$]

$\Rightarrow T_1 = 80$

$\therefore T_2 = (80 - 4) = 76$

25. To lift a load of 1500N, an effort of 200N is applied to a machine. The load is lifted by a distance of 0.2m while the effort is moved by a distance of 2m. Find the VR, MA and efficiency of the machine. Find also the ideal effort, effort lost due to friction, ideal load and frictional load. [WBSCTE 2022]

Answer:

Load lifted (W) = 1500 N

Effort (P) = 200 N

Distance moved by load (x) = 0.2 m

Distance moved by effort (y) = 2 m

$\therefore$ (i) V.R. = $\frac{y}{x} = \frac{2}{0.2} = 10$

(ii) MA = $\frac{W}{P} = \frac{1500}{200} = 7.5$

(iii) Efficiency of the machine, $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{7.5}{10} = 0.75 = 75\%$

(iv) Ideal effort (P_i) = $\frac{W \times x}{y} = \frac{1500 \times 0.2}{2} = 150 \text{ N}$

(v) Effort lost due to friction = $(P - P_i) = 200 - 150 \text{ N} = 50 \text{ N}$

(vi) Ideal load, $W_i = (P \times \text{V.R.}) = 200 \times 10 = 2000 \text{ N}$

(vii) Frictional load, $F_w = (P \times \text{V.R.}) - W = (200 \times 10) - 1500 \text{ N} = 500 \text{ N}$

26. a) In a double purchase crab winch, no of teeth on pinions are 20 & 30 and that of spur gears are 60 & 80. Radius of the load drum is 200 mm and length of the handle is 300mm. Find the efficiency of the machine, if the effort 100N is required to lift a load of 750N.

b) What are the advantages of a Lifting Machine?

[WBSCTE 2022]

Answer:

a) Length of the handle (L) = 300 mm

Radius of load drum = 200 mm

$\therefore$ Diameter of load drum = $(2 \times 200) = 400 \text{ mm}$

No. of teeth on the pinions are, $T_1 = 20$ & $T_3 = 30$

No. of teeth on the spur gears are, $T_2 = 60$ & $T_4 = 80$,

Effort (P) = 100 N

Load lifted (W) = 750 N

$\therefore$ Velocity Ratio (V.R.) = $\left(\frac{2L}{D} \times \frac{T_2}{T_1} \times \frac{T_4}{T_3} \right) = \frac{2 \times 300}{400} \times \frac{60}{20} \times \frac{80}{30} = 12$

$$\therefore \text{Efficiency, } \eta = \frac{M.A}{V.R}$$

$$\text{where as, } M.A = \frac{W}{P} = \frac{750}{100} = 7.5$$

$$\therefore \eta = \frac{7.5}{12} = 0.625 = 62\%$$

$\therefore$ The efficiency of the machine is 62 %.

b) Refer to Article No. 2 of Unit at a Glance.

27. A string is wound round a pulley of 0.30m diameter. A weight attached to the free end of the string, describes a distance of 8m after starting from rest within 4 secs. Find the angular velocity of the pulley. Also find the total distance travelled by the weight to make the pulley rotate at 400rpm. [WBSCTE 2022]

Answer:

1st Part:

Given, $s = 8$ m

$t = 4$ sec

Diameter of the pulley = 0.30 m

Radius of the pulley = $\frac{0.30}{2}$ m = 0.15 m

$$\therefore \text{Linear velocity, } v = \frac{s}{t} = \frac{8}{4} = 2 \text{ m/sec}$$

We know, $v = r\omega$

$$\Rightarrow 2 = 0.15 \times \omega \quad [\because \omega = \text{Angular velocity}]$$

$$\Rightarrow \omega = 13.33 \text{ radian/sec.}$$

$\therefore$ The angular velocity of the pulley is 13.33 rad/sec.

2nd Part:

Here, $\omega = 400 \text{ rpm}$

$r = 0.15$ m

$t = 4$ sec.

Let, The total distance travelled by the weight to make pulley = s .

$$\theta = \omega t \quad [\text{Here, } 400 \text{ rpm} = (400 \times 0.1047) \text{ rad/sec}]$$

$$= 400 \times 0.1047 \times 4 = 167.52$$

$$s = r\theta = 0.15 \times 167.52 = 25 \text{ m}$$

$\therefore$ The total distance travelled by the weight to make the pulley is 25 m.

Unit 7: Motion in a Plane

Unit at a glance:

7.1 Kinematical Equations in one Dimension

Rectilinear Motion:

When a particle moves along a straight line, then the motion of the particle is known as rectilinear motion.

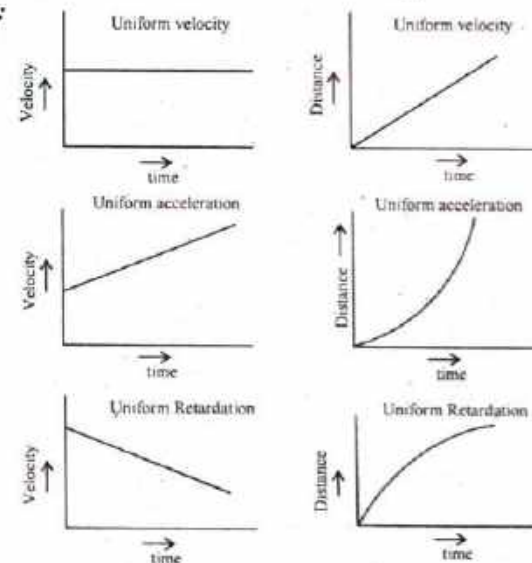
Suppose a particle is started with velocity u and constant acceleration a . Then after t sec the particle moves the path s .

$$\text{Then, } s = ut + \frac{1}{2}at^2$$

After t second the velocity will be $v = u + at$ also it can be shown that $v^2 = u^2 + 2as$

Distance travelled by the particle in n^{th} second is, $s_n = u + \frac{a}{2}(2n-1)$

Time diagrams:



Kinematics equations for motion under gravity:

Those equations will be same as the previous case, but a will be replaced by g which is the acceleration due to gravity. Its value is 32 ft/s or 9.81 m/s², so the equation will be always note the g is towards the centre of the Earth.

If you consider from the ground to a point above the ground as positive direction then the value of g will be negative.

So the equations will be –

$$s = ut + \frac{1}{2}gt^2$$

$$v = u + gt$$

$$v^2 = u^2 + 2gs$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Here downward direction, towards the center of the Earth is taken as positive. So the value of g is positive.

7.2 Newton's Laws of Motion

Newton's First Law of Motion states that a particle either remains at rest or continues to move in a straight line with a constant velocity provided that there is no unbalanced force acting on it.

Newton's Second Law of Motion states that the acceleration of a particle is proportional to the resultant force acting on it and its direction is in the direction of this force.

The mathematical expression of this law is

$$F = ma \quad \dots (1)$$

Here F is the force that causes an acceleration of a to the body of mass m .

Newton's Third Law of Motion states that the forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction and act along the same line of action (collinear). In short, every action is an equal and opposite reaction.

7.3 Momentum and Impulse of Force

Momentum is the product of the mass of a body and its velocity. Let the mass of the body is m and velocity is v then the momentum of the body is mv .

Its dimension is $[MLT^{-1}]$ and S.I unit is kilogram meter per second (kg-m/s).

Impulse of force is the product of the average force multiplied by the time it is exerted.

Also its dimension is $[MLT^{-1}]$ and SI unit of impulse is the newton second (N-s) or, in base units, the kilogram meter per second (kg-m/s)

Change in momentum is known as impulse of force.

7.4 Conservation of Linear Momentum

The total momentum of two bodies remains constant after their collision or any other mutual action.

Mathematically, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Where m_1 & m_2 are mass of first & second body u_1 & u_2 are initial velocity of first & second body. v_1 & v_2 are final velocity of first & second body.

7.5 Rotational Motion

Angular displacement: It is the total angle, through which a body has rotated, it is usually denoted by θ . It is defined as the angle in radians through which a point on line has been rotated in specified sense about a specified axis.

Mathematically it is defined as $\theta = \omega t$

(where ω = angular velocity; t = time; θ = angular displacement)

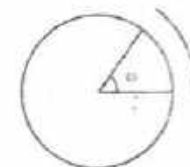
Angular Velocity: It is defined as the rate of change of angular displacement of a body. Angular displacement is always measured in terms of angle covered by the body from the initial position.

Angular displacement = $\angle AOB = \theta$

Time taken = t

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{time}} = \frac{\theta}{t}$$

Mathematically, it is defined as, $\omega = \frac{d\theta}{dt}$



Angular acceleration: It is defined by the rate of change of angular velocity & unit of angular acceleration is radius per second² (rad/sec²) and it is usually denoted by α . It may be constant as variable. It follows the relation: $\alpha = a/r$

$\Rightarrow$ (Angular Acceleration) = (Linear acceleration $\times$ radius) $\dots$

7.6 Relation between a Linear Velocity & Angular Velocity

The relationship between the linear and angular velocity is, $v = r\omega$ where v is the linear velocity, r is the radius of the curve and ω is the angular velocity.

7.7 Relation between a Linear Acceleration & Angular Acceleration

Let Angular velocity of a body in rad/sec = ω

Radius of the circular path in meter = r

Linear velocity of the particle on the periphery in m/sec = v

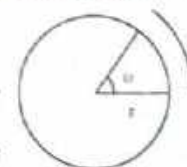
After one sec, the particle will move v meters along the circular path & the angular displacement will be ω radian.

$\therefore$ We know that length of the arc = Radius of arc $\times$ Angle subtended in radian (if the angle subtended is very small).

Consider a body rotating about its axis with a constant angular acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}(\omega r) = r \times \frac{d\omega}{dt} = r \times \alpha$$

where α = Angular acceleration in rad/sec² and is equal to $\frac{d\omega}{dt}$



7.8 Centripetal Force

It is a force which keeps a body moving with a uniform speed (not velocity) along a circular path and is directed along the radius towards the centre of the curve.

7.9 Centrifugal Force

It is a outward force that draws a rotating body away from the centre of rotation and it is caused by the inertia of the body.

Let the velocity at point P is $\vec{V}_1$ & point Q is $\vec{V}_2$ and it subtended an angle of $\Delta\theta$. Let the magnitude of velocity is fixed at v .

The value of centrifugal force = $m\omega^2 r$

As centrifugal force is equal & opposite of centripetal force, so –

Centrifugal force = $m\omega^2 r$

Where m is the mass of the rotating body, r is the radius of the curve and ω is the angular velocity.

7.10 Work

The work done by a constant force of magnitude F on a point that moves a distance d in the direction of the force is the product, $W = F \cdot d$.

S.I. unit of it is Joule.

7.11 Power

It is the rate of work. It is the amount of work done in unit time. S.I. unit of it is Watt.

7.12 Energy

Energy is the capacity of a physical system to perform work. Energy exists in several forms such as heat, kinetic or-mechanical energy, light, potential energy, electrical, or other forms.

S.I unit of it is Joule.

Short Answer Type Questions

A. Choose the correct answer from the given alternatives in each of the following:

1. The angular velocity of the second hand of a clock is – [Model Question]
(a) 2π rad/s (b) $(\pi/3)$ rad/s (c) $(\pi/30)$ rad/s (d) $(\pi/1800)$ rad/s

Answer: (c)

2. Impulse (I) of a force (P) acting on a particle for time t is given by – [Model Question]

- (a) $I = \frac{P}{t}$ (b) $I = \frac{t}{P}$ (c) $I = P \times t$ (d) none of these

Answer: (c)

3. A body on or near the earth's surface is seen to be attracted towards the earth but the earth is not, because – [Model Question]

- (a) the body does not attract the earth
(b) the attraction in earth due to the body is very small
(c) Newton's third law is not applicable in this case
(d) none of above is true

Answer: (b)

4. The distance traveled by an object along a straight line in time t is given by $s = 3 - 4t + 5t^2$, the initial velocity of the object is [Model Question]
(a) 3 unit (b) -3 unit (c) 4 unit (d) -4 unit

Answer: (d)

5. A body is revolving with uniform speed ' v ' in a circular of radius ' r '. The tangential acceleration is [Model Question]
(a) v/r^2 (b) v^2/r (c) v/r (d) zero

Answer: (b)

6. A car moving with a speed of 50 km/hr can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is – [Model Question]
(a) 12 m (b) 18 m (c) 24 m (d) 6 m.

Answer: (c)

7. A man weighs 80 kg. He stands on a weighting scale in the lift, which is moving upwards with a uniform acceleration of 5m/s^2 . What would be reading on the scale? $(g = 10\text{m/s}^2)$ – [Model Question]
(a) Zero (b) 400 N (c) 800 N (d) 1200 N

Answer: (b)

8. If the radius of the circular path of particle going around the circle is doubled without changing its frequency of rotation, the centripetal force on it is – [Model Question]
(a) unchanged (b) doubled (c) halved, (d) quadrupled

Answer: (b)

9. For a particle moving uniformly around a circular path of radius r which of the following is true? – [Model Question]

- (a) $w = \frac{v}{r}$ (b) $w = v \times r$ (c) $w = \frac{r}{v}$ (d) none of these

Answer: (a)

10. In velocity-time graph, area enclosed with the time axis represents [Model Question]
(a) displacement (b) velocity (c) force (d) distance

Answer: (a)

11. Time displacement graph of two particles, the angle between graphs with time axis are respectively 30° and 60° . The ratio of their velocity is – [Model Question]

(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$ (c) $3:1$ (d) $1:3$

Answer: (d)

12. A particle is moving along a straight path with constant acceleration of 4.0 m/s^2 . If the initial velocity of the particle be 20 m/s , the distance travelled by it in the 5^{th} second is – [Model Question]

(a) 150 m (b) 40 m (c) 38 m (d) none of these

Answer: (c)

13. A body of mass 4.0 kg is hanged at the lower end of a spring balance whose upper end is fixed at the ceiling of a lift. If the lift moves up with an acceleration of 6.0 m/s^2 , the reading of the spring balance is (Take $g = 9.8 \text{ m/s}^2$) – [Model Question]

(a) 15.2 N (b) 24.0 N (c) 39.2 N (d) 63.2 N

Answer: (d)

14. Which of the following remains constant for a particle executing uniform circular motion? [Model Question]

(a) Linear speed (b) Linear velocity
(c) Linear acceleration (d) None of these

Answer: (a)

15. The retardation (f) a particle moving along a straight path varies with time (t) as $f = 4t$. If the initial velocity of the particle be zero, find the time which the velocity increase to 32 ms^{-1} [Model Question]

(a) 16 s (b) 8 s (c) 4 s (d) 2 s

Answer: (b)

16. An electric fan is running at an angular speed of 120 rpm . The angular retardation developed after switching off the fan is $\pi/2 \text{ rad s}^{-2}$. Find the time in which the angular speed decreases to 30 rpm – [Model Question]

(a) 57.3 s (b) 6.0 s (c) 3.0 s (d) 0.96 s

Answer: (c)

17. Power is defined as – [Model Question]

(a) capacity of a system to do work (b) work done by a force
(c) rate of work done by a system (d) none of these

Answer: (c)

18. Work is a [Model Question]

(a) Vector (b) scalar (c) tensor (d) none of these

Answer: (b)

B. Fill in the blanks in the following statements:

19. The maximum range of projectile is u^2/g , where u is the initial velocity of the particle g is the Earth's gravitational acceleration. [Model Question]

20. Acceleration is the rate of change of velocity. [Model Question]

21. A force of 10 N acts on a mass of 10 kg , then the mass is moving with an acceleration of 1 m/sec^2 . [Model Question]

22. The relationship $S = ut + \frac{1}{2}at^2$ is applicable when the body is moving with uniform acceleration. [Model Question]

23. The acceleration of a body may be defined as the rate of Change of speed. [Model Question]

24. The first law of Newton's laws of motion is known as law of Inertia. [Model Question]

25. Watt is the S.I. unit of Power. [Model Question]

26. B.O.T. is the unit of energy. [Model Question]

C. State whether the following statements are True or False:

27. For a particle of mass m and moving with uniform speed v along a circle of radius r , there is no force acting on the particle. [Model Question]

Answer: False, centrifugal force is acting.

28. Momentum is a scalar quantity. [Model Question]

Answer: False

29. Principle of rocket launching is based on Newton's second law of motion. [Model Question]

Answer: False

Long Answer Type Questions

- 1. (a) A body is moving with uniform acceleration and it covers 20 m in 4^{th} second and 30 m in 8^{th} second. Find (i) the initial velocity, (ii) the acceleration of the body.

(b) Define Velocity and Acceleration. [WBSCTE 2022]

Answer:

a) Let the initial velocity = u
and acceleration is = a

For the 1st part of motion travelled 20 m in 4 seconds,

$$\therefore S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 20 = 4u + \frac{1}{2}a \cdot 16 = 4u + 8a$$

$$\therefore 4u + 8a = 20 \quad \dots (i)$$

For the 2nd part of motion travelled 30m in 8 seconds,

$$\therefore 20 + 30 = u \times 8 + \frac{1}{2}a(8)^2$$

$$\Rightarrow 50 = 8u + 32a$$

$$\Rightarrow 8u + 32a = 50 \quad \dots (ii)$$

Solving (i) & (ii), we get

$$u = \frac{15}{4} \text{ m/s} = 3.75 \text{ m/s}$$

Putting u value in equation (i), we get,

$$4 \cdot \frac{15}{4} + 8a = 20$$

$$\Rightarrow 8a = 20 - 15 \quad \therefore a = \frac{5}{8} \text{ m/s}^2 = 0.625 \text{ m/s}^2$$

b) Difference between Velocity and Acceleration:

Sl. No.	Velocity	Acceleration
1.	Velocity is the rate of change of displacement.	Acceleration is the rate of change of velocity.
2.	Velocity is a vector quantity because it consists of both magnitude and direction.	Acceleration is also a vector quantity as it is just the rate of change of velocity. As velocity is a vector quantity, obviously, acceleration is also.
3.	Velocity may be positive or negative or zero.	Acceleration may be positive or negative. It cannot be zero as it is defined as the change in velocity or speed. This explains that there should be some motion for acceleration.
4.	SI unit is m/s or ms ⁻¹ .	SI unit is m/s ² or ms ⁻² .
5.	Formula for velocity = displacement/time	Formula for acceleration = velocity/time

2. State Newton's law of motion.

Answer: Refer to Unit at a Glance 7.2.

[Model Question]

3. Prove that Newton 1st law & 3rd law of motion are special case of 2nd law.

[Model Question]

Answer:

Newton's first law from 2nd law: From Newton's 2nd law we know, (m) be the mass of the particle, when we applied force (P) on a body then it must be the momentum (f), so, $P = mf$.

The initial velocity u and after t time velocity v , $f = \frac{v-u}{t} \therefore P = m \cdot \frac{u-v}{t}$

If the force which is not applied on a body, when $P = 0$. Since m and t not zero, therefore $P = 0$ be $(v-u) = 0$.

(i) $u = 0$ and $v = 0$ means if no force is applied the body at rest will remain at rest.

(ii) $v = u$ means if no force is applied the body's states of if the first two of motion. Uniform motion in a straight line does not change.

3rd Law: If m_1 and m_2 be the masses of two particles, with uniform motion are in the straight line. When they collide with each other, velocity and momentum will be changed. Let, in small time δt , momentum change in the mass $m_1 = \delta p_1$, Momentum change in mass $m_2 = \delta p_2$. From the principle of conversation of momentum, the linear momentum of the system remains constant or the change of momentum is zero.

Divided time by δt , $\frac{\delta p_1}{\delta t} + \frac{\delta p_2}{\delta t} = 0$ at the end point, when $\delta t \rightarrow 0$, we get, $\frac{\delta p_1}{\delta t} = -\frac{\delta p_2}{\delta t}$, so

force applied on the body of mass $m_1 = -(\text{force applied on the body of mass } m_2)$

or action = - reaction.

It is the 3rd law of motion.

4. State the principle of conservation of linear momentum.

[Model Question]

Answer:

Principle of Conversation of Linear Momentum:

If no external force acts on a system the momentum of the system remains constant.

Let two particles of masses m_1 & m_2 move with velocities u_1 & u_2 and collide after which velocities become v_1 & v_2 respectively.

Then we have $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

5. Define centripetal force.

[Model Question]

Answer:

Centripetal force: It is the force which is continuously needed to move a particle in a circular path with constant angular velocity and which is always directed towards the

centre and perpendicular to the direction of the particle. It is $F = \frac{mv^2}{r} = m\omega^2r$ where m is

the mass of the body, r is radius of the path v is velocity & ω is angular velocity.

Q. Define SI unit of force from Newton's second law of motion. [Model Question]

Answer:

From Newton's 2nd law of motion we get $P = mf$

where P is the force m is the mass of the body f is the acceleration.

From the principle of dimensional homogeneity we get –

The dimension of R. H. S. is $mg = [M][L T^{-2}]$

So dimension of L.H.S. = $[MLT^{-2}]$

So the S.I. unit is, kg – meter/second² which is commonly known as Newton.

Q. 7. Write down the name of the quantity playing the same role in curvilinear motion as mass in linear motion. What is Parking orbit? What is the time period of the satellite revolving in this orbit? [Model Question]

Answer:

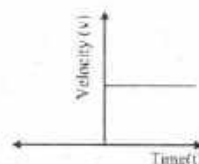
The name of the quantity is moment of Inertia which plays the same role in Curvilinear motion as mass works in linear motion.

The Geo-stationary orbit of a satellite is also called parking orbit.

The time period of the satellite revolving in the orbit is 24 hours.

Q. 8. Draw the velocity-time graph of a body moving with uniform velocity and starting from rest. [Model Question]

Answer:



Q. 9. What is the rotational analog of mass? [Model Question]

Answer:

Moment of inertia is a rotational equivalent of mass. It is a property of a body of significant size which depends on mass, shape, distribution of mass and axis of revolution. Mathematically, $I = \int r^2 dm$ where dm is the infinitesimal element of mass of the body and r is the separation of said element of mass from the axis of revolution.

Q. 10. (a) What do you mean by impulse of a force? Derive its relation with momentum.

(b) A force of 12N produces an acceleration of $3m/s^2$ on a body and an acceleration $4m/s^2$ on another body of different mass. What will be the acceleration if the same force is applied on the combined mass? [Model Question]

Answer:

a) Refer to Chapter at a Glance 7.3.

b) We know, $P = ma$

Where, P = force in N, a = acceleration in m/sec^2 , m = mass in kg.

1st case: Mass of the 1st body = $\frac{P}{a} = \frac{12}{3} = 4$ kg

2nd case: Mass of the 2nd body = $\frac{12}{4} = 3$ kg

Now, if the same force is applied on the combined mass, the acceleration will be

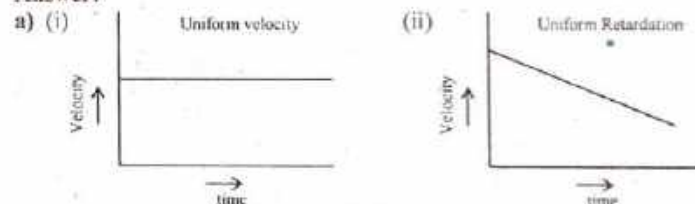
$$a = \frac{P}{m} = \frac{12}{(3+4)} = \frac{12}{7} \text{ m/sec}^2$$

Q. 11. (a) A train is moving along straight rails. Draw velocity - time graph of the train in the following cases - (i) When the train is moving with constant velocity (ii) When the train is moving with constant retardation.

(b) A particle travels 35 cm and 33 cm during fifth and seventh second of its motion. What will be the velocity of the particle after 9 second of motion?

(c) What is linear momentum of body? [Model Question]

Answer:



b) Distance travelled by the particle in 5th second is $25 = u + \frac{a}{2}(10-1)$

$$\text{or, } 50 = 2u + 9a$$

$$\therefore 2u + 9a = 50 \quad \dots\dots\dots (1)$$

Distance travelled by the particle in 7th second is $33 = u + \frac{a}{2}(14-1)$

$$\therefore 2u + 13a = 66 \quad \dots\dots\dots (2)$$

$$\therefore \text{Acceleration (a)} = 4m/sec^2$$

$$\therefore \text{Velocity of the particle (4)} = 7 \text{ cm/sec}$$

$$\therefore \text{The velocity of the particle after 9 second of monitor} = 7 \text{ cm/sec}$$

c) Momentum is the product of the mass of a body and its velocity. Let the mass of the body is m and velocity is v then the momentum of the body is mv . Its dimension is $[MLT^{-1}]$ and S.I unit is kilogram meter per second (kg·m/s).

12. (a) Write the relation between torque and angular momentum.
 (b) A machine gun has a mass 5 kg. It fires 50 gram bullets at the rate of 30 bullets per minute at speed of 400 m/s. What force is required to keep the gun in position? [Model Question]

Answer:

a) Relation between Torque and Angular Momentum

It is torque (τ) = rate of change of angular momentum (dL/dt)

Where L is the angular momentum.

b) Let the mass of the machine gun, $M = 5$ kg

Mass of the bullet, $m = 50$ g = 0.05 kg

Velocity of the bullet, $v = 400$ ms⁻¹

Number of bullets fired per second, $n = \frac{30}{60} = \frac{1}{2}$

Velocity of recoil of the gun, $V = ?$

By the law of conservation of momentum,

$$MV + nmv = 0$$

$$\text{or, } V = \frac{nmv}{M} = \frac{\frac{1}{2} \times 0.05 \times 400}{5} = -2 \text{ ms}^{-1}$$

-ve sign shown that the gun recoils.

$\therefore$ Force required to hold the gun in position.

$$F = \frac{\text{Change in momentum}}{\text{Time}} = \frac{nmv}{t} = \frac{\frac{1}{2} \times 0.05 \times 400}{1}$$

$$\text{or, } F = 10 \text{ N}$$

13. (a) If the velocity of a car moving along a straight path be doubled, the distance by which it can be stopped becomes four times. Explain.
 (b) Can be velocity-time graph of an object be a straight line parallel to velocity axis? Explain.
 (c) A force of 12N produces an acceleration of 3m/s² on a body and an acceleration 4m/s² on another body of different mass. What will be the acceleration if the same force is applied on the combined mass? [Model Question]

Answer:

a) Let the initial velocity of car is u and acceleration f .

$\therefore$ We know that

$$v^2 = u^2 - 2fs$$

$$\therefore 0 = u^2 - 2fs$$

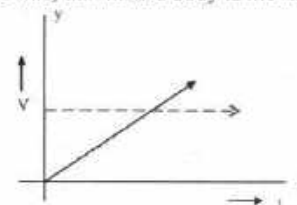
$$\therefore S = \frac{u^2}{2f}$$

$\therefore$ If we doubled the initial velocity the $u_{\text{new}} = 2u$

$$\therefore S_{\text{new}} = \frac{4u^2}{2f} = 4s \quad [\text{Proved}]$$

b) The time-velocity graph of an object parallel to the velocity (y -axis) never possible, because parallel to velocity axis means $\perp$ to time axis.

This if the graph is parallel to velocity axis then we can say that at a particular instance of time, the velocity increase from some initial value to final value. This means that the velocity change occurs at same time and hence slope of the graph will be infinity. Thus, acceleration of the body is infinity, which is not possible. Whereas it is possible that the graph can be parallel to time axis; that means body is moving with a uniform velocity.



c) We know that

$$F = mf$$

For first case,

$$F_1 = 12 \text{ N}$$

$$f_1 = 3 \text{ m/s}^2$$

$$\therefore m_1 = \frac{F_1}{f_1} = \frac{12}{3} = 4 \text{ kg}$$

For second case,

$$F_2 = 12 \text{ N}$$

$$f_2 = 4 \text{ m/s}^2$$

$$\therefore m_2 = \frac{F_2}{f_2} = \frac{12}{4} = 3 \text{ kg}$$

If we combine these two object then total mass become 7 kg and the force is same.

$$\therefore F = 12 \text{ N}$$

$$m = 7 \text{ kg}$$

$$\therefore f = \frac{12}{7} = 1.7 \text{ m/s}^2$$

14. (a) Write down the principle of conservation of linear momentum.

(b) A bullet moving with a speed 200 ms⁻¹ can penetrate 4 cm thick wooden board. What will be suitable speed of the bullet to penetrate 16 cm thick same wooden block?

(c) A stone is dropped from the top of the tower 50m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25 ms⁻¹. At what distance from the top and after how much time the stones cross each other? [Model Question]

Answer:

a) The principle of conservation of momentum states that the total momentum of a system of colliding objects remains constant provided no resultant external force acts on the system.

b) Initial speed of bullet, $v = 200$ m/s.

$$S = 4\text{cm} = 0.04\text{m}$$

$$V = 0$$

We know that,

$$v^2 = u^2 - 2fs$$

$$0 = 200^2 = 2 \times f \times 0.04$$

$$\therefore f = \frac{200 \times 200}{2 \times 0.04} = 500000 \text{ m/s}^2$$

Second case

$$S = 16\text{cm} = 0.16\text{m}$$

$$v = 0$$

$$f = 500000 \text{ m/s}^2$$

$$\therefore v^2 = u^2 - 2fs$$

$$0 = u^2 - 2 \times 500000 \times 0.16$$

$$u = \sqrt{2 \times 500000 \times 0.16} = \sqrt{2 \times 50000 \times 16} = \sqrt{10000 \times 16} = 400$$

$\therefore$ Initial speed of bullet is 400 m/s to penetrate the 16 cm wooden block.

c) Let, after time t they will cross each other at h from the top.

$$\text{First case, } h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8t^2 \quad \dots (i)$$

$$\text{Second case, } 50 - h = 25t - \frac{1}{2} \times 9.8 \times t^2 \quad \dots (ii)$$

By equating (i) & (ii)

$$50 - \frac{1}{2} \times 9.8 \times t^2 = 25t - \frac{1}{2} \times 9.8t^2$$

$$t = 2\text{s}$$

$$\text{From (i), } h = \frac{1}{2} \times 9.8 \times 2^2 = 19.6$$

$\therefore$ After 2s they will meet 19.6m from the top.

15. (a) A force acts on a rest body of mass 2 kg for 5 seconds and then force ceases to act and the body moves 80 m during 4 seconds. Find how much force was applied on the body?

(b) Explain – (i) why gun recoils at the time of firing? (ii) Why bird cannot fly in vacuum?

[Model Question]

Answer:

$$a) S = vt + \frac{1}{2}at^2$$

$$\text{Here, } S = 80\text{m}$$

$$v = 0$$

$$t = 4\text{sec}$$

$$\therefore 80 = 0 + \frac{1}{2}a \cdot 16$$

$$a = 10\text{m/s}^2$$

$$\therefore \text{Force } F = ma \quad [\because m = 2\text{kg}]$$

$$= 2 \times 10 = 20\text{N}$$

b) i) When we fire a gun obviously an explosion takes place inside of the bullets casing. The result of that explosion is the actual bullet itself being propelled down the barrel and out towards whatever direction the gun happened to be pointed in when it fired.

When we feel the gun's kick more appropriately called the RECOIL affect, all that is the effect of the shell kicking backwards because of the explosion. So yes it's mainly just the reaction of the now empty shell kicking back or recoiling rearward. That's basically all it really is. But by the time that you are able to feel the recoil by that time the bullet is already long gone.

ii) Birds fly by displacing air, the same way you swim by displacing water. The two main forces acting during flight are drag and lift. Lift is the responsible for making birds not fall down to the ground. Drag is a force opposed to the bird's motion.

16. a) A particle is moving along a straight path with uniform acceleration a . Its initial velocity is u and final velocity after time t is v . Write down the relation between u , v , a and t and draw the v - t curve.

b) A constant force of 6.0 N acts on a body for 4.0 seconds. Draw the force vs. time graph and calculate the change in momentum of the body from the graph.

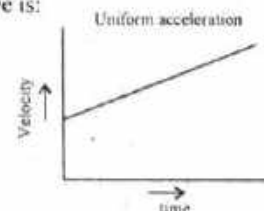
[Model Question]

Answer:

a) After t time the velocity will be expressed as:

$$v = u + at$$

The v - t curve is:

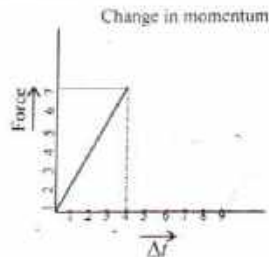


1) If we combine Newton's second law, with the definition of acceleration, $a = \frac{\Delta v}{\Delta t}$.

$$\text{This gives } F = \frac{\Delta(mv)}{\Delta t}$$

$$\text{So, } \Delta(mv) = F \cdot \Delta t = 6.0 \times 4.0 = 24.0 \text{ N seconds}$$

Thus the change in momentum is 24 N seconds.



17. (a) Establish the relation between linear speed and angular speed of a particular executing circular motion.

(b) A body is released from the top of a tower. It reaches the ground after 3 seconds. Calculate the height of the tower and the velocity of the body just before touching the ground. Also calculate the distance travelled by the body in the last second of its journey. ($g = 10.0 \text{ m s}^{-2}$). [Model Question]

Answer:

a) Linear motion are classified by s (displacement), v (velocity) and a (acceleration). Similarly for angular motion we have θ (theta), ω or omega (angular velocity) and α (angular acceleration) and the most important parameters specifies the angular movement is Radius (r).

So according to the question - Every angular motion attributes is accompanied with radius vector (r) when their linear parameters are calculated.

Hence, displacement and radius are related as $-s = (r) \times (\theta)$ angular velocity and linear velocity are related as $-v = (r) \times (\omega)$

angular acceleration and linear acceleration are related as $-\alpha = (r) \times (\alpha)$

where the product is vector product and all the parameters defined are vectors.

Changing the parameters positions for product will result into change in sign or change in direction of the resultant.

b) 1st part:

$$v^2 = u^2 - 2gh$$

$$u = \text{Initial velocity} = 0$$

$$h = (v^2 - u^2)/2g = (29.4)^2/(2 \times 9.8) = 149.028 \text{ m}$$

2nd part:

Here,

v = final velocity

u = Initial velocity

s = displacement

So, in equation $v^2 - u^2 = 2as$, s is displacement.

$$v^2 = (2) \times 9.8 \times 44.1 - 0$$

$$v = 29.4 \text{ m/s}$$

3rd part:

In this case we will use second equation of motion $S = ut + (1/2)at^2$

where, s = distance travelled (height)

u = initial velocity (here $u = 0$)

a = acceleration due to gravity (9.8 ms^{-2})

t = time taken

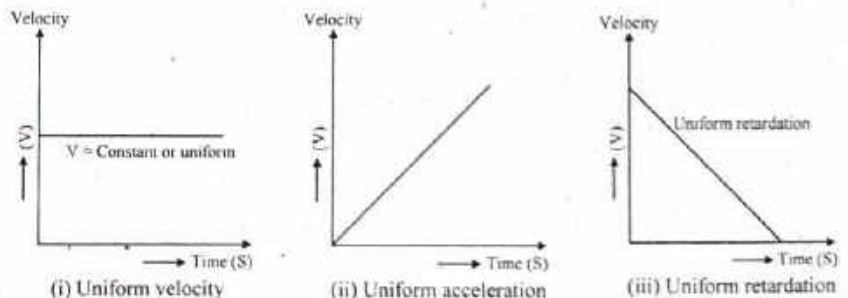
$$s = 0 \times 3 + (1/2) \times 9.8 \times 3 \times 3 = 44.1 \text{ m}$$

Equations of motion are valid only when acceleration is constant and motion is constrained to a straight line.

18. Draw velocity-time diagram for particle travels with [Model Question]

(i) uniform velocity, (ii) uniform acceleration, (iii) uniform retardation.

Answer:



19. (a) A man of mass 45 kg stands on a lift. When the lift - (i) is moving up with acceleration 4.9 m/s^2 , (ii) when the lift is descending with acceleration 4.9 m/s^2 , find out the reactionary force on the man in each case. [Model Question]

Answer:

(i) When lift moving up with acceleration ' a ', apparent weight will be increased.

$$R = m(g + a) = 45(9.8 + 4.9) = 45 \times 14.7 = \frac{661.5}{9.8} \text{ kg wt}$$

Apparent weight of the man = 67.5 kg wt.

(ii) When the lift is descending with acceleration 'a'

$$R = m(g - a) = 45(9.8 - 4.9) \text{ N} = 45 \times 4.9 = \frac{220.5}{9.8} = 22.5 \text{ kg wt.}$$

19. (b) A stone is freely dropped into a well. Its sound of impact at the bottom of the well is heard 6 seconds after it is dropped. If the velocity of sound is 320 m/s, what is depth of the well. [Model Question]

Answer:

Suppose the depth of the well = h metre.

Consider the time which the stone dropped into the bottom of the well = t_1 sec. and the sound take time = t_2 sec. Therefore it can be said, $t_1 + t_2 = 6 \dots$ (i) [u = initial velocity]

We know,

$$h = ut + \frac{1}{2}gt^2$$

$$\text{or, } h = \frac{1}{2}gt_1^2$$

$$\text{or, } t_1 = \sqrt{\frac{2h}{g}} \dots \text{(ii)}$$

The velocity of sound in air = 320 m/s.

$$\therefore h = 320 \times t_2$$

$$\text{or, } t_2 = \frac{h}{320} \dots \text{(iii)}$$

From eq. (i) we get, $t_1 + t_2 = 6$

$$\text{or, } \sqrt{\frac{2h}{g}} + \frac{h}{320} = 6 \text{ or, } 0.45 \times \sqrt{h} + \frac{h}{320} = 6$$

$$\text{or, } 144.56\sqrt{h} + h = 1920$$

$$\text{or, } \sqrt{h} = \frac{-144.56 \pm \sqrt{(144.56)^2 - 4 \times 1 \times (-1920)}}{2 \times 1} = \frac{-144.56 \pm 169}{2}$$

$$\therefore \sqrt{h} = \frac{169 - 144.56}{2} = 12.24 \text{ [(-) value can be neglected]}$$

$$\therefore h = 149.94 = 150 \text{ metre}$$

$\therefore$ The depth of the well is 150 metre.

20. Prove that: $s = ut + \frac{1}{2}ft^2$

[Model Question]

Answer:

We know acceleration is rate of change of velocity.

So, $f = \frac{dv}{dt}$, where f is acceleration, v is velocity.

$v = \frac{ds}{dt}$, where s is displacement.

$$\text{So } f = \frac{d^2s}{dt^2}$$

Integrating w.r.t. t we get -

$$\frac{ds}{dt} = ft + c, \text{ where } c \text{ is a constant}$$

$$\text{at } t = 0, \frac{ds}{dt} \text{ as velocity} = u \text{ (initial velocity)}$$

$$\text{So, } c = u \therefore \frac{ds}{dt} = ft + u$$

Again integrating we get - $s = ft^2 + ut + c_1$

At $t = 0, s = 0$

$$\text{So, } c_1 = 0 \therefore s = ut + \frac{1}{2}ft^2 \text{ (Proved)}$$

Question Paper 2017 (December)

1. A. Fill in the blanks (any ten):

- i) Equilibrant force is equal and opposite to the resultant of several forces acting on the body.
- ii) A resultant force is a single force which can replace two or more forces and produce the same effect as the force.
- iii) The process of finding out the resultant force of two or more given forces is called composition of forces.
- iv) The turning or rotational tendency of a force is called torque.
- v) A couple has no resultant force but has resultant moment.
- vi) Principle of moments is based on Varignon's theorem.
- vii) Dynamic or kinetic friction is always less than the static friction.
- viii) Frictional force is directly proportional to normal reaction.
- ix) Velocity ratio of first system of pulley is 16.
- x) The C.G. of a solid circular cone divides the height in the ratio 1:4.
- xi) The maximum mechanical advantage of a lifting machine is 1/m.
- xii) The diagram which shows all forces including reactions at the contact surface after removing supports is known as free body diagram.

B. State True or False (any ten):

- i) If the arm of a couple is doubled, its moment will be same. [False]
- ii) Moment of a force is a scalar quantity. [False]
- iii) The components of a force along two mutually perpendicular lines are called rectangular components. [True]
- iv) The resultant of two like parallel forces acts at a point between their points of application. [True]
- v) The distance of C.G. from its base of a right circular cone of height ' h ' is $h/4$. [True]
- vi) Equivalent couple is one where moment and direction are different. [False]
- vii) At the point of reversing the efficiency of a simple lifting machine is 60%. [True]
- viii) Velocity ratio of a simple machine is constant. [True]
- ix) In actual machine $MA > VR$. [False]
- x) Load vs Effort graph is a curve. [True]
- xi) A machine is said to be self-locking when its efficiency is less than 50%. [True]
- xii) Semi-central angle in cone of friction is equal to angle of friction. [False]

Group-A

2. a) Give geometrical representation of moment of a force.
 - b) Two men carry a concrete block weighing 400 N using a light wooden plank on their shoulders. The stronger man carries 225 N. How should the block be supported so that the stronger man gets his share if the length of the plank between the shoulders is 3.5 meter?
- a) See Unit: Moments and Couples, Question No. 10 of Long Answer Type Questions.
b) See Unit: Moments and Couples, Question No. 3 of Long Answer Type Questions.

Question Papers

Me.137

3. a) Write short notes on equivalent couple.
 - b) A horizontal bridge AB of 18 meter long weighing 18 kN and rest on two supports at its end. What will be the pressure on each support when a car of weight 5 kN starting from A is two third of the way across the bridge.
- a) See Unit: Condition of Equilibrium, Question No. 19 of Short Answer Type Questions.
b) See Unit: Moments and Couples, Question No. 2 of Long Answer Type Questions.

4. a) State, explain and prove Lami's theorem.

- b) A string AC of length 24 cm is attached to a point on a smooth vertical wall and to a point on the surface of a sphere of radius CO is 12 cm. The sphere whose weight is 100 kN hangs in equilibrium against the wall. Find the tension in the string and the reaction of the wall. (Fig. 1)

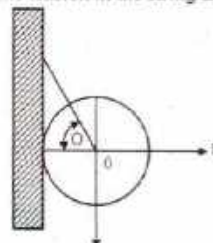
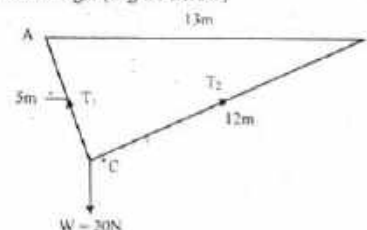


Fig: 1

See Unit: Condition of Equilibrium, Question No. 15 of Long Answer Type Questions.

5. a) State and explain triangular law of equilibrium.

- b) A body of weight 20 N is suspended by two strings 5 meter and 12 meter long and other ends being fastened to the extremities of a rod of length 13 meter. If the rod is to remain horizontal, determine the tensions in the strings. (Figure below)



See Unit: Condition of Equilibrium, Question No. 16 of Long Answer Type Questions.

Group-B

6. a) Define limiting friction and normal reaction.
 - b) A body resting on a horizontal plane required a pull of 80 N inclined to 30° to horizontal just to move it. It was also found that a push of 100 N inclined at 30° to the horizontal just moved the body. Find the weight of the body and co-efficient of friction.
- See Unit: Friction, Question No. 19 of Long Answer Type Questions.

7. a) Prove that angle of repose is equal to the angle of friction.

b) A load W acts on an inclined plane of inclination θ to the horizontal. If the co-efficient of friction is μ , show that the horizontal force necessary to move the load up the plane is

$$\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \times W$$

See Unit: Friction, Question No. 20 of Long Answer Type Questions.

8. a) Define the terms 'Coefficient of friction' and 'Angle of friction'.

b) A uniform ladder of weight 1000 N, inclined 45° to the horizontal, rests with upper extremity against a rough vertical wall and its lower extremity on the ground. Find the least horizontal force which will move the lower end towards the wall. Given 0.25 and 0.20 are the co-efficient of friction at the lower and upper ends respectively.

a) See Unit: Friction, Question No. 21 of Long Answer Type Questions.

b) See Unit: Friction, Question No. 8 of Long Answer Type Questions.

9. a) Find out the centroid of the uniform triangular lamina by method of integration.

b) Find out the centroid of an I-section has bottom flange $20 \text{ cm} \times 2 \text{ cm}$, top flange $10 \text{ cm} \times 2 \text{ cm}$ and web $15 \text{ cm} \times 2 \text{ cm}$.

a) See Unit: Centroid and Centre of Gravity, Question No. 2 of Long Answer Type Questions.

b) See Unit: Centroid and Centre of Gravity, Question No. 14 of Long Answer Type Questions.

Group-C

10. a) Define mechanical advantage, velocity ratio, efficiency and effort of a simple machine.

b) For a differential wheel and axle, the diameter of the wheel is 240 mm. The larger and the smaller diameter of the differential axle are 80 mm and 70 mm respectively. An effort of 320 N is applied to lift a load of 8000 N. Determine – (i) V.R. (ii) M.A. (iii) Efficiency of the machine, and (iv) Effort lost in friction.

See Unit: Simple Lifting Machines, Question No. 20 of Long Answer Type Questions.

11. a) Derive the conditions maximum efficiency of a lifting machine.

b) In a lifting machine, the effort required to lift a load of 250 N and 375 N are 60 N and 70 N respectively. If the velocity ratio of the machine is 20. Determine – (i) law of machine, (ii) efficiencies corresponding to load of 250 N and 375 N, (iii) effort lost in friction in both cases.

See Unit: Simple Lifting Machines, Question No. 21 of Long Answer Type Questions.

12. a) A simple screw jack has a thread of pitch of 10 mm, find the mechanical advantage and efficiency, if an effort of 20 N is applied at the end of an arm 500 mm long to lift a load 2.6 kN.

b) The number of teeth on the worm wheel of a double threaded worm and worm wheel is 25. The effort handle is 25 cm long and load drum is of 15 cm diameter. Find the efficiency of the machine, if an effort of 300 N can lift load of 3.25 kN.

See Unit: Simple Lifting Machines, Question No. 22 of Long Answer Type Questions.

Question Paper 2018 (December)

1. A. Fill in the blanks (any ten):

i) If a number of forces are acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces. This is known as Parallelogram law.

ii) If the two forces P and Q are acting at an angle θ and their resultant acting at an angle α with P . Then $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$.

iii) The effect of a couple acting on a rigid body is to rotate it without moving it as a whole.

iv) If a body is in equilibrium. We may conclude that the resultant of all the forces acting on it is zero and the moment of the forces about any point is zero.

v) The Lami's Theorem is applicable only for three coplanar and concurrent and non-collinear forces.

vi) The centre of gravity of a right circular cone of diameter (d) and height (h) lies at a distance of $\frac{h}{4}$ from the base measured along the vertical radius.

vii) C.G. is the point through which the whole weight of a body may be assumed to act.

viii) The efficiency of a screw jack may be increased by increasing its pitch.

ix) The kinetic friction is slightly less than the limiting friction.

x) A simple machine has only one point for the application of effort and load.

xi) Instead of simple wheel and axle, differential wheel and axle gives $\frac{2D}{d_1 - d_2}$ V.R.

xii) The amount of effort required to overcome the machine friction is given by actual effort-ideal effort.

B. State whether the following statements are True or False (any ten):

i) The efficiency of a screw jack is maximum when the helix angle is equal to $\left(45^\circ - \frac{\theta}{2}\right)$. [True]

ii) The force of friction between two bodies in contact is always normal to the surface of their contact. [False]

iii) Centroid of a triangle lies at $\frac{2h}{3}$ from the base, where h is the height of a triangle. [False]

iv) A body is said to be in equilibrium, if it has no linear motion. [False]

v) A couple can be balanced by a force equal to its magnitude. [False]

vi) Angle of repose is equal to angle of friction. [True]

vii) A body isolated from all other supports which are in contact to it, is called a Free body. [True]

viii) Turning the cap of a pen is not an example of a couple. [False]

ix) In case of a lifting machine load vs. Effort graph is a curve. [False]

x) If the law of lifting machine is $P = 0.04W + 20$, the maximum mechanical advantage

Ans. 25

- xi) Mechanical advantage is the ability of a machine to multiply force. [True]
 xii) The velocity ratio of a single purchase crab which can be increased by increasing number of teeth on the pinion. [False]

Group-A

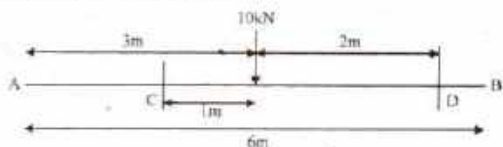
2. a) Two concurrent forces M & N are acting at an angle α , derive the expression of their resultant force.
 b) The resultant of two forces M & N is R . If N is doubled, R is doubled and if M is reversed again doubled. Show that $M : N : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$

See Unit: Basics of Mechanics & Force System, Question No. 15(a) & (b) of Long Answer Type Questions.

3. a) State and prove Varignon's theorem.

See Unit: Moments and Couples, Question No. 1 of Long Answer Type Questions.

- b) Replace the 10kN force by two downward force acting at C and D respectively, which will disturb the overall external effect on the beam.

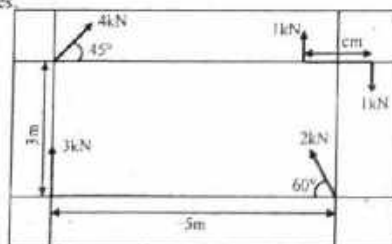


See Unit: Basics of Mechanics & Force System, Question No. 16 of Long Answer Type Questions.

4. a) Define the properties of a couple.

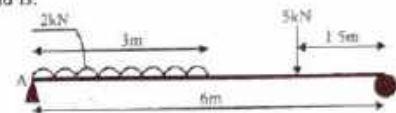
See Unit: Moments and Couples, Question No. 11 of Long Answer Type Questions.

- b) Find the magnitude of resultant force and also the angle through which the resultant acts for following system of forces.



See Unit: Basics of Mechanics & Force System, Question No. 17 of Long Answer Type Questions.

5. a) Describe briefly the types of loading and types of support of the beam.
 b) A simply supported beam AB of 6m span is subjected to loading as shown in Fig. Find the support Reaction at A and B.



See Unit: Basics of Mechanics & Force System, Question No. 18(a) & (b) of Long Answer Type Questions.

Group-B

6. a) Write short note on: Laws of static friction.

See Unit: Friction, Question No. 5 of Long Answer Type Questions.

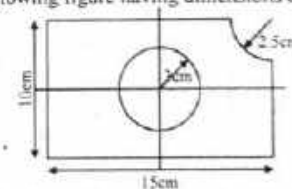
- b) A metal block of weight 200 N is resting on a horizontal surface. The co-efficient of friction between the surfaces being 0.3. A pull is exerted on the block by means of a string inclined at 30° to the horizontal which is sufficient to cause motion of the block. Calculate the magnitude of force exerted by the string.

See Unit: Friction, Question No. 22 of Long Answer type Questions.

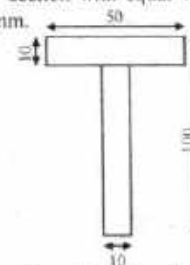
7. a) What is Centroid and Centre of Gravity?

See Unit: Centroid and Centre of Gravity, Question No. 15(a) of Long Answer Type Questions.

- b) Find the Centroid for the following figure having dimensions as shown.



Find the centre of Gravity of a 'T' section with equal width throughout having dimensions as shown in fig. All dimensions are in mm.



See Unit: Centroid and Centre of Gravity, Question No. 15(b) of Long Answer Type Questions.

Group-C

8. a) What is the law of a machine. From equation of law of machine find the maximum Mechanical advantage and maximum efficiency of a simple lifting machine.
 b) In a differential wheel and axle the larger axle is 200 mm in diameter and the smaller one is 150 mm in diameter. The effort handle is 400 mm long. Find the Velocity Ratio. If an effort of 150 N lifts a load of 1500 N, find the efficiency.

See Unit: Simple Lifting Machines, Question No. 23(a) & (b) of Long Answer Type Questions.

9. a) Deduce the expression for the condition of self-locking of a lifting machine.
 b) In a Weston's differential pulley clock, the difference in the number of teeth of the two pulleys is 4. If the efficiency of the machine is 50% and an effort of 100 N just lifts a load of 2 kN, find the number of teeth of the two pulleys.

See Unit: Simple Lifting Machines, Question No. 24(a) & (b) of Long Answer Type Questions.

Question Paper 2022 (January)

1. A. Fill in the blanks (any ten):

- i) Coefficient of friction depends upon nature of the surfaces
- ii) Example of vector quality is velocity.
- iii) The product of either force of couple with the arm of the couple is called moment of the couple.
- iv) A heavy string attached at two ends at same horizontal level and when central dip is very small approaches the following curve Parabola.
- v) The centre of gravity a T-section 100mm×150mm×50mm from its bottom is 87.5.
- vi) The mechanical advantage of a lifting machine is the ratio of load lifted to the effort applied.
- vii) From a circular plate of diameter 6cm is cut out a circle whose diameter is a radius of the plate. Find the C.G. of the remainder from the centre of circular plate 0.5 cm.
- viii) In ideal machines, mechanical advantage is numerically equal velocity ratio.
- ix) Frictional force encountered after commencement of motion is called dynamic friction.
- x) The angular velocity (in rad/s) of a body rotating at N revolutions per minute is $\frac{\pi N}{30}$ NRAD/S.
- xi) A machine having an efficiency greater than 50%, is known as reversible machine.
- xii) The algebraic sum of moments of the forces forming couple about any point in their plane is equal to the moment of the couple.
- xiii) The skidding away of the vehicle on a level circular path can be avoided if the force of friction between the wheels and ground is less than the centrifugal force.
- xiv) The slope on the road surface generally provided on the curves is known as angle of banking.
- xv) On a ladder resting on smooth ground and leaning against vertical wall, the force of friction will be upward at its upper end.
- xvi) The velocity of a body on reaching the ground from a height h, is $\sqrt{2Hg}$.
- xvii) The ratio of static friction to dynamic friction is always greater than one.

xviii) A flywheel on a motor goes from rest to 1000 rpm in 6 sec. The number of revolutions made is nearly equal to 50.

xix) The angle which an inclined plane makes with the horizontal when a body placed on it is about to move down is known as angle of repose.

Group-A

2. a) Define Scalar & Vector quantity.
 b) The following forces are acting at a point. (i) 20N inclined at 30° towards north of east. (ii) 25N towards north. (iii) 30N towards north of west and (iv) 35N inclined at 40° towards south of west. Find the magnitude and resultant of the force system.
 a) See Unit: Basics of Mechanics & Force System, Question No. 43 of Short Answer Type Questions.
 b) See Unit: Basics of Mechanics & Force System, Question No. 2 of Long Answer Type Questions.

3. a) The resultant of a force P of 40N and an unknown force Q, is known to be perpendicular to P. If P is replaced by a force of 70N in the same line of action, the resultant would be inclined at 45° to the force of 70N. Find the magnitude and direction of unknown force Q.

b) State and show by a sketch Varignon's Theorem.

- a) See Unit: Moments and Couples, Question No. 12 of Long Answer Type Questions.
- b) See Unit: Moments and Couples, Question No. 1 of Long Answer Type Questions.

4. a) State the conditions of equilibrium.

b) Forces each of 4N in magnitude act along AB and CD of a square ABCD and forces each of 6N in magnitude act along the sides AD and CB respectively in the same square. Find the moment of the couple which will produce equilibrium. Length of each sides of the square is 80mm.

See Unit: Condition of Equilibrium, Question No. 17 of Long Answer Type Questions.

5. a) The resultant of two forces is 10N and it is inclined at 60° to one of the forces whose magnitude is 5N. Find the magnitude and direction of the other force.

b) State Triangle Law and Parallelogram Law of forces.

See Unit: Basics of Mechanics & Force System, Question No. 19 of Long Answer Type Questions.

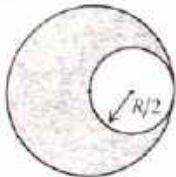
Group-B

6. a) State Lami's Theorem.

b) ABCDEF is a rectangular hexagon, forces of 3, P, Q, 5 and 4N are acting along AB, CA, AD, AE and FA respectively and are in equilibrium. Determine the value of P & Q

- a) See Unit: Condition of Equilibrium, Question No. 1 of Long Answer Type Questions.
- b) See Unit: Condition of Equilibrium, Question No. 18 of Long Answer Type Questions.

7.



a) Determine the CG of the section above.

b) Define CG of any section.

a) See Unit: Centroid and Centre of Gravity, Question No. 16 of Long Answer Type Questions.

b) See Unit: Centroid and Centre of Gravity, Question No. 34 of Short Answer Type Questions.

Group-C

8. a) What is Law of machine?

b) To lift a load of 1500N, an effort of 200N is applied to a machine. The load is lifted by a distance of 0.2m while the effort is moved by a distance of 2m. Find the VR, MA and efficiency of the machine. Find also the ideal effort, effort lost due to friction, ideal load and frictional load.

a) See Unit: Simple Lifting Machines, Question No. 4 of Long Answer Type Questions.

b) See Unit: Simple Lifting Machines, Question No. 25 of Long Answer Type Questions.

9. a) In a double purchase crab winch, no of teeth on pinions are 20 & 30 and that of spur gears are 60 & 80. Radius of the load drum is 200 mm and length of the handle is 300mm. Find the efficiency of the machine, if the effort 100N is required to lift a load of 750N.

b) What are the advantages of a Lifting Machine?

See Unit: Simple Lifting Machines, Question No. 26 of Long Answer Type Questions.

10. a) A body is moving with uniform acceleration and it covers 20m in 4th second and 30m in 8th second. Find (i) the initial velocity, (ii) the acceleration of the body.

b) Define Velocity and Acceleration.

See Unit: Motion in a Plane, Question No. 1 of Long Answer Type Questions.

11. A string is wound round a pulley of 0.30m diameter. A weight attached to the free end of the string, describes a distance of 8m after starting from rest within 4 secs. Find the angular velocity of the pulley. Also find the total distance travelled by the weight to make the pulley rotate at 400rpm.

See Unit: Simple Lifting Machines, Question No. 27 of Long Answer Type Questions.